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CONTENTS OF VOL. VII., SECOND SERIES.

JANUARY, 1916.

Reflection of Slow Moving Electrons by Copper. ALBERT W. HULL 1

Einstein's Photoelectric Equation and Contact Electromotive Force. R. A. MILLIKAN. 18

The Absorption Spectra of the Blue Solutions of Sodium and Magnesium in Liquid Ammonia. G. E. GIBSON and W. L. ARGO 33

The Free Ions in the Atmosphere of the Tropics. J. R. WRIGHT and O. F. SMITH 49

On the Construction of Sensitive Photoelectric Cells. JAKOB KUNZ and JOEL STEBBINS 62

Reflection from, and Transmission through Rough Surfaces. A. F. GORTON 66

The Distance between two Parallel Transparent Plates. C. BARUS 79

Comparison of the Fall of a Droplet in a Liquid and in a Gas. O. W. SILVEY 87

The Fall of Mercury Droplets in a Viscous Medium. O. W. SILVEY 106

A Method for the Measurement of Small Capacities. WILL C. BAKER 112

The Effect of Magnetization on the Opacity of Iron to Rontgen Rays. A. H. FORMAN 119

An Investigation of the Transmission, Reflection and Absorption of Sound by Different Materials. F. R. WATSON 125

The Radioactive Deposit from the Atmosphere on an Uncharged Wire. S. J. M. ALLEN 133

Proceedings of the American Physical Society 139

Minutes of the Eightieth Meeting; Negative Resistance, *A. W. Hull;* Planck's Radiation Formula Deduced from Hypotheses Suggested by X-Ray Phenomena, *William Duane;* The Value of "h" Determined Photoelectrically from the Ordinary Metals, *W. H. Kadesch and A. E. Hennings;* The Relations of the Photo Potentials Assumed by Different Metals when Stimulated by Light of a Given Frequency, *A. E. Hennings and W. H. Kadesch;* Demonstration of a Model Visibly Representing Gas Molecules, *E. F. Northrup;* The Theory of the Flicker Photometer. II. Unsymmetrical Conditions, *Herbert E. Ives and E. F. Kingsbury;* The Velocity of Polymorphic Changes under Pressure, *P. W. Bridgman;* Heat Conductivity of Tungsten at High Temperatures and the Wiedemann-Franz-Lorenz Relation, *Irving Langmuir;* Radiation from Tungsten Filaments and the Mechanical Equivalent of Light, *Irving Langmuir;* Equipartition of Energy and Radiation Theory, *W. F. G. Swann;* The Maximum Frequency of X-Rays at Constant Voltages between 30,000 and 100,000, *Albert W. Hull.*

New Books 159

FEBRUARY, 1916.

A Spectral Luminosity Curve Equation and Its Use. E. F. KINGSBURY 161

The Hall Effect and Allied Phenomena in Tellurium. PETER I. WOLD 169

The Effect of Temperature upon the Coefficient of Absorption of Certain Glasses of Known Composition. K. S. GIBSON 194

The X-Ray Spectrum of Tungsten. W. S. GORTON 203

The Temperature Coefficient of Contact Potential. K. T. COMPTON 209

On the Effect of General Mechanical Stress on the Temperature of Transition of Two Phases, with a Discussion of Plasticity. P. W. BRIDGMAN 215

A New Relation Derived from Planck's Law. PAUL D. FOOTE 224

A Study of Ripple Wave Motion. F. R. WATSON and W. A. SHEWHART 226

The Distribution of Energy in the Normal Radiation Spectrum. LEIGH PAGE........ 229
Cathode Measurements with Glow Current in Hydrogen. W. L. CHENEY............ 241
Cathode Measurements with Glow Current in Oxygen and Nitrogen. W. NEUSWANGER 253
Tests of Thermo-electric Formulæ Based on Bismuth and Bismuth-Tin Alloys. A. E. CASWELL........ 268
Proceedings of the American Physical Society........ 279
Minutes of the Eighty-first Meeting; On A Supposed Allotropy of Copper, *G. K. Burgess and I. N. Kellberg;* Sensitive Photoelectric Cells and a Photoelectric Relay, *Jakob Kunz and J. Stebbins;* The Distribution of Electrons in a Substance that can Exist in Both the Conducting and the Non-Conducting States, *L. E. Dodd;* Talbot's Bands and the Resolving Power of Spectroscopes, *Thomas E. Doubt;* The Measurement of Optical Rotation in the Infra-red Spectrum, *L. R. Ingersoll;* The Cause of the Variation of the Emanation Content of Spring Water, *R. R. Ramsey;* A Theory of the Initial Conditions of the Corona, *Jakob Kunz;* On the Optical Properties of an Isolated Crystal of Selenium, *Charles H. Skinner;* Atomic Models in Whose Behavior a Magnetic Field is an Important Factor, *Ernest Merritt;* The Reflection of Sound, *F. R. Watson;* The Exceptions to the Law of Dulong and Petit, *J. E. Siebel.*

MARCH, 1916.

The Light Sensitiveness of Copper Oxide. A. H. PFUND........ 289
The Characteristics of Tungsten Filaments as Functions of Temperature. IRVING LANGMUIR........ 302
On Ewell's Method of Measuring Single Potential Differences. S. J. BARNETT....... 321
A Mice X-Ray Spectrometer. W. S. GORTON........ 334
A Physical Study of the Thermal Conductivity of Solids. ARTHUR H. COMPTON...... 341
On the Location of the Thermal Energy of Solids. ARTHUR H. COMPTON............ 349
A Direct Photoelectric Determination of Planck's "h". R. A. MILLIKAN............ 355
The Constants of Radioactivity. GERALD L. WENDT........ 389
The Fluorescing Sodium Uranyl Phosphate. H. L. HOWES and D. T. WILBER........ 394
Proceedings of the American Physical Society........ 397
An Attempt to Detect a Change in the Specific Heat of Selenium with a Change in the Illumination, and also with the Application of an Electric Field, *L. P. Sieg;* Measurements of Electric Currents by their Heating Effect, *S. Leroy Brown;* On the Properties of Matter at Low Temperatures, *Jakob Kunz;* A New Law Relating Ionization Pressure and Current in the Corona of Constant Potentials, *Earle H. Warner;* Unipolar Induction and Absolute Rotation, *E. H. Kennard;* An Investigation of the Acoustical Properties of the Armory at the University of Illinois, *F. R. Watson;* The Tribo-Luminescence of Manganese-Zinc Compounds, *C. W. Waggoner;* The Emission Quanta of Characteristic X-Rays, *David L. Webster;* The Reflecting Power of Alkali Metals in Contact with Glass, as Determined by the Photo-electric Cell, *J. B. Nathanson;* A Powerful Source of Constant High Potential, *Albert W. Hull;* On the So-called Magnetic Rays of Righi, *James E. Ives;* Arcs in Gases between Non-Vaporizing Electrodes, *G. M. J. McKay and C. V. Ferguson;* On the Velocity of Sound and the Value of the Ratio of the Specific Heats for Hydrogen, *Karl K. Darrow;* Some Devices used in Evaluating Certain Trigonometric and Exponential Functions Occurring in Photometry, *Irwin G. Priest;* The Relation between the Pressure Effect and the Light Effect in Selenium Crystals, *E. O. Dieterich.*
New Books........ 416

APRIL, 1916.

A Study of Apparent Specific Volume in Solution. L. D. WELD........ 421
The Heat Losses from Incandescent Filaments in Air. L. W. HARTMAN............ 431

Certain Cases of the Variation of Sound Intensity with Distance. G. W. Stewart 442
Distribution of Current between a Gas Column and a Metallic Shunt. R. W. Earhart 447
Selective Radiation from Osmium Filaments. E. F. Barker 451
The Optical Activity of Liquids and Gases. Frank Gray 472
The Theory of Ionization by Collision. I. Distribution of the Velocity of the Electrons. K. T. Compton 489
Proceedings of the American Physical Society 497
The True Temperature Scale for Tungsten and Its Emissive Powers at Incandescent Temperatures, *A. G. Worthing;* The X-Ray Spectrum of Tungsten, *Arthur H. Compton;* Note on the End Effect in the Electrostriction of Cylindrical Condensers, *Edwin C. Kemble;* On Possible Planetary Magnetic Effects, *L. A. Bauer.*

MAY, 1916.

The Theory of Ionization by Collision. II. Case of Inelastic Impact. K. T. Compton. 501
The Theory of Ionization by Collision. III. Case of Elastic Impact. K. T. Compton. 509
Some Characteristics of Crystal Detectors. Victor A. Hunt and Laurens E. Whittemore 518
The Magnetic Susceptibility of Gases. W. P. Roop 529
Note on the Damping Factor used in Ballistic Constants of Moving-Coil Galvanometers. Paul E. Klopsteg 543
The Diffraction Ring Pattern in the Shadow of a Circular Object. Mason E. Hufford 545
The Effect of Temperature on the Resistance, the Light-Sensitiveness, and the Rate of Recovery of Certain Crystals of Metallic Selenium. Kathryn Johnstone Dieterich 551
Sub-Helmholtzian Vibrations of a Rubbed String. Harry Clark 561
A Note on the Spectrum of Rontgen Rays. O. J. Zobel 580
Proceedings of the American Physical Society 583
A New Mercury Vapor Pump, *H. B. Williams;* A Resonance Method for Measuring the Phase Difference of Condensers, *H. L. Dodge;* A Synchrono-phosphoroscope, *Edward L. Nichols and H. L. Howes.*

JUNE, 1916.

The Interferences of the Direct Spectra of Two Gratings. Carl Barus 587
Experiments on the Emission Quanta of Characteristic X-Rays. David L. Webster .. 599
Note on the End Effect in the Electrostriction of Cylindrical Condensers. Edwin C. Kemble 614
Retrograde Rays from the Cold Cathode. Orrin H. Smith 625
On the "Current-Deflection" Method for Determining Ballistic Constants of Moving Coil Galvanometers, with a Note on the Non-Uniformity of Magnetic Fields in such Instruments. Paul E. Klopsteg 633
The Correction for Thermoelectric Current to be Applied to the Throw of a Ballistic Moving-Coil Galvanometer. Paul E. Klopsteg 640
A Recording X-Ray Spectrometer, and the High Frequency Spectrum of Tungsten. Arthur H. Compton 646
A New Method for Enlarging Photographs without the Use of a Lens. Alfred J. Lotka 660
The Effect of a Magnetic Field on the Conductivity of Flames. C. W. Heaps 663
Proceedings of the American Physical Society 676
Minutes of the Eighty-second Meeting; Minutes of the Eighty-third Meeting; Convection Currents and Calorimeter Design, *Walter P. White;* Some Effects of Magnetization on the Length of a 35.25 Nickel Steel, *A. W. Gray, D. W. Sweet and L. W. Schad;* American Nickel Steels of Low Thermal Expansivity, *A. W. Gray, D. H. Sweet and L. W. Schad;* The E.M.F. Generated by the Rotation of a Cylindrical Magnet about its Axis, *G. F. Hull and A. B. Meservey;* The Low Potential

Discharge Spectrum of Mercury Vapor in Relation to Ionization Potentials, *John T. Tate;* The Calculation of Planck's Constant C_2, *J. H. Dellinger;* Note on the Action of Calcium in a Discharge Tube, *H. B. C. Allison;* A Chemically Active Modification of Hydrogen Produced by Alpha-Rays, *William Duane and G. L. Wendt;* An alternating Current Method of Measuring Mutual Inductance, *Harvey L. Curtis;* The Coefficient of Total Radiation of a Uniformly Heated Enclosure, *W. W. Coblentz and W. B. Emerson;* The Constants of Radiation of a Black Body, *W. W. Coblentz.*

Errata and Addenda 696

Index 697

Second Series. *January, 1916* *Vol. VII., No. 1.*

THE

PHYSICAL REVIEW.

THE REFLECTION OF SLOW-MOVING ELECTRONS BY COPPER.

By Albert W. Hull.

Introduction.

THE following is a report of experiments on the reflection by a rough copper surface—electrons having velocities from 0.4 volt to 50 volts. The copper was originally polished, but when taken out at the end of the experiment had a dull, smooth *gray* appearance, due probably to a deposit on it of copper distilled from hotter parts of the apparatus during the prolonged heating. This condition of the surface is to be especially borne in mind in interpreting the results.

The most striking result of the experiment is the smallness of the reflection observed—only 10 per cent. at 0.4 volt velocity, increasing to 40 per cent. at 50 volts velocity. Interesting evidence is also obtained regarding the cause of the spreading of a beam of slow-moving electrons, and the energy relations in the production of delta rays.

Previous Experiments.

The best values of reflection at low velocities are those of von Baeyer[1] and Gehrts.[2] Their results may be summarized as follows: At about 2 volts velocity, the fraction of incident electrons reflected varies from 35 to 50 per cent., depending on the condition of the surface, rather than the material, of the reflector. The reflection increases with increase of velocity up to about 5 volts velocity, then decreases with further increase of velocity to 11 volts velocity, after which it increases rapidly and reaches 100 per cent. at about 30 volts velocity. At velocities above 30 volts more electrons leave the receiving plate than strike it. Hence in addition to reflection there must be a secondary emission of electrons by the bombarded plate, and this secondary emission is assumed to begin at 11 volts, the point where rapid increase in the apparent reflection begins.

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[1] Ber. d. D. Phys. Ges., 10, 96–111, and 953–967, 1910.

[2] Ann. d. Phys., 36, 1000, 1911.

The picture would be much simpler if, instead of an initial increase in reflection with increasing velocity, followed first by a decrease and then an increase, a constant decrease from the lowest velocities up to eleven volts had been observed, with subsequent increase. For the process could then be considered as the superposition of two, viz., a reflection, which is large for low velocities and decreases continually with increasing velocity, and a secondary emission which begins at about 11 volts and increases continually with the velocity. This view of the process von Baeyer actually adopts, considering, with good reason, that the smaller values of reflection which he observed at velocities below 8 volts could be explained by the poor vacuum which he had. Gehrts follows von Baeyer's example in drawing the same conclusion and discrediting his own data at low velocities, but with less justification, as his vacuum seems to have been excellent.

Some time ago the writer[1] had occasion, in the course of another research, to measure the reflection from a silver-plated brass surface, of photo-electrons produced by Shumann light, the velocities of which lie between 0 and 3 volts. The result was a surprise, as the reflection was found to be zero within the limits of measurement. Here again, however, it must be remembered that the surface was in a peculiar condition, namely that of an electrolytic deposit. For although the brass was highly polished before being plated, and the silver plating was carried on very slowly, so that a fine mirror surface resulted, yet this surface was purposely not polished, and all electrolytic surfaces, however fine-grained, are undoubtedly granular. Such a surface, being composed of little Faraday cages, may reflect very differently from a polished or burnished surface.

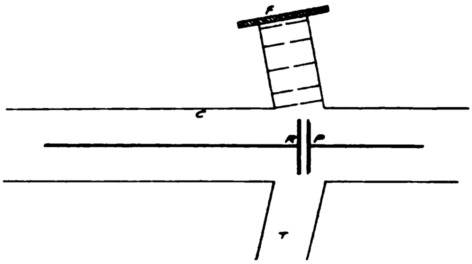

Fig. 1.

The method used in this experiment may be briefly stated here, as it differs from the methods that have been used by other experimenters and is free from the objection to which these methods are open.

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[1] PHYS. REV., 35, 400, 1912.

An aluminum plate (Fig. 1) P, 4 cm. in diameter is supported by amber in a glass tube 5 cm. in diameter and can be illuminated by a carefully diaphragmed narrow beam of ultra-violet light through a fluorite window F, the reflected light passing out freely through a side tube T. Opposite P is the "reflector" R, in this case a silver plated brass disc, which is movable along the tube by means of a magnet. The inside of the tube is lined with copper foil covered by fine copper gauze, both being blackened with lamp black to diminish reflection.

When the reflector R is 4 mm. from P (in which position no light falls on it) all the electrons from P will strike R, since the beam of light is only 2 mm. in diameter and strikes accurately the center of P. Part of these will be reflected back to P, and after multiple reflections, assuming none escape to the cylinder walls, the net loss by P will be $N((1 - r)/(1 - rs))$ where r is the coefficient of reflection of the reflector R, *i. e.*, the fraction of the electrons striking it which it reflects back to P, s that of the plate P, and N the total number of electrons leaving P in the first place. Where R is drawn back to the farther end of the cylinder, 15 cm. from P, only a negligible fraction of the N electrons return to P. The number escaping is N. Hence if s is very small, as has usually been found true for aluminum, the ratio of these currents from P is $1 - r$; if s and r are equal the ratio is $1/(1 + r)$.

Since P, R and the black coating on the cylinder walls were all of different material, it was necessary to compensate their contact difference of potential. This was determined by the Kelvin capacity method, as follows: When R was only 1 or 2 mm. from P the capacity of P was influenced only by R. By small movements of R it was easy to determine within .01 volt the potential it must have in order that the electrometer connected to P should not be disturbed. If R was now moved back several centimeters the electrometer was again disturbed, since P was no longer screened by R, and its capacity was influenced by the blackened walls of the cylinder. Hence by *large* movements of R it was possible to ascertain when the cylinder was at the same potential as P, and this adjustment could be made within about 0.1 volt.

When the potentials of the three surfaces had been thus equalized, it was found that the electron current from P was the same whether R was near or far away, in other words, there was no reflection.

As a check upon the adjustment of the potentials, the whole enclosure was coated with silver by making the silver reflector cathode in a high tension discharge for two hours. A determination of the surface potentials by the method given above showed that they were all equal within 0.1 volt. Measurements on reflection again gave the same result, the current from P was independent of the position of R. In this case, since both surfaces were silver, we have $r = s$, and the ratio of the currents from P when R is near and far away respectively should be $1/(1 + r)$. The value actually found differed from unity by less than 1 per cent.

APPARATUS.

In the construction of apparatus two objects were especially borne in mind: (1) To obtain a beam of electrons as nearly as possible homogeneous with respect to speed and direction. The admixture in the beam of reflected and secondary electrons from the walls or edges of diaphragms was most to be feared, but it was also anticipated that the shape of the electric field would have a marked effect. To avoid the first of these errors, sharp edged diaphragms (D, Fig. 2) in a wide tube were used instead of a narrow tube to limit the beam; to study the second a "focus plate" (F, Fig. 2) was placed between the filament and diaphragms. By charging this plate to different potentials, both positive and negative, the electric field near the diaphragms could be varied over a wide range. A copper ring r, concentric with the face of the filament and connected to its negative terminal, improved the focusing and enabled large currents to be obtained even at low voltages. (2) To avoid all substances capable of yielding condensible vapors or oils into the vacuum. For such oily films not only change the reflecting surface from metal to oil, thereby changing entirely its reflecting power, as Norman Campbell[1] has clearly demonstrated, but when bombarded by electrons these surfaces assume potentials from 10 to 50 volts above the potential of the metal on which they are deposited.[2] For this reason the apparatus was constructed entirely of glass and copper, and after being evacuated

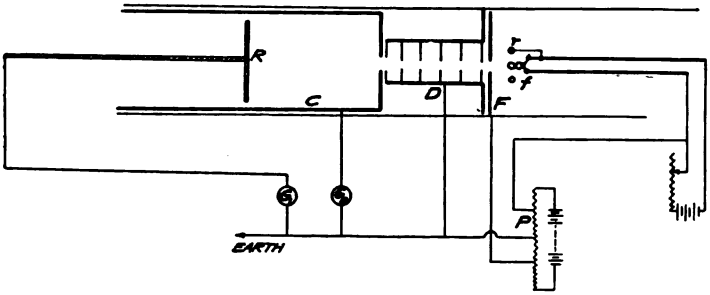

Fig. 2.

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[1] Phil. Mag., 28, 286–302, 1914.

[2] Stuhlmann and Compton, PHYS. REV., 2, 199, 1913; Seeliger, Phys. Z., 14, 1237, 1913.

was surrounded by an electric furnace and heated to 350° C. for forty-eight hours, with the Gaede pump working continually. A tube immersed in solid carbon di-oxide and ether prevented mercury and oil vapors from the pump and McLeod gauge from diffusing into the apparatus.

The apparatus is shown in Fig. 2. Electrons from a filament *f* passed through a 6 mm. hole in a copper "focus plate" *F*, through a series of 4 mm. holes in the copper diaphragms *D*, into a copper cylinder *C*, 5 cm. in diameter and 10 cm. long, which was lined with fine copper gauze to decrease the reflection of electrons from it.[1] Here they fell upon the reflector *R*, a polished[2] copper disc, 4 cm. in diameter, which was movable along the axis of the cylinder by means of a magnet. A potentiometer *P* enabled filament and focus plate to be charged to any desired potentials, and galvanometers G_1 and G_2 measured the currents from reflectors and cylinder to earth. For convenience they were adjusted to the same sensitiveness by means of shunt resistance.

Method of Experiment.

The procedure was to connect the negative end of the filament to a definite negative potential, which is taken as a measure of the velocity of the electrons when they entered the cylinder; charge the focus plate

Table I.

Reflection of 8-Volt Electrons.

Potential of focus plate, + 18 volts. Pressure of gas .0006 mm.

Distance of *R* from End of Cylinder.	i_R Amp. $\times 10^{-7}$.	i_c Amp. $\times 10^{-7}$.	*i* Total Amp. $\times 10^{-7}$.	Scattering $\frac{i_{R,.1}-i_{R,d}}{i_{R,.1}}$.	Coefficient of Reflection. $\frac{i\text{ Total}-i_{R,.1}}{i\text{ Total}}$.
.1	2.00	.25	2.25	0.0	
6.0	1.59	1.01	2.60	.20	.23
.15	2.00	.27	2.27	0.0	
.25	2.00	.34	2.34	0.0	
.50	2.00	.43	2.43	0.0	
.75	1.98	.51	2.49	.01	
1.00	1.95	.60	2.55	.02	
2.00	1.87	.72	2.59	.07	
3.0	1.79	.83	2.62	.10	
4.0	1.71	.91	2.62	.15	
5.0	1.64	1.00	2.64	.18	
6.0	1.58	1.06	2.64	.21	
.1	2.02	.22	2.24		.23

---

[1] Lamp black was avoided on account of contact potential difference.

[2] At the end of the experiment this disc had a dull "gray" appearance, probably due to a deposit of copper distilled from hotter parts of the apparatus during the long heating.

to a carefully determined potential;[1] and measure the currents from cylinder C and reflector R to earth for a series of positions of R, ranging from 1 mm. to 6 cm. from the end of the cylinder. The difference between the total current $i_{C,d} + i_{R,d}$ at any distance, d, greater than 4 cm., and the current $i_{R,1}$ to R at the distance .1 cm., divided by the total current, gives the fraction of the electrons reflected, or the "coefficient of reflection."[2] Tables I. and II. give typical sets of readings which are shown graphically in Figs. 3 and 4.

TABLE II.

Reflection of 8-Volt Electrons.

Potential of focus plate, − 7 volts. Pressure of gas .0002 mm.

Distance of R from End of Cylinder.	i_R Amp. × 10^{-7}.	i_c Amp. × 10^{-7}.	i Total Amp. × 10^{-7}.	Scattering $\frac{i_{R,.1} - i_{R,d}}{i_{R,.1}}$.	Coefficient of Reflection. $\frac{i\ \text{Total} - i_{R,.1}}{i\ \text{Total}}$.
.1	1.35	.17	1.52	0.0	
6.0	1.20	.58	1.78	.11	.24
.1	1.35	.18	1.53	0.0	
1.0	1.32	.38	1.70	.02	
2.0	1.30	.45	1.75	.03	
3.0	1.27	.50	1.77	.06	
4.0	1.24	.52	1.76	.08	
5.0	1.21	.56	1.77	.10	

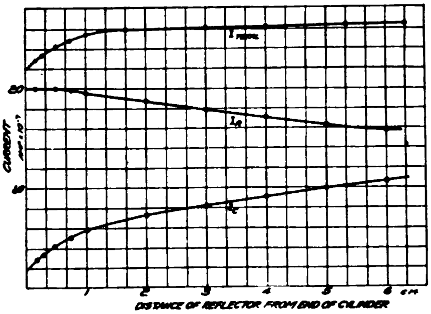

Fig. 3.

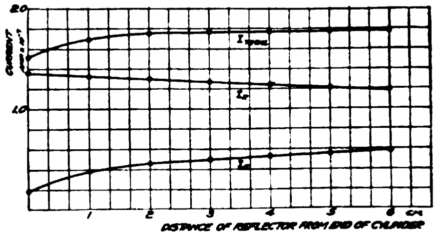

Fig. 4.

EXPERIMENTAL RESULTS. COEFFICIENT OF REFLECTION.

The final values obtained for the coefficient of reflection are collected in Table II., together with the conditions of experiment. The values for velocities below 50 volts were taken at different times and in varying sequences, and represent the maximum variations observed. The low values were invariably observed at the beginning of a series, after the apparatus had been lying idle over night, and are to be attributed to gas adsorbed by the reflecting plate. As this gas is gradually removed by the electron bombardment the reflection increases. This effect, generally much more marked, has been observed by all experimenters in this field. The readings for velocities above 50 volts were taken somewhat hastily, as gas was generally evolved by the metal parts of the apparatus under the bombardment, and no great accuracy is claimed for them. The last column gives the amount of "scattering" of the beam, *i. e.*, the fraction of it which fails to strike the plate when the latter is 6 cm. from the last diaphragm. Those experiments in which the scattering was small are the more reliable. This is discussed below under "scattering."

---

[1] The optimum potential for the focus plate had to be determined by a long series of measurements on the divergence of the electron stream. This is discussed below under "scattering."

[2] It is here assumed that all the electrons reflected from R in the 1 mm. position escape. The validity of this assumption is discussed below under "scattering."

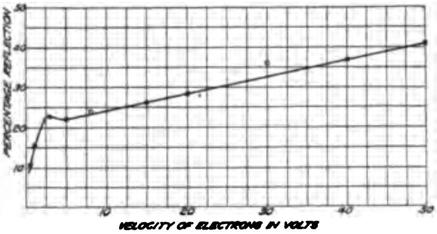

Fig. 5.

Variation of reflection with velocity.

The variation of reflection with velocity, as given in Table III., is shown graphically in Fig. 5. The curve is almost identical with that obtained by other investigators for lamp black[1] and platinum black,[2] and suggests very strongly that the small reflectivity in all these cases is due to the microscopic roughness of the surface. The fact that aluminum reflects less than other metals[3] would be accounted for in the same way, and the irrecoverable loss in the reflecting power of a platinum strip by prolonged heating, observed by Campbell,[4] would be satisfactorily explained as the result of crystallization of the platinum. The small reflectivity of electrolytic silver observed by the writer (p. 2, above) is another instance, and indicates, in harmony with chemical data, the extreme ultra-microscopic roughness of such surfaces compared with burnished surfaces. It is to be anticipated that for receiving

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[1] Gehrts, l. c., p. (1005).

[2] Von Baeyer, l. c., p. (965).

[3] Von Baeyer, l. c., p. (965).

[4] N. Campbell, Phil. Mag., 28, 286, 1914.

TABLE III.

Variation of Reflection with Velocity.

Gas Pressure, Mm.	Velocity in Volts.	Potential of Focus Plate, Volts.	i_R at 1 Cm. Amp. $\times 10^{-8}$.	i (Total) at 6 Cm. Amp. $\times 10^{-8}$.	Coefficient of Reflection.	Scattering at 6 Cm.
.00015	0.4	+ 5	2.93	3.29	.106	
.00013	0.4	+ 5	2.85	3.19	.107	.63
.00015	0.4	+ 5	3.29	3.64	.098	
					Mean .104	
.00013	1.0	+ 5	2.45	2.90	.155	.62
.00035	1.0	+ 5	3.30	3.85	.160	.45
.00035	1.0	+ 5	3.40	4.10	.170	.45
.00038	1.0	+ 3	1.82	2.12	.147	.23
.00038	1.0	+ 3	1.86	2.20	.154	.22
					Mean .157	
.00050	3.0	− 2	3.22	4.17	.225	.37
.00052	3.0	− 2	3.01	3.90	.227	.38
.00052	3.0	− 2	3.10	4.00	.225	.23
					Mean .226	
.00052	5.0	− 3.5	7.96	10.39	.230	.14
.00023	5.0	− 3.5	8.20	10.65	.230	
.00023	5.0	− 4.0	6.35	8.10	.216	.26
.00060	5.0	+12	8.8	11.1	.203	.27
					Mean .220	
.00023	8.0	− 7	8.00	11.23	.26	.13
.00016	8.0	− 7	14.2	18.6	.25	.10
.00018	8.0	− 7	14.1	18.7	.25	.09
.00020	8.0	− 7	13.5	17.8	.245	.11
.00026	8.0	− 7	13.0	16.7	.22	.15
.00060	8.0	− 7	14.9	19.4	.23	.17
.00060	8.0	+18	20.2	26.4	.235	.21
.00060	8.0	+18	20.0	26.0	.230	.20
					Mean .240	
.00018	15	−14	17.0	22.8	.26	.16
.00018	20	−19	18.0	25.7	.28	.17
.00018	30	−29	19.1	29.8	.36	.20
.00018	40	−35	117	185	.37	.26
.00018	50	−43	159	269	.41	.27
.00016	50	0	21.6	36.3	.40	.31
.00016	50	0	21.8	37.9	.42	.35
					Mean .41	
.00016	60	0	53.0	103	.49	.38
.00016	80	0	36.9	74.5	.51	.43
.00016	100	0	20.9	46.1	.55	.47
——	150	0	8.6	27.0	.68	.59
——	150	0	7.8	19.5	.60	.62
					Mean .64	
——	200	0	6.1	14.6	.59	.67
——	200	0	11.2	30.7	.63	.67
					Mean .61	

electrodes in photo-electric and other similar experiments a surface formed by vacuum distillation or a fine-grained electrolytic deposit would be quite as good a "black body" for absorbing electrons as soot or platinum black.

Velocity Distribution.

It is important to know what fraction of electrons in the beam denominated "1 volt" in the above table actually had that velocity. This was investigated in the usual manner by charging the reflecting plate to different negative potentials and observing how many electrons were able to reach it. For most of these measurements the plate was brought close to the end of the cylinder at which the electrons entered (the "1 mm." position). But for low velocities these measurements were checked by others in which the plate was at the other end of the cylinder, and the total current to plate and cylinder observed. The two methods gave identical results. For high velocities the latter method could not be used on account of the insufficient insulation of the cylinder.

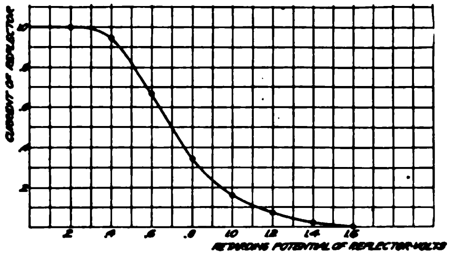

Fig. 6.

Velocity distribution of 1-volt electrons.

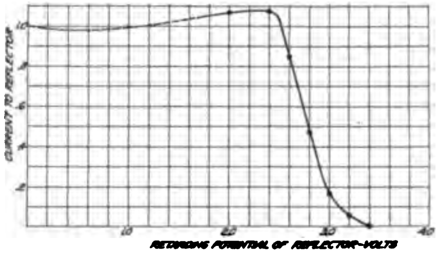

Fig. 7.

Velocity distribution of 3-volt electrons.

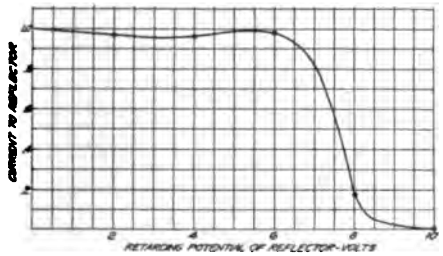

Fig. 8.

Velocity distribution of 8-volt electrons.

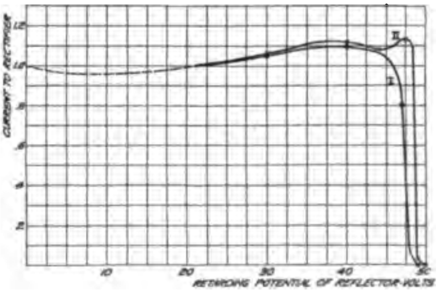

Fig. 9.

Velocity distribution of 50-volt electrons. Curve I., reflector alone at retarding potential. Curve II., both reflector and cylinder at retarding potential.

The results are shown in Figs. 6 to 9. The homogeneity of the beams is very satisfactory, especially at the lower velocities. This is in striking contrast to the velocity distribution (Fig. 12) in the preliminary experiments when mercury vapor (about .001 mm.) was present, which is discussed below under "Delta Rays." The curves for 50-volt electrons are very remarkable. They show the paradoxical fact that a plate upon which electrons fall receives more electrons the more negatively it is charged with respect to its surroundings. The same is evident in Fig. 7 for 3-volt electrons, and is still more strikingly shown in the curves of Fig. 10, where the plate was at the far end of the cylinder. It is readily explained by the change of "reflection coefficient" with velocity, and is a striking demonstration of the reality of this change. Curve 2, Fig. 10, is especially convincing, as here all influences, including surface leakage over the glass insulation, which becomes considerable at high negative potentials of the plate, combine to decrease the number of electrons striking the plate. Yet when the plate is 47 volts negative it still receives more electrons than when it is at zero potential.

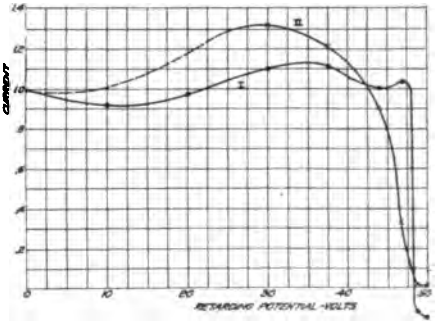

Fig. 10.

Velocity distribution of 50-volt electrons. Curve I., reflecting plate alone charged to retarding potential; Curve II., both reflector and cylinder at retarding potential.

SCATTERING.

The method employed in these experiments assumes a nearly cylindrical beam, and it is important to know to what extent this condition was fulfilled. If the electrons move in straight lines after entering the system of diaphragms the cone of rays emerging should have an angle of only 11°. Its maximum diameter, after travelling 6 cm. in the chamber, would be 1.5 cm. Thus the whole beam ought to strike the 4 cm. plate at all times, and the current received by the plate should be constant. The data given in the last column of Table III. show that this was never the case, a fact which must now be accounted for.

This "scattering" of a beam of slow-moving electrons has been observed by all experimenters, and is generally attributed to the effect of gas-molecules. As the calculated mean free path is often large compared with the dimensions of the apparatus, it has been necessary to assume a special attraction between molecule and electron, which can be effective at distances much greater than the "radius of molecular action." There are, however, three other very plausible explanations of this scattering, viz., (2) mutual repulsion of the electrons in the beam; (3) divergence of the beam caused by the strong electrostatic field near the edges of the diaphragms; (4) multiple reflections of electrons between the reflector and cylinder.

Two facts speak very strongly against the first explanation, that of scattering by gas molecules, namely, that the observed scattering was independent of gas pressure from .12 to .65 μ, and that it was greater for fast electrons than for slow ones (Table III., columns 1, 2 and 7).

The scattering due to mutual repulsion can be calculated approximately. Assume a cylindrical beam of radius R and uniform density ρ. The electric intensity at the edge of the beam is given by Poisson's equation,

$$\frac{d}{dr}\left(r\frac{dV}{dr}\right) = -4\pi\rho r;$$

which gives

$$\left(\frac{dV}{dr}\right)_R = -2\pi\rho R = \frac{2i}{Rv}$$

since

$$-\rho = \frac{i}{\pi R^2 v},$$

where i is the total current, and v the velocity of the electrons. For small displacements δr, this force will be sensibly constant, so that

$$\begin{aligned}\delta r &= \tfrac{1}{2}\frac{dV}{dr}\frac{e}{m}\frac{x^2}{v^2}\\ &= \tfrac{1}{2}\frac{2i}{Rv}\frac{e}{m}\frac{x^2}{v^2}\\ &= \frac{i}{R}\frac{e}{m}\frac{x^2}{v^3},\end{aligned}$$

where x is the distance travelled parallel to the axis of the beam, and δr the radial displacement of an electron on the edge of the beam. This gives an upper limit for δr, since an increase in the diameter of the beam and a decrease in its density near the edge, as compared with the center, will both have the effect of decreasing dV/dr at the edge.

For the 3 volt electrons of Table III., $i = 10^{-7}$ amp., $v = 10^8$ cm./sec.,

$R \geq .2$, $x = 6$, which gives $\delta r \leq .028$ cm. For the 1 volt electrons, $v = 6 \times 10^7$, $i = 4 \times 10^{-8}$ amp., hence $\delta r \leq .053$ cm. In the case of the .4 volt electrons the reflecting plate was charged to $-$ .6 volt. The electrons entered the chamber at 1 volt velocity, and travelled the greater part of its length at this velocity so that the spreading would be not very different from that calculated for the 1-volt electrons. Thus in all the above experiments the scattering due to this cause was negligible.

The effect of the electric field near the edges of the diaphragms is hard to calculate, but it can be studied experimentally by varying the potential of the focus plate. When the focus plate is at zero or negative potential, the electric field near the edges of the diaphragms, which are always earthed, is an accelerating one for electrons; when the focus plate is positive this field is a retarding one for electrons. In the first case the field will tend to make the beam divergent, in the second case convergent. Hence if there is any scattering due to this cause it should be greater in the first case than in the second. This was found to be true. When the scattering was large both it and the calculated coefficient of reflection depended very much on the potential of the focus plate. Table IV.

TABLE IV.

Effect of Focusing Upon Reflection and Scattering.

Velocity of Electrons. Volts.	Potential of Focus Plate. Volts.	Distance of R from End of Cylinder, Cm.	i_R Amp. $\times 10^{-7}$.	i_c Amp. $\times 10^{-7}$.	i Total Amp. $\times 10^{-7}$.	Percentage Reflection.	Percentage Scattering
50	0	6	1.60	2.25	3.85		
"	"	.1	2.34	.69	3.03		
"	"	5	1.67	2.20	3.87		
						39.3	32
50	−2	5	1.61	2.72	4.33		
"	"	.1	2.34	.63	2.97		
"	"	6	1.51	2.80	4.31		
"	"	.1	2.32	.60	2.92		
						46	35
50	0	.1	2.35	.60	2.95		
"	"	6	1.60	2.30	3.90		
"	"	.1	2.36	.59	2.95		
						39.7	32
50	0	.1	2.47	.60	3.07		
"	"	6	1.64	2.39	4.03		
"	"	.1	2.42	.70	3.12		
						39.3	33
50	−2	.1	2.40	.60	3.00		
"	"	6	1.50	2.94	4.50		
						46.6	35

gives a series of readings with 50-volt electrons, where a change of 2 volts in the potential of the focus plate made a change from 32 per cent. to 35 per cent. in the scattering, and from *39 per cent. to 46 per cent. in the reflection coefficient.* This is an extreme case and shows how serious the scattering due to this cause can be. In the final experiments this source of error was largely avoided by adjusting the potential of the focus plate until the scattering was a minimum, and in most cases quite small. There were in general two potentials of the focus plate, one positive and one negative, which satisfied this condition, and both gave the same value for the coefficient of reflection. Tables I. and II. are excellent examples, where for 8-volt electrons, changing the focus potential from + 18 to − 7 had no effect on the value found for reflection. In Tables I., II. and III. the values of scattering, measured by the percentage difference $(i_{.1} - i_6)/i_{.1}$ in the current to the plate at 1 mm. and 6 cm. distance respectively, is given as a measure of the reliability of the results.

The fourth explanation mentioned above, multiple reflection between the plate and cylinder, is probably responsible for the residual 10 per cent. apparent scattering at low velocities which was the minimum that could be obtained by focusing, and for the large values at high velocities. For if there is reflection at all velocities, then some of the electrons reflected from the plate to the cylinder walls will be re-reflected, and the fraction of these twice-reflected electrons that strikes the plate will be greater when the plate is near the closed end of the cylinder than when it is 6 cm. back. An estimate of the difference to be expected can be easily obtained if we assume, as has been shown to be at least very nearly true, that the reflected electrons leave the plate with random directions, that is, are distributed, as regards direction, uniformly over the whole hemisphere.

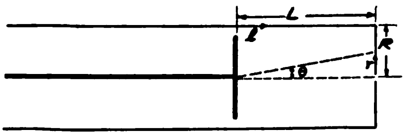

Fig. 11.

Let N_0 be the number of electrons striking the reflecting plate, ρ the coefficient of reflection of the plate, and ρ' that of cylinder walls. If the primary beam is narrow the number of electrons which, after reflection from the plate, strikes the end of the cylinder (Fig. 11) in the circular zone between r and $r + dr$ is $(\rho N_0 \cos \theta r dr)/(r^2 + L^2)$. Of these the fraction that is re-reflected and strikes the plate again is $(\rho' A \cos \theta)/(2\pi(r^2 + L^2))$ approximately where A is the area of the plate. Hence the total number returned to the plate by the zone between r and $r + dr$ is

$$\frac{\rho\rho' N_0 A \cos^2\theta r dr}{2\pi(r^2+L^2)^2} = \frac{\rho\rho' N_0 A L^2 r dr}{2\pi(r^2+L^2)^3}$$

and the total number returned by the end of the cylinder is

$$N_e' = \frac{\rho\rho' N_0 A L^2}{2\pi}\int_0^R \frac{r dr}{(r^2+L^2)^3} = \frac{\rho\rho' N_0 A}{8\pi}\left[\frac{1}{L^2} - \frac{L^2}{(L^2+R^2)^2}\right].$$

Similarly the number returned to the plate by the element of the cylindrical surface between l and $l + dl$ is

$$\frac{\rho N_0 2\pi R \sin\theta dl}{2\pi(R^2+l^2)} \frac{\rho' A \cos\theta}{2\pi(R^2+l^2)} = \frac{\rho\rho' N_0 A R \sin\theta \cos\theta\, dl}{2\pi(R^2+l^2)^2} = \frac{\rho\rho' N_0 A R^2 l dl}{2\pi(R^2+l^2)^3}$$

and the total number returned to the plate by the cylindrical surface is

$$N_c' = \frac{\rho\rho' N_0 A R^2}{2\pi}\int_0^L \frac{l dl}{(R^2+l^2)^3} = \frac{\rho\rho' N_0 A}{8\pi}\left[\frac{1}{R^2} - \frac{R^2}{(L^2+R^2)^2}\right].$$

The total number of electrons returned to the plate by reflection from both the end and the cylindrical surface is therefore

$$N' = N_e' + N_c' = \frac{\rho\rho' N_0 A}{8\pi}\left(\frac{1}{L^2} + \frac{1}{R^2} - \frac{1}{L^2+R^2}\right).$$

Substituting the values of A and R from the dimensions of the apparatus, and taking $\rho = \rho' = .25$, we have

$$\text{for } L = 1 \text{ cm.},\ N_1' = .05\, N_0;$$
$$\text{for } L = 6 \text{ cm.},\ N_6' = .008\, N_0.$$

Hence the reflecting plate will show a current about 4 per cent. smaller when it is 6 cm. from the end of the cylinder than when it is 1 cm. from the end, which is not far from the value observed in the best experiments.

If $\rho = \rho' = .50$ then

$$N_1' = .12\, N_0,$$
$$N_6' = .02\, N_0.$$

In this case the current to the plate should decrease 10 per cent. in moving from 1 to 6 cm.[1]

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[1] No account has been taken of multiple reflections of higher order, which, however, become very important when the reflection coefficient is large. If the plate is not too large in comparison with the cylinder, so that the solid angles subtended at a point on the plate by the elements of area of the cylinder are approximately the same for all points on the plate, then the number of electrons returned by the cylinder to the plate at the second reflection will obey the same law as for the first reflection, and we shall have, denoting the multiplier of N_0 in the equation for N' above by σ, $N' = \sigma N_0$; $N'' = \sigma N'$; $N''' = \sigma N''$, etc., and the net current received by the plate will be

$$N = N_0 + N' + N'' + N''' + \ldots = N_0(1 + \sigma + \sigma^2 + \sigma^3 + \ldots) = \frac{N_0}{1-\sigma}.$$

Delta Rays.

In the preliminary experiments, in which mercury vapor was not excluded from the apparatus, large currents were obtained of electrons of "zero velocity," that is, when the receiving plate and cylinder were at the same potential as the most negative end of the filament, the focus plate alone being at a positive potential. The electrons from the filament were thus accelerated as they approached the focus plate, retarded as they left it, and reached the cylinder with only their initial velocity of thermal emission. When the velocity distribution of these electrons was measured, it was found that a very considerable fraction of them had velocities as high as 9 volts. Tables V. and VI. give the observed distribution. In the first the retarding potential was applied through the galvanometer to the plate alone, which was placed close to the opening of the cylinder; in the second the plate was at the far end of the cylinder and connected to it, and the current to both measured at the corresponding potentials. Fig. 12 shows this distribution graphically.

Table V.

Velocity Distribution of "Zero-Volt" Electrons.

Distance of reflector from end of cylinder 1 mm. Potential of focus plate 12.4 volts. Reflector alone charged to retarding potential. Gas pressure .00025 mm. + Hg at 16° C.

Retarding Potential of Reflector.	i_R Amp.× 10^{-7}.	i_c Amp.× 10^{-7}.	i_R Per Cent.
0	5.20	.50	1.00
1	4.75	.60	.91
2	4.12	.82	.79
3	3.32	1.13	.64
4	2.60	1.40	.50
5	1.90	1.68	.365
6	1.23	1.92	.236
7	.72	2.12	.138
8	.25	2.13	.048
9	.09	2.07	.017
10	0	1.95	0
0	5.23	.50	

As these velocities are very much higher than could be accounted for by thermal emission, great care was taken to make sure their reality. Many similar series of readings were taken under widely varying conditions. Currents were obtained with the negative end of the filaments at − 6 volts, all other parts of the apparatus except the focus plate being earthed, and the velocity distribution of these was similar to that of Fig. 12. The maximum velocity could be increased by increasing the potential of the focus plate, but was always smaller, measured in volts, than

TABLE VI.

Velocity Distribution of "Zero-Volt" Electrons.

Distance of reflector from end of cylinder 1 mm. Potential of focus plate 12.4 volts. Reflector and cylinder both charged to retarding potential. Gas pressure .00025 mm. + Hg at 16° C.

Retarding Potential of Reflector and Cylinder.	i Total Amp. × 10^{-7}.	i Total Per Cent.
0	6.80	1.00
1	6.31	.93
2	5.62	.83
3	4.71	.69
4	3.70	.542
5	2.76	.406
6	1.90	.280
7	1.12	.164
8	.48	.070
9	.10	.015
10	−.09	−.014
0	6.80	1.00

that potential. Such sources of error as surface leakage were, of course, carefully investigated, and it was easily proven, by changing the temperature of the filament, that the electrons came, directly or indirectly, from the hot filament.

The best evidence regarding the nature, as well as the reality, of these high velocity electrons, is *that they entirely disappeared when the U tube leading to the pump was immersed in solid carbon di-oxide and ether.* They are evidently delta rays, produced at the filament by the bombardment of the mercury ions. There is no record, so far as I am aware, of the production of such relatively high speed delta rays by the impact of slow positive ions, though they have been observed by Dr. Langmuir in some experiments not yet published. The most striking characteristic of these rays is *that their maximum initial energy, 9 volts, is nearly the*

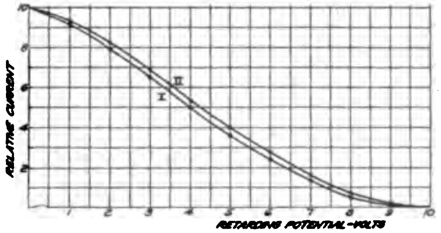

Fig. 12.

Velocity distribution of "zero-volt" electrons. Curve I., reflecting plate alone at retarding potential. Curve II., both reflector and cylinder at retarding potential.

same as that (12.4 volts) of the ions which produce them, which suggests that the interchange of energy between ions and electrons, as well as that between electrons and radiation, may be governed by quantum relations. An initial energy greater than that of the ions, such as would violate this law, was never observed.

CONCLUSION.

The electron reflection from a copper surface roughened by prolonged heating at 350° C. in vacuum is, like that of electrolytic silver, very small, being only 10 per cent. for electrons having a velocity of 4/10 volt. This is explained by the extreme ultra-microscopic roughness of these surfaces. Such surfaces should make excellent electric "black bodies" for receiving electron currents.

The scattering of a beam of slow-moving electrons has been shown to be due to deflection by strong electric field, and multiple reflection between the plate and its surroundings, rather than to gas molecules or mutual repulsion of the electrons.

Delta rays have been produced by mercury ions which have fallen through only 12.4 volts. The energy of the fastest of these delta rays is approximately the same as that of the ions, suggesting that the energy exchanges between ion and electron are governed by quantum relations.

These experiments were made at the physical laboratory of the Worcester Polytechnic Institute during the winter of 1913–14. Further experiments are in progress at the research laboratory of the General Electric Company, and will be reported in the near future.

EINSTEIN'S PHOTOELECTRIC EQUATION AND CONTACT ELECTROMOTIVE FORCE.

By R. A. Millikan.

§ 1. Introductory.

EINSTEIN'S photoelectric equation for the maximum energy of emission of a negative electron under the influence of ultra-violet light, namely,

$$\tfrac{1}{2}mv^2 = Ve = h\nu - p \qquad (1)$$

cannot in my judgment be looked upon at present as resting upon any sort of a satisfactory theoretical foundation. Its credentials are thus far purely empirical, but it is an equation which, if correct, is certainly destined to play a scarcely less important rôle in the future development of the relations between radiant electromagnetic energy and thermal energy than Maxwell's equations have played in the past.

I have in recent years been subjecting this equation to some searching experimental tests from a variety of viewpoints and have been led to the conclusion that, whatever its origin, it actually represents very accurately the behavior, as to both photoelectric and contact E.M.F. relations, of all the substances with which I have worked. The precision which I have been able to attain in these tests has been due to the following precautions.

1. I have made simultaneous measurements in extreme vacua of photo-currents and contact E.M.F.'s and have thus been able to eliminate the considerable influence which these latter have on photo-potentials.

2. I have worked with surfaces newly formed in extreme vacua and with very large photo-currents of saturation value of the order 20,000 scale divisions in 30 secs. so that I have thus been able to locate the intercept of the photo-current curve on the *PD* axis with much precision.

3. I have used substances which are photo-sensitive practically throughout the whole length of the visible spectrum and have thus been able to use a large range of wave-lengths all of which were above the long wave-length limit of the receiving Faraday cylinder—a matter of no little importance.

4. I have, with the aid of filters, carefully chosen for the principal lines of the mercury spectrum, eliminated from the photo-current-

potential curves corresponding to the longer wave-lengths, the effects of the scattered short wave-length light, which not infrequently falsifies entirely the shape of these curves in the neighborhood of the intercept on the potential axis.

My conclusions, however, reported briefly last year[1] and this[2] are directly at variance with results recently reported in a very notable paper by Ramsauer[3] who finds that there is no definite maximum velocity of emission of corpuscles from metals under the influence of ultra violet light. Before considering, then, any of the theoretical consequences of Einstein's equation it is necessary to first present such evidence as exists for believing that, in spite of Ramsauer's results, there is in fact a definite and accurately determinable maximum velocity of emission for each exciting wave-length.

§ 2. Proof of the Existence of a Maximum Energy of Emission of Photoelectrons and Discussion of Ramsauer's Experiments.

Ramsauer's method is notable in that he makes the first direct measurement by a magnetic deflection, of the velocity of emission of photo-corpuscles. By this method he is able to choose those corpuscles which emerge in one particular direction only, for example, the direction of the normal to the surface, and he finds that these have a certain distribution about a most frequent value. This distribution he finds the same for all wave-lengths of the incident light, for all substances, and for all angles of emission. His source, like Kadesch's, and like my own in much of my former work, is a powerful condensed spark between zinc electrodes. This source I discarded in my most recent tests on Einstein's equation because the mercury arc was found to give greater reliability in the settings and to have greater monochromatism in its lines.

The substances which Ramsauer studies are gold, brass, zinc and carbon. His range of wave-lengths, obtained with a quartz spectrometer, is quite narrow, being the same as that used by Hughes, namely, from 186 $\mu\mu$ to 256 $\mu\mu$ for gold, and 186 $\mu\mu$ to 334 $\mu\mu$ for zinc. As is well known lines of wave-length below 220 $\mu\mu$ pass with great difficulty through a quartz spectrometer. Ramsauer further works quite largely with waves which are shorter than the long wave-length limit of his receiving surface, and in fact his corrections for "falsches licht" (this term actually covers several different effects such as the emission of corpuscles from the illuminated surface itself by stray light, and the emission of corpuscles from the surrounding walls by light reflected to them from the illuminated surface) are very large, amounting to as much as 1/10 of the maximum photo-currents. This seems to me to rob the lower parts of his velocity distribution curves of practically all significance. His procedure differs from mine most vitally in that while I measure very precisely, as I think, the maximum velocity of emission he measures instead the most frequent velocity of emission. This was the quantity from which Richardson and Compton[1] drew most of their conclusions and upon which they placed their chief reliance, though their method of obtaining it differed from Ramsauer's.

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[1] Phys. Rev., IV., p. 73, '14.

[2] Phys. Rev., VI., p. 55, '15.

[3] Ann. der Phys., 45, p. 1120, 1914. Also 45, p. 961.

In his results Ramsauer agrees with Richardson and Compton in finding this most frequent velocity (expressed in energy units) a linear function of the frequency and in finding that the long wave-length limit is given by the intercept of this line on the frequency axis. Question may be raised regarding the certainty with which Richardson and Compton could determine this intercept, since its location involves the contact E.M.F. and they made no contact E.M.F. measurements in vacuo. Ramsauer eliminates contact E.M.F. entirely by surrounding the emitting surface by walls of the same metal as the emitter itself. He agrees with Richardson and Compton[1] also in finding the slopes of the volt-frequency lines differing among themselves by 20 or more per cent. and like Richardson and Compton he finds these slopes all lower than h/e by large per cents., which vary in his case, from 35 to 50 per cent. *None of these results so far are at variance either with my work or with Einstein's equation, for measurements on the most probable velocity of emission are not capable of furnishing a test of Einstein's equation.*

But Ramsauer's results are at variance with mine and with Einstein's equation in that he finds no definite maximum velocity of emission at all for when he plots energies of emission as ordinates and deflecting magnetic field strengths as abscissæ he finds these curves run off asymptotically to the axis of abscissæ. In my judgment this is because the "falsches licht" errors mask entirely the phenomenon under investigation in the region corresponding to the lower parts of his velocity distribution curves. My own experiments seem to me approximately 1,000 times better adapted to the testing of this point than are Ramsauer's, since my maximum currents are about 1,000 times larger than his, as measured in scale divisions of deflection, and if his distribution curve is the correct one, I should obtain very large currents at potentials at which, in fact, I get none at all. Thus in the case of the mercury lines 2535Å, if the potential applied to my lithium surface was .02 volt to the left of the intercept shown in Fig. 1, there was not a trace of deflection in 30 seconds.

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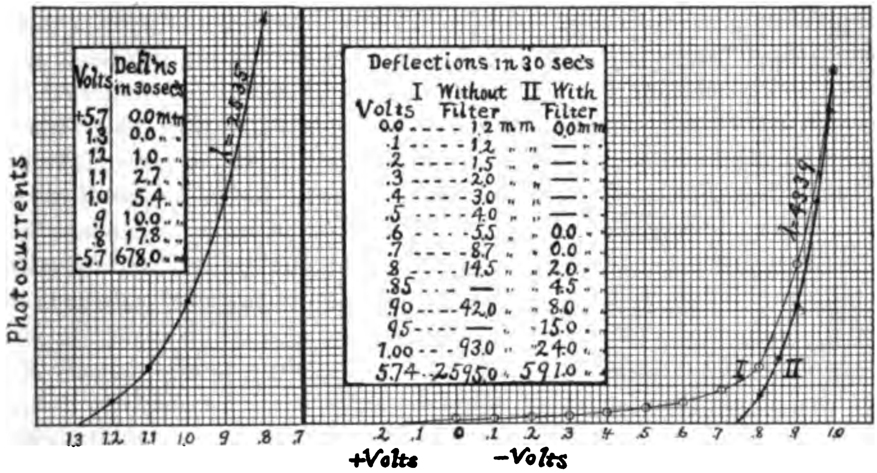
[1] Phil. Mag., 24, p. 572, 1912.

But with lines 5461 and 4337 in the case of sodium, and line 4339 in the case of lithium, although I used a Hilgar monochromator and a narrow slit (.01 inch) I did obtain definite indications of deflections due to stray short-wave-length light, *which, however, disappeared entirely as soon as I used filters which cut out all lines of shorter wave-length than that under examination.* Fig. 1 furnishes a very good illustration of this effect. Without a filter the curves corresponding to line 4339 seemed to approach the axis asymptotically, as in Ramsauer's experiments (note the curve marked I.), but with a filter of æsculin in a glass trough which cut out entirely all lines below 4339 including the strong adjacent line 4047, the asymptotic character disappeared completely and the curve shot suddenly into the axis and gave no indications whatever of deflections either at − .6 or at − .7 with volts (see Fig. 1). The curves shown in the figure and a great many other similar ones which I have taken seem to me to establish beyond question the contention that *there is a definite maximum velocity of emission of corpuscles from a metal under the influence of ultra-violet light*, or, in other words, that the curves due to a particular spectral line do plunge sharply into the potential axis and do not approach it asymptotically.



Volts	Defl'ns in 30 secs
+5.7	0.0 mm
1.3	0.0 " "
1.2	1.0 " "
1.1	2.7 " "
1.0	5.4 " "
.9	10.0 " "
.8	17.8 " "
−5.7	678.0 " "

Deflections in 30 secs

Volts	I Without Filter	II With Filter
0.0	1.2 mm	0.0 mm
.1	1.2 " "	— " "
.2	1.5 " "	— " "
.3	2.0 " "	— " "
.4	3.0 " "	— " "
.5	4.0 " "	— " "
.6	5.5 " "	0.0 " "
.7	8.7 " "	0.0 " "
.8	14.5 " "	2.0 " "
.85	— " "	4.5 " "
.90	42.0 " "	8.0 " "
.95	— " "	15.0 " "
1.00	93.0 " "	24.0 " "
5.74	2595.0 "	591.0 " "

Fig. 1.

The work on the photoelectric determination of *h* will be reported more fully in another paper but the data furnished in Fig. 1 suffices to determine *h* from lithium with no little precision. Thus, since the frequencies of 2535 and 4339 are 118.2×10^{13} and 69.1×10^{13} respectively, we see from the intercepts of the figure that the slope

$$\frac{dV}{dv} = \frac{h}{e} = \frac{129 + .74}{(118.2 - 69.1) \times 10^{13}} = 4.13 \times 10^{-15} \frac{\text{Volt}}{\text{Frequency}}; \quad (2)$$

$$\therefore h = \frac{4.13 - 10^{-15} \times 4.774 \times 10^{-10}}{300} = 6.58 \times 10^{-27} \frac{\text{Erg}}{\text{Frequency}}.$$

The error here can scarcely exceed .02 volt in 2 volts or 1 per cent.

The second conclusion of Ramsauer's which seems at variance with Einstein's equation is that each corpuscle liberated by a given wave-length does not leave the atom with a constant energy but that a given wave-length may liberate corpuscles from a given kind of atom with a large range of energies. But if the p in Einstein's equation is indeed a characteristic constant of the material as he assumed it to be, then the corpuscles are all expelled from the atom with a constant speed and any differences which may be shown by the velocities of the corpuscles which have escaped from the surface of the metal at a given angle are due to differences in the retardations which they have encountered in getting out from different depths beneath the surface. Ramsauer, however, concludes from the fact that he apparently gets the same curve of distribution of velocities for all wave-lengths and for all angles of emission that his observed external distribution of velocities is the same as the "internal distribution," that is that the corpuscles are emitted from the atoms themselves with precisely the same distribution of speeds as that which he measures outside the metal.

Now I am not at all convinced that Ramsauer's results actually do show that the distribution of velocities is the same for different wave-lengths, for the range of wave-lengths (185 $\mu\mu$ to 256 $\mu\mu$ for gold) seems to me too small and the experimental uncertainties too large to permit of such a conclusion. According to his own statement the curve corresponding to 256 $\mu\mu$, is badly falsified by stray short-wave-length light (the maximum deflections obtained with this wave-length were but 17 mm. as against the 20,000 mm. which I have used in my work with sodium). The same is true of all his curves corresponding to the longer wave-lengths. The point in question could be convincingly tested only by using widely different wave-lengths like those corresponding to the lines 2535 and 5461 as I have done in the work with sodium.

Secondly, even if the large experimental uncertainties should be reduced 10 times, and the distribution of velocities for different wave-lengths and different angles then shown to be the same, I should still consider Ramsauer's argument for the identity of the external and internal distribution of velocites to be quite unconvincing. For even though the corpuscles make perfectly elastic impact with the atoms, as Ramsauer assumes, according to the Maxwell-Boltzmann law their energy of agitation must decrease continually with successive impacts until they are in temperature equilibrium with the atoms. In other words the corpuscles which have made many impacts before emerging in a given direction must have a smaller velocity than those which have made few. And as a matter of fact Ramsauer's observed velocity distribution curve, ignoring the asymptotic portion, is one which differs from all his suggested energy distribution curves in being too steep on the high velocity side just as would be the case if all the corpuscles had started with a common velocity and only those which came from appreciable depths beneath the surface had fallen below this velocity.

Thirdly Ramsauer in identifying the internal and external distribution of velocities appears to me to overlook the fact that the mere phenomenon of a free charge remaining on a charged conductor necessitates the existence of a surface force which prevents its escape. This is the force which Helmholtz conceived of as arising from "the specific attraction of matter for electricity." It is not a force which in any way impedes the free movement of electricity over or through the conductor, else the body would not act like a conductor, and it is with conductors alone that we are here concerned. The force considered is then one which acts on the conduction electrons, that is, on the so-called free electrons as distinguished from those which are permanent constituents of the atoms. Hence, even after an electron has escaped from the interior of an atom it cannot escape from the metal until this force is overcome. It is this force which is responsible for about 999 thousandths of the contact E.M.F. which we measure between metals. The other thousandth, measured by the Peltier effect, has a kinetic, instead of a static, origin. These relations have been the occasion of much confusion among writers on contact effects, though they have been stated with admirable clearness by Kelvin, Helmholtz and others.[1]

I am inclined to think then that neither Ramsauer's of new conclusions, (1) that there is no definite maximum energy of emission and (2) that the external and internal distribution of velocities are the same can, possibly stand. At any rate the correctness of the second has in no way been demonstrated, while the incorrectness of the first seems to me to have been established.

§ 3. THE RELATION OF CONTACT E.M.F. AND EINSTEIN'S EQUATION.

The precise tests which I have reported of Einstein's equation consist in showing (1) that there is a very exact linear relation between the maximum P.D. and the frequency, (2) that the slope of this line yields very accurately Planck's h, and (3) that the intercept of this line on the frequency axis is the frequency at which the metal first becomes photosensitive. In order to test this last point it was necessary to displace in the direction of positive potentials the observed P.D.ν. line by the exact amount of the measured contact E.M.F. When this was done the observed long wave-length limit, as directly determined, agreed quite accurately with the intercept (see Fig. 2). This means that in Einstein's equation p represents not the amount of work necessary to remove the corpuscle entirely from the influence of the metal, that is, to carry it out beyond the influence of the latter's contact field, but rather the work necessary to just free it from the surface so that a relatively large accelerating field can then remove it, for it is in just this way that we actually make the test. The quantity p, then, is the work necessary to just detach a corpuscle from the surface of the metal and we have, by putting in (1) $v = 0$

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[1] A remarkably lucid presentation is found in Wüllner's Experimental Physik, Vol. III., pp. 736–755. See also Kelvin, Phil. Mag., 46, p. 82, 1898.

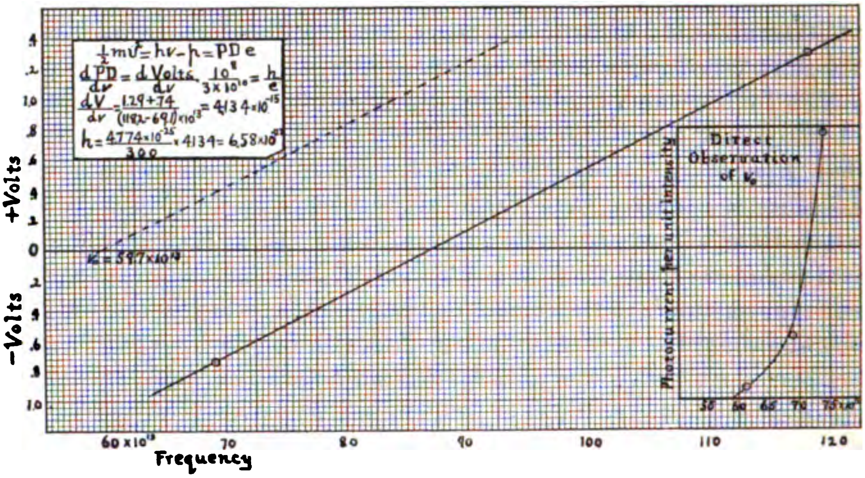

Fig. 2.

$$p = h\nu_0. \tag{3}$$

Now since both the independence of photo-emission upon temperature, and also the fact that gases show the photo-effect, indicate that the electrons which are ejected by light from metals are not the free electrons of the metal, but rather electrons which are constituents of the atoms, we would naturally consider p as made up of two parts, (1) the work p_1 necesary to detach the electron from its parent atom and make it a free electron of the metal, and (2) the work p_2 necessary to detach this free, or conduction, electron from the surface of the metal.

If we consider two opposed metal surfaces, for example one of pure zinc and one of pure copper separated only by the ether, and imagine that they have been put initially into the same electrical condition, so that no electrical field exists between them, then if a wire of copper be run from the copper plate to the zinc plate, we find by experiment that upon making contact an electric field is established between the plates. We say that the P.D. which now exists has arisen because of a contact E.M.F. at the junction of copper and zinc which causes an electrical flow from copper to zinc until equilibrium is set up, and we measure this contact E.M.F. by the observed P.D. which it creates, taken of course with the opposite sign. By definition then the contact E.M.F. is the amount of work which, before any electrical field exists, would be required to transfer one unit of free positive electricity from the zinc over to the copper against the superior attraction of zinc for this unit. After the contact has been made and equilibrium set up, it, of course, requires no work to carry electricity across the zinc-copper junction other than that represented by the Peltier effect, which has another cause and is of an altogether different order of magnitude. Writing then the above definition in symbols we have

$$\text{Contact E.M.F.} = \frac{p_2 - p_2'}{e}. \tag{4}$$

in which p_2 relates to the zinc and p'_2 to the copper.

If, as in the case we are considering, e is negative, then this contact E.M.F. is negative. Now if we write for each of any two opposed metals

$$h\nu_0 = p = (p_1 + p_2)$$

and

$$h\nu_0' = p' = (p_1' + p_2')$$

and subtract we obtain

$$h\nu_0 - h\nu_0' = (p_1 - p_1') + (p_2 - p_2') \tag{5}$$

which in view of (4) becomes

$$\text{Contact E.M.F.} = \frac{h\nu_0 - h\nu_0' - (p_1 - p_1')}{e}, \tag{6}$$

an expression which shows that Einstein's equation does not at all demand that the contact E.M.F., even between two pure metals, be equal to the difference between the frequencies corresponding to long wave-length limits of the two metals multiplied by h/e. If this latter relation is found by experiment to hold for any two metals it is an exceedingly interesting and important fact which, however, has no bearing on the validity or invalidity of Einstein's equation. If this equation is correct such a result would simply mean that $(p_1 - p_1') = 0$ which in turn might mean that the energy of *escape* of the corpuscle from the atom is always equal to $h\nu$, the absorbed energy being, contrary to the physical theory which guided Einstein, greater than $h\nu$ or it might mean that, though the absorbed energy is but $h\nu$, the works necessary to detach corpuscles from the atoms of the two metals are the same, or that these works are both so small in comparison with the works necessary to detach conduction electrons from metallic surfaces that $(p_1 - p_1')$ is in any case negligible in comparison with $(p_2 - p_2')$.

Now the experimental situation is as follows: Richardson and Compton,[1] although they made their contact E.M.F. and their photoelectric measurements under different conditions, namely the former in air and the latter in vacuo, yet found that for any two metals $h/e(\nu_0 - \nu_0')$ was at least of the same order of magnitude as the contact E.M.F. between these same two metals, and last year I also found that in the case of sodium and copper oxide, while the measured contact P.D. between them was 2.51 volts, h/e times the difference in the frequencies corresponding to the long wave-length limits was 2.79 volts, a result which seemed almost near enough to be in accord with Richardson and Compton's conclusion. Furthermore if Einstein's equation is correct, this conclusion can be tested very accurately without any contact potential measurement at all by a method which has already been tried a number of times, though the results have been quite discordant, and so far as I know the relation of the result to contact E.M.F. has not been clearly pointed out. This relation appears at once as soon as we determine just what are the demands which Einstein's equation imposes on contact E.M.F.

Consider a corpuscle ejected from any conducting surface by light of frequency ν into a Faraday cylinder made for example of a metal more electronegative than the emittor. Then, if the corpuscle is to be brought to rest just as it reaches the wall of the Faraday cylinder, the energy of ejection must just equal the work done against the applied positive potential plus the contact potential and this by Einstein's equation is equal to $h\nu - p$. Thus denoting by V_0 the *observed* maximum positive potential, and by K the contact E.M.F. between the Faraday cylinder and the emittor, and remembering that $p = h\nu_0$ in which, if the emitting surface is inhomogeneous ν_0 is the long wave-length limit of the most electropositive element in the surface, that is, the element which loses negative electrons most easily, we have

$$\tfrac{1}{2}mv^2 = (V_0 + K)e = h\nu - h\nu_0.$$

Writing a similar equation for an electron ejected by light of the same frequency from the surface of any other conducting material into the same Faraday cylinder we obtain

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[1] Phil. Mag., 24, p. 592, 1912.

$$\tfrac{1}{2}mv^2 = (V_0' + K')e = h\nu - h\nu_0'.$$

By subtraction we find since the contact E.M.F. between the two metals used as emittors is $K' - K$.

$$\text{Contact E.M.F.} = \frac{h}{e}(\nu_0 - \nu_0') - (V_0 - V_0'). \qquad (7)$$

If then Einstein's equation is correct this should furnish a perfectly general way of measuring contact E.M.F. between any two conducting surfaces, pure or impure, and if it can be experimentally verified, then we have one more proof of the correctness of Einstein's equation. This is the equation which I have been submitting to careful experimental test in the case of the alkali metals, and I have thus far found it in perfect agreement with experiment (see below).

If (7) may be regarded as established by the work which follows, then it will be obvious that we can test whether or not, with any particular metals,

$$\text{Contact E.M.F.} = \frac{h}{e}(\nu_0 - \nu_0') \qquad (8)$$

by simply observing whether these metals, when placed before the same Faraday cylinder and stimulated by a given wave-length, show the same value of the observed maximum positive potential, *i. e.*, whether $V_0 - V_0' = 0$.

What I wish to point out then is that, though Einstein's equation does demand that certain relations exist between contact E.M.F.'s and photo-potentials, namely those stated in equation (7) it does not demand the relation (8) suggested by the experimental work of Richardson and Compton, and with about the same sort of roughness, by my results on sodium. If however (7) is a correct and perfectly general relation, as it must be if Einstein's equation is a rigorous one, then (8) can be tested for any particular substances by seeing whether, for these substances, the last term of (7) vanishes. This point was tested carefully in 1906 by Millikan and Winchester[1] who found marked differences in the V_0's for eleven different metals and whose results therefore seem to conflict with (8). It was again tested by Page in 1913[2] who found that with freshly scraped surfaces of copper, aluminum and zinc the V_0's were all alike. Kadesch[3] in 1914 measured accurately the V_0's corresponding to freshly cut sodium and potassium in front of the same receiving chamber, and although he did not discuss the question here under consideration, a glance at his curves shows that for a given wave-length these V_0's differ by as much as .85 volt. Last winter I found that the directly measured contact E.M.F. in vacuo between lithium and copper oxide was more than a volt less than the observed value of $h/e(\nu_0 - \nu_0')$ in which ν_0 was the observed long wave-length limit of lithium freshly cut in vacuo and ν_0' the observed long wave-length limit of CuO in the same vessel. (It is to be remembered that e is here negative.) These results appear to show conclusively that (8) does not in general hold, and in a paper presented at the April meeting of the American Physical Society[1] I suggested that the failure of (8) in some cases and its apparent validity in others might be explained by the influence of surface inhomogeneities. For obviously, in measuring $(h/e)(\nu_0 - \nu_0')$ one is always making his long wave-length limit tests on the most photosensitive, *i. e.*, the most electropositive constituent of a given surface, while in measuring contact E.M.F.'s one is testing the *mean* effect outside the surface of all the surface constituents, so that in the case of a lithium surface which is discharging electrons into a copper oxide cylinder, if the lithium surface were a mixture of substances, some of which were much more electropositive than others, the measurement of $(h/e)(\nu_0 - \nu_0')$ would correspond, if (8) were correct for homogeneous surfaces, to a determination of the contact E.M.F. of the most electropositive element in the lithium surface. Hence the measured contact E.M.F. would be expected to fall below the value of $(h/e)(\nu_0 - \nu_0')$ as my results showed that it did.

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[1] Phil. Mag., 14, p. 201, 1907.

[2] Am. Jr. Sci., 36, p. 501, 1913.

[3] Phys. Rev., III., p. 367, May, 1914.

Nevertheless this was not a necessary cause of the failure of (8), for equation (6) shows that there is no reason other than an experimental one for supposing that (8) ever holds. Accordingly, shortly after the above-mentioned meeting, I suggested to Doctors Kadesch and Hennings that they reëxamine the point tested first by Millikan and Winchester and last by Page, using as nearly as possible the latter's experimental conditions in order to find out whether the difference between the two sets of results was due to the fact that we worked in these early experiments with old surfaces while Page had tested newly scraped metals, or whether new surfaces of the ordinary metals do actually show differences in the V_0's which escaped Page's detection, as equation (6) indicates that they might well do. They found that they could use with some modification the apparatus on which Dr. Hennings had worked with contact E.M.F.'s in this laboratory some years previously. Their results are given in papers which follow and seem to support Page's conclusion for the ordinary metals when newly scraped. It is useless, however, to attempt to put any interpretation upon these results until Einstein's equation is shown to be a reliable tool with which to work, that is, until equation (7) is shown to be able to predict accurately and invariably observed contact E.M.F.'s. The following results show that in all the cases thus far examined in which (8) breaks down completely (7) nevertheless yields the most beautiful agreement.

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[1] PHYS. REV., VI., p. 55, '15.

Thus in the experiments reported in April to the Physical Society[1] the measured contact E.M.F. between lithium and copper oxide was found to be 1.52 volts. The long wave-length limit of the lithium was found to correspond accurately to $\nu_0 = 57.0 \times 10^{13}$. This was determined most reliably by displacing the P.D., ν line toward positive potentials by the amount of the measured contact E.M.F. and then taking the intercept of this line on the ν axis. Direct observation checked closely however the value thus obtained. The long wave-length limit of the receiving copper oxide cylinder was directly determined at line 2535 with an uncertainty of perhaps 50 Å. This corresponds to $\nu_0' = 118.2 \times 10^{18}$. These figures give (see equation (2))

$$\frac{h}{e}(\nu_0 - \nu_0') = 4.13 \times 10^{-15}(118.2 - 57.0) \times 10^{13} = 2.53 \text{ volts.}$$

Now line 2535 was just at the long wave-length limit of the copper oxide, so that for this line V_0' between CuO and a Faraday cylinder of CuO was zero. On the other hand, for line 2535 the V_0 between the lithium and the CuO Faraday cylinder was found to be just + 1.00 volts, so that (7) becomes

$$1.52 = 2.53 - 1.00 = 1.53.$$

These measurements were made on a newly cut lithium surface. Several months later the measurements were repeated and the contact E.M.F. between the then old lithium surface and the Faraday cylinder had changed to 1.11 volts. The V_0 between the lithium and cylinder for line 2535 had changed to 1.29 volts (these are the measurements shown in Fig. 1) and the long wave-length limit ν_0 was now measured at 59.7×10^{13}. The ν_0' had not changed. These figures give

$$\frac{h}{e}(\nu_0 - \nu_0') = 4.13 \times 10^{-11}(118.2 - 59.6) \times 10^{13} = 2.42 \text{ volt}$$

and equation (7) now becomes

$$1.11 = 2.42 - 129 = 1.13.$$

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[1] Phys. Rev., VI., p. 55, '15.

Although the agreements in both these cases are exceedingly close the uncertainties in the long wave-length limit of the CuO amount to possibly 50 Å. so that $\nu_0 - \nu_0'$ is uncertain by as much as 3 per cent., or possibly a trifle more.

Again, in the case of the sodium I stated above that the results of the measurements gave contact E.M.F. = 2.51 volts, while $h/e(\nu_0 - \nu_0')$ came out 2.79 volts, which looked at first like fair agreement with equation (8), but the agreement is well nigh perfect when the second term of equation (7) is taken into account, for the ν_0' for this Faraday cylinder (it was a different one from that used with the lithium) corresponded to $\lambda_0 = 2685$ Å. instead of $\lambda_0 = 2535$ Å. This gives $\nu_0' = 111.8 \times 10^{13}$. For the sodium ν_0 came out accurately 43.9×10^{13}. From the relation $(d \text{ volts})/d\nu = 4.13 \times 10^{-15}$ we can compute the maximum energy of emission V_0 of corpuscles under the influence of line 2535 from a surface for which this energy is zero at $\lambda = 2685$. It is

$$4.13 \times 10^{-15}(118.2 - 111.8) \times 10^{13} = .26 \text{ volt.}$$

This is V_0'. With line 2535 the observed V_0 for the sodium was .52 volt, so that

$$(V_0 - V_0') = .52 - .26 = .26 \text{ volt.}$$

Then equation (7) becomes

$$2.51 = 2.79 - .26 = 2.53$$

which is but one per cent. in error. All of these results then, are in perfect agreement with the demands of equation (7) although they are definitely opposed to the generality of the conclusion that contact E.M.F. = $h/e(\nu_0 - \nu_0')$, which is identical with the conclusion that the observed photo-potentials of different metals are all the same when light of a given wave-length ejects corpuscles from metals into the same Faraday cylinder. Whether the differences between the alkali metals and ordinary metals in regard to equation (8) are due to surface inhomogeneities, as I stated last April, or that it might be to intrinsic properties of the metals must be determined by further experiments. Some data has already been accumulated, but its presentation will be deferred to another occasion.

Since we are here concerned primarily with the testing of Einstein's equation, the important result already attained and so far as I am aware not hitherto shown is that Einstein's equation *appears to predict accurately and generally the relations between Contact E.M.F.'s and photo-potentials.*

§ 4. Contact E.M.F.'s and Temperature.

There are some interesting relations between Einstein's photoelectric equation and the effect of temperature on contact E.M.F.'s which, so far as I am aware, have not hitherto been pointed out. In 1906 Millikan and Winchester[1] and Lienhop[2] independently established the lack of dependence upon temperature of the v of equation (1). Since ν is not dependent on temperature it follows from (1) that $p(=h\nu_0)$ also is not a function of temperature. It follows then from equation (7), since all the terms on the right were definitely shown in Millikan and Winchester's experiments to be independent of temperature, that contact E.M.F. must also be independent of temperature.

Now W. Schottky[3] has recently made measurements on contact E.M.F.'s at incandescent temperatures and obtained results which are the same, within the rather wide limits of uncertainty, as those commonly obtained at ordinary temperatures, that they are the same as at ordinary temperatures, so that, so far as experiment has now gone, Einstein's photoelectric equation, whatever may be said of its origin, seems to stand up accurately under all of the tests to which it has been subjected.

§ 5. Summary.

The tests of Einstein's photoelectric equation which I have considered and, save in the case of the last, subjected to accurate experimental verification are:

1. The existence of a definite and exactly determinable maximum energy of emission of corpuscles under the influence of a given wave-length.
2. The existence of a linear relationship between photo-potentials and the frequency of the incident light. (This has not been shown in the present paper.)
3. The exact appearance of Planck's h in the slope of the potential-frequency line. The photoelectric method is one of the most accurate available methods for fixing this constant.
4. The agreement of the long wave-length limit with the intercept of the P.D., ν line, when the latter has been displaced by the amount of the contact E.M.F.
5. Contact E.M.F.'s are accurately given by

$$\frac{h}{e}(\nu_0 - \nu_0') - (V_0 - V_0').$$

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[1] Phys. Rev., 24, p. 16, and Phil. Mag. (6), 14, 188.

[2] Ann. d. Phys. (4), 21, 284.

[3] Ann. d. Phys., 44, p. 1011, 1914. Phys. Zeit., 15, p. 624, June 15, 1914.

6. Contact E.M.F.'s are independent of temperature. This last result follows from Einstein's equation taken in conjunction with the experimentally well established fact of the independence of photo-potentials on temperature. If the surface changes in the heating so as to change the photoelectric currents, the contact E.M.F. should change also, otherwise not.

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September 15, 1915.

THE ABSORPTION SPECTRA OF THE BLUE SOLUTIONS OF SODIUM AND MAGNESIUM IN LIQUID AMMONIA.

By G. E. Gibson and W. L. Argo.

I. Introduction.

THE blue color of solutions of metals in liquid ammonia and alkyl amines has been observed and several of their physical properties have been investigated by Kraus and others. In order to account for the electrical properties of these solutions Kraus[1] assumes that in all cases the metal dissociates on dissolving according to the equation

$$Me = Me^+ + \theta,$$

where Me^+ is the cation present in solutions of salts of the metal and θ is an electron. Both Me^+ and θ may be combined to a greater or less extent with the solvent molecules. From measurements of the electromotive force of concentration cells Kraus[2] concludes that at least one third of the current in a normal solution of sodium is carried by the unsolvated electron.

The color of these metallic solutions appears to be independent of the metal and the solvent used. They are intensely blue to transmitted light, and, as far as the eye can judge, the shade of color is the same in all cases. In daylight .001 normal solutions of potassium and sodium in ammonia can be matched against ammoniacal copper solutions. This is also true of a saturated solution of magnesium in liquid ammonia.

Similar blue solutions are obtained when sodium is dissolved in molten sodamide[1] and when calcium is dissolved in a molten mixture of potassium and sodium chloride.[3]

The blue color of these metallic solutions may be due (*a*) to un-ionized molecules (or atoms) of the metal, (*b*) to solvated electrons, (*c*) to unsolvated electrons.

(*a*) We should expect the color of the un-ionized metal atoms to vary with the metal and to be similar to that of the vapor of the metal, provided the metal atoms were not affected by the presence of the solvent.

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[1] J. Am. Chem. Soc., *29*, 1557 (1907).

[2] J. Am. Chem. Soc., *36*, 864 (1914).

[3] Titherley, J. Chem. Soc., *65*, 508 (1894).

[4] A description of this experiment will be published shortly.

Sodium vapor has a purple color. The absorption spectrum of the vapor, however, has been investigated by Wood and others, and is in no way similar to that of the solution in ammonia.

If the un-ionized metal atoms combined with the solvent molecules we should expect the color of the compound to vary with the solvent.

(*b*) If the color were due to solvated electrons we should expect an absorption band differing from solvent to solvent. It is unlikely that electrons bound to molecules so different in character as ammonia and fused alkali chlorides should produce the same color.

On the whole, therefore, the experimental evidence hitherto obtained favors the assumption (*c*) that the color is due to unsolvated electrons. In this paper the latter assumption will be tested quantitatively by comparing the theoretical absorption spectrum deduced from this hypothesis with the aid of Drude's theory of metallic dispersion with the actual absorption spectrum determined by experiment.

II. THE RELATION BETWEEN EXTINCTION COEFFICIENT AND WAVE LENGTH ACCORDING TO DRUDE'S THEORY.

Let us make the same assumptions for the unsolvated electron in the blue solutions that Drude[1] employs in his electron theory of metallic dispersion and conduction, *i. e.*, that the resistance to the motion of the electron through the medium is proportional to its velocity.

The equation of motion of an unsolvated electron is, therefore,

$$ma = eE - re^2v. \tag{1}$$

The term on the left is the product of the mass m of the electron and its acceleration a. E is the intensity of the electric field, e the charge on the electron, and r is such a quantity that re^2 is the resistance to the motion of an electron moving with unit velocity through the medium. Electrostatic units will be used throughout in this paper.

Equation (1) combined with the equations of the electromagnetic field, leads to the equations

$$\begin{aligned} n^2(1 - \kappa^2) &= 1 + \Sigma \frac{\theta_h}{1 - (\tau_h/\tau)^2} - 4\pi \frac{m'\mathfrak{N}}{r^2 + (m'/\tau)^2}, \\ n^2\kappa &= 2\pi\tau \frac{r\mathfrak{N}}{r^2 + (m'/\tau)^2}, \end{aligned} \tag{2}$$

where n is the refractive index of the solution. The index of absorption κ is defined by the equation

$$J = J_0 e^{-\frac{4\pi\kappa}{\lambda} d} \tag{3}$$

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[1] Lehrbuch d. Optik, Leipzig, 1912, p. 386.

where J_0 is the intensity of the incident ray of wave-length λ and J is the intensity after a layer of thickness d has been traversed. The symbol m' is the ratio of the mass of the electron to the square of its charge ($m' = m/e^2$). $\mathfrak{N}$ is the number of unsolvated electrons in a cubic centimeter of the solution. The radian time τ is equal to the periodic time divided by 2π. θ_h and τ_h are constants of the bound electrons.

Equations (2) are the general equations of Drude for an absorbing medium containing both bound and unbound electrons. The term under the sign of summation in the first of equations (2) represents the influence of the bound electrons.

For a colorless solvent both κ and τ_h/τ are very small in the visible spectrum. The refractive index of the pure solvent, therefore, varies very slightly with the wave-length, and will be assumed to be constant in the following. In terms of the symbols of equations (2) its value would be given by

$$n_0^2 = 1 + \Sigma\theta_h.^1 \tag{4}$$

Neglecting τ_h/τ in equations (2) and eliminating n we obtain

$$\frac{1-\kappa^2}{\kappa} = \left(\frac{n_0^2 r}{2\pi\mathfrak{N}} - \frac{2m'}{r}\right)\frac{1}{\tau} + \frac{n_0^2 m'^2}{2\pi\mathfrak{N}r} \cdot \frac{1}{\tau^3}. \tag{5}$$

In terms of the wave-length $\lambda = 2\pi c\tau$ and the extinction coefficient $\epsilon = (4\pi\kappa \log_{10} e)/\lambda$ this becomes

$$\frac{a^2}{\epsilon} - \epsilon\lambda^2 = 2\pi ac\left(\frac{n_0^2 r}{2\pi\mathfrak{N}} - \frac{2m'}{r}\right) + \frac{4\pi^2 ac^3 n_0^2 m'^2}{\mathfrak{N}r} \cdot \frac{1}{\lambda^2}, \tag{5a}$$

where a is a contraction for $4\pi \log_{10} e$.

Except for very large values of ϵ (such as occur in pure metals) the term $\epsilon\lambda^2$ may be neglected in the visible spectrum, and (5a) may therefore be written

$$\frac{a^2}{\epsilon} = b_1 + \frac{b_2}{\lambda^2}, \tag{6}$$

where b_1 and b_2 are independent of the wave-length, *i. e.*,

$$\begin{aligned} b_1 &= 2\pi ac\left(\frac{n_0^2 r}{2\pi\mathfrak{N}} - \frac{2m'}{r}\right), \\ b_2 &= \frac{4\pi^2 ac^3 n_0^2 m'^2}{\mathfrak{N}r}. \end{aligned} \tag{7}$$

Equation (6) expresses the theoretical relationship between the extinction

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[1] The refractive index of liquid ammonia n_0 has not been determined. In the calculations n_0^2 was taken to be equal to 2 as the refractive index of liquid ammonia cannot be far from that of water.

coefficient ϵ and the wave-length λ in the visible spectrum. According to (6) a^2/ϵ is a linear function of $1/\lambda^2$.

If $2m'/r$ is negligible compared with $(n_0^2 r)/2\pi\mathfrak{N}$ and if λ is small enough to make $\epsilon\lambda^2$ negligible compared with a^2/ϵ the extinction coefficient ϵ for a given wave length will be proportional to the concentration $\mathfrak{N}$ of the unsolvated electrons. These are therefore necessary conditions that Beer's law shall hold.

For sufficiently large wave-lengths we see from (5a) that

$$\frac{a^2}{\epsilon} - \epsilon\lambda^2 = 0,$$

or

$$\epsilon = \frac{a}{\lambda}.$$

In other words ϵ is independent of the concentration when λ is large. Hence Beer's law must cease to hold when the wave-length exceeds a certain amount determined by the values of b_1 and b_2. The extinction coefficient vanishes for infinite wave-lengths in all cases.

For very short wave-lengths (5a) reduces to

$$\frac{a^2}{\epsilon} = \frac{4\pi^2 a c^3 m''^2 n_0^2}{r\mathfrak{N}} \cdot \frac{1}{\lambda^2},$$

so that in this case ($b_1 = 0$) Beer's law is always obeyed.

If b_1 is negative we must have $2m'/r > (n_0^2 r)/2\pi\mathfrak{N}$ and Beer's law cannot hold, since b_1 is no longer inversely proportional to $\mathfrak{N}$.

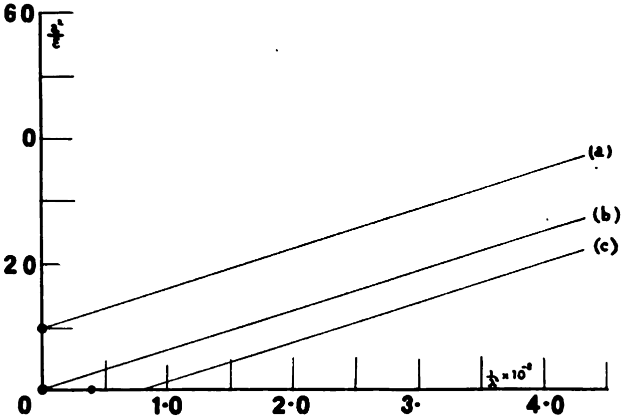

Fig. 1.

The graphs of a^2/ϵ against $1/\lambda^2$ as they follow from the general equation (5a) are drawn in Fig. 1. There are three types of curve, a, b, and c, according as b_1 is positive, zero, or negative. The equation of the asymptote in all cases is equation (6) viz.,

$$\frac{a^2}{\epsilon} = b_1 + \frac{b_2}{\lambda^2}.$$

On the scale of Fig. 1 the remainders of the three curves *a*, *b*, *c* are indistinguishable from the axes. The minima of a^2/ϵ are marked with circles.

A more elaborate theory of metallic dispersion has recently been developed by Jaffé[1] in which the frictional resistance r is assumed to be due to collisions between the free electrons and the atoms of the medium. In place of Drude's equations (2) his theory leads in our case to the equations

$$\begin{aligned} n^2(1 - \kappa^2) &= n_0^2 + \frac{2\sqrt{\pi^3}\sigma}{\nu}\psi_1, \\ n^2\kappa &= \frac{2\pi\sigma}{\nu}\psi_2, \end{aligned} \tag{8}$$

where ψ_1 and ψ_2 are functions of the argument $z = (3\sqrt{\pi}/4)(m'/r)\nu$. For large values of z we obtain from Jaffé's theory

$$\psi_1 = -\frac{3}{2}\cdot\frac{1}{z}$$

$$\psi_2 = \frac{2}{z^2}.$$

Substituting these values of ψ_1 and ψ_2 in (8), and eliminating n, we obtain in place of equation (5*a*)

$$\frac{a^2}{\epsilon} = -\frac{9\pi^2}{8}\frac{am'c}{r} + \frac{9\pi^3}{8}\frac{n_0^2 am'^2 c^3}{\mathfrak{N} r}\cdot\frac{1}{\lambda^2}.$$

When $(n_0^2 r)/2\pi r$ is negligible this equation is identical in form with (5*a*) and differs only slightly in the values of the constants. Jaffé's theory therefore leads to the same conclusions as the theory of Drude.

III. THE THEORIES OF DRUDE AND JAFFÉ IN VIEW OF THE EXPERIMENTAL DETERMINATIONS.

With a view to testing the hypothesis that the blue color is due to unsolvated electrons, the absorption spectra of solutions of sodium and magnesium in liquid ammonia were determined in the visible spectrum. The results of these experiments will be discussed now and compared with the requirements of the theories of Drude and Jaffé while the details of the experiments will be found in Section IV.

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[1] Ann. d. Physik [4], 45, 1217, 1914.

In Fig. 2 the values of a^2/ϵ calculated from our determinations (See col. IX. and X., Table II., and col. XI. and XIII., Table VII.) are plotted against $1/\lambda^2$. There are three curves for sodium at different concentrations, Na(1), Na(2) and Na(3) (circles), and one for a nearly saturated solution of magnesium (triangles).

The concentration of sodium is 2.8×10^{-3} normal in series (1) and 1.3×10^{-3} normal in series (2). The concentration in series (3) is less than 1.3×10^{-3} normal, but was not determined exactly. No attempt was made to determine the concentration of the magnesium solution.

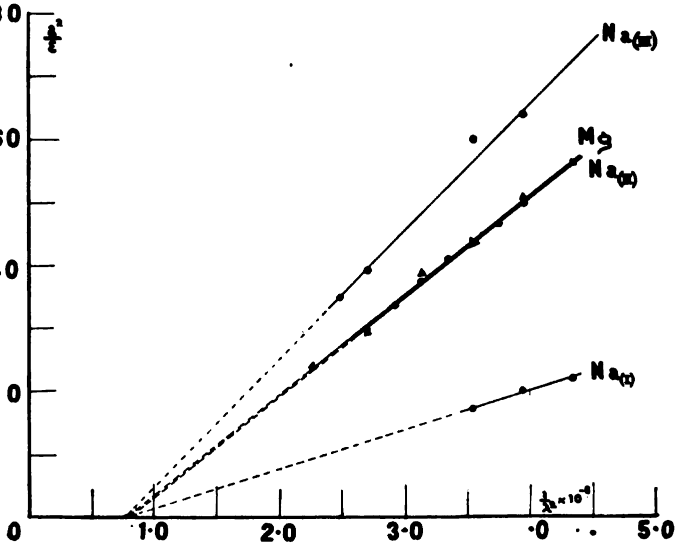

Fig. 2.

The curve for magnesium (Fig. 2) coincides, within the limits of experimental error, with one of the sodium curves. *This is strong evidence in favor of the assumption that the coloring principle is the same for both metals.*

Within the limits of error the graphs are all straight lines and intersect at a common point on the axis of abscissæ. In other words, the ratio of the ordinates of the various curves at a given wave-length is independent of the wave-length. While this does not prove that Beer's law is obeyed, it is one of the necessary conditions for the proportionality of the extinction coefficient and the concentration of the coloring molecules.

If Beer's law were true for the total dissolved sodium, the concentration N of sodium multiplied by the value of a^2/ϵ should be constant for all wave-lengths. This is approximately confirmed by our experiments. At wave-length $\lambda = 502\mu\mu$ we have $(a^2/\epsilon)N = 0.56$ for series (1) and

$(a^2/\epsilon)N = 0.65$ for series (2). The difference between these two values can be accounted for by the loss of sodium due to the amide reaction in the time which elapsed between the two measurements (see Section IV., *d*). Beer's law therefore holds approximately for the total dissolved sodium.

The equation (5 a) obtained from Drude's theory in Section II. may be tested now in the light of these measurements.

The values of b_1 and b_2 obtained by the method of least squares from the experimental data are given in columns II. and III. of Table I.

TABLE I.

I. Series.	II. b_1(Exp.).	III. $b_2 \times 10^8$ (Exp.).	IV. $\mathfrak{N} \times 10^{-20}$ (Calc.).	V. r (Calc.).	VI N(Mols per Liter) (Calc.).	VII. N(Exp.) (Mols. per Liter).	VIII. $K \times 10^{-5}$. (Calc.).	IX. $K \times 10^3$ (Kraus).
Na(1)	− 4.57	6.18	1.672	1801	0.271	0.00281	1.03	2.92
Na(2)	−12.83	15.97	1.715	641	0.295	0.00130	3.14	1.35
Na(3)	−15.38	20.28	1.816	535	0.278		3.56	
Mg	−12.21	15.88	1.738	674	0.281		2.86	

Columns IV. and V. contain the values of the number of electrons $\mathfrak{N}$ per cubic centimeter and the frictional resistance constant r as they would follow from Drude's theory. Column VI. contains the concentration of free electrons in mols per liter calculated from the values of $\mathfrak{N}$ in column IV. Column VII. contains the concentration of sodium determined by experiment. Column VIII. contains the values of the conductivity K (ohm^{-1}cm.$^{-1}$) calculated from the values of $\mathfrak{N}$ and r in columns IV. and V., and column IX. the specific conductivity derived from the experiments of Kraus.[1]

Drude's theory, therefore, gives values for the conductivity which are about thirty million times too large. Thus the specific conductivity K of a .0028 normal solution of sodium is $2.92 \times 10^{-3}\omega^{-1}$cm.$^{-1}$ while the conductivity calculated from Drude's theory for the same solution is $1.03 \times 10^5\omega^{-1}$cm.$^{-1}$ (see Table I.).

Drude's theory also leads us to conclude that Beer's law would not be even approximately true for the free electrons. The calculated concentrations of free electron are very great and independent of the concentration of sodium. For each sodium atom in a .001 normal solution Drude's theory would require about one thousand electrons, and we should have to ascribe the increased depth of color on adding more sodium to an increased frictional resistance to the motion of the electrons without any increase in their number.

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[1] J. Am. Chem. Soc., *36*, 877 (1914).

If, therefore, we grant the validity of Drude's theory we are forced to conclude that the blue color of these solutions is not wholly due to unsolvated electrons.

It is important to determine the effect on the coloring principle of changing the metal and the solvent. With this end in view we propose to continue our measurements with other metals such as potassium and lithium and with other solvents such as methylamine, ethylamine, hydrazine and possibly with fused salts.

IV (*a*). The Spectrophotometer.

The absorption spectrum was measured by means of a spectrophotometer of the type described by Grünbaum and Martens.[1] A source of light *A* (Fig. 3) illuminates the ground-glass window *B*. The light from *B* passes through two lenses *C*, which produce an image of *B* on the slits *E*, *E'*. The two beams pass respectively through a cell *R* containing the solution to be examined, and a cell *L* containing water which is used to compensate for the loss of light due to reflection from the windows of the cell. After passing through the slits *E*, *E'* the two beams traverse successively the collimator *F*, the prism *G*, the polarizer *P* and the analyzer *K*. The analyzer is rotated until the fields illuminated by the two beams appear of equal intensity.

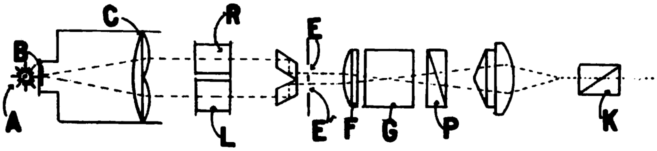

Fig. 3.

The calculation of the extinction coefficient from the angle of rotation is discussed in section IV (*d*).

The source of illumination is an acetylene flame, one inch wide and shielded by a metal chimney three inches in diameter and about two feet high. The flame, when so arranged, burns steadily and illuminates the ground glass more uniformly than a tungsten filament lamp.

The arbitrary wave-length scale of the spectrophotometer had already been calibrated by Adams and Rosenstein[2] in this laboratory. The calibration was repeated for the sodium *D* lines and agreed exactly with their determination.

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[1] Ann. der Phys., *12*, 984 (1903).

[2] J. Am. Chem. Soc., *36*, 1452 (1914).

IV (*b*). The Absorption Cell.

The ordinary metal absorption cell with plane glass windows cannot be used for solutions of metals in liquid ammonia. In the first instance, the cell must be perfectly airtight, as the least trace of moisture is sufficient to destroy the blue color. It must also be capable of withstanding a pressure of eight to ten atmospheres, which is the vapor pressure of liquid ammonia at room temperatures. The first cells used were made of copper with glass windows pressed tightly against a thin flange of the metal. Although it was possible to obtain a vacuum-tight joint in this way, the cells were difficult to clean, and slight traces of impurity, such as solder or brass, catalyzed the amide reaction and made it difficult to obtain a permanent blue color.

The cell finally adopted was made entirely of quartz. It is shown in completed form in Fig. 4. The two plates *A*, *A*, one centimeter in diameter, were made of optical quartz free from bubbles. They were ground plane parallel and were fused into the quartz tube *H*, by means of the oxygen gas flame, in the following manner:

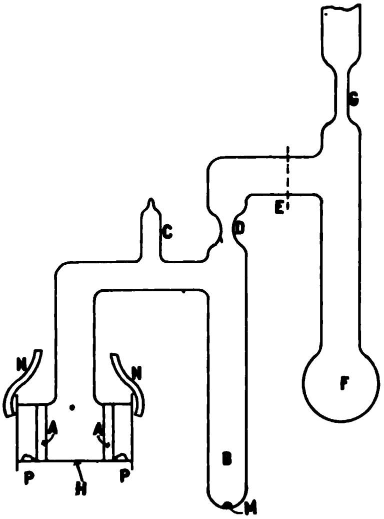

Fig. 4.

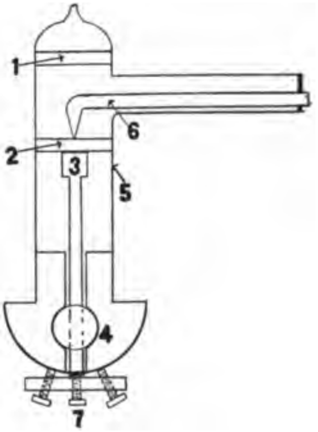

Fig. 5.

The window 1, Fig. 5, was first sealed into the quartz test tube 5 in a position as nearly perpendicular to the axis as possible. The window 2 was then adjusted parallel to 1 by means of the appliances shown. A cylindrical block of quartz 3 was ground plane on its upper surface and connected by a quartz rod with the universal joint 4. By manipulating the three screws, 7, the upper surface of the block could be adjusted to a position making any desired angle with the axis of the tube. The window

2 was kept in close contact with the block, 3, by means of a quartz pointer, 6, which was depressed by a glass spring not shown in the figure. The window 2 was then adjusted until the image of a point source of light reflected from its surface coincided exactly with the image of the same source reflected from the window, 1. The source, which consisted of a small loop of incandescent platinum wire, was placed in the line of the normal to the two windows at a distance of about 150 cm. from the cell. The reflected images were observed through the incandescent loop. The two windows could be adjusted in this way to within 10′ of arc. Window 2 was then sealed into position by means of the oxygen gas flame.[1]

The two ends of the quartz tube were then cut off, so as to leave an edge of about 3 mm. beyond the windows. The completed cell is shown in Fig. 4.

IV (*c*). THE METHOD OF FILLING THE CELL.

The system *ABCD* in Fig. 4 was made entirely of quartz. Beyond the Khotinsky cement seal *E* glass was used throughout. The whole system was first evacuated and left in contact with phosphorus pentoxide for several hours. After the apparatus had been washed out several times with dry ammonia gas a quantity of ammonia sufficient to fill the cell was distilled into the bulb *F* by immersing it in alcohol cooled to −60° C. The bulb *F* contained a small piece of sodium which dissolved in the ammonia and removed the last traces of moisture. The constriction *G* was then sealed and the ammonia in *F* distilled into the tube *B*, which contained a portion *M* of the metal to be examined. After the cell had been sealed off at the constriction *D* by means of the oxygen-gas flame it was ready for the measurements.

IV (*d*). MEASUREMENTS WITH SODIUM IN LIQUID AMMONIA AT —33.5° C.

The quartz cell was surrounded with a copper jacket *A* (Fig. 6), which could be filled with boiling liquid ammonia through the tube *B*. The ammonia vapor was led out through the tube *C* to a large flask filled with water where it was absorbed. Copper filings were placed between the quartz cell and the copper jacket in order to secure good conduction of heat. The jacket was then fitted into a wooden cylinder *D* which served the double purpose of a heat insulator and a support for the cell in the spectrophotometer.

To prevent the deposition of moisture on the cold quartz windows, cover glasses were cemented with Khotinsky cement to the projecting edges of the cell, in the manner shown in Fig. 4. The air chambers were dried by small pellets of partly deliquesced phosphorus pentoxide, *p*, *p*. Jets of dry air were directed against the outer surfaces of the cover glasses by means of the tubes *N*, *N*. When the air jets were adjusted correctly they prevented entirely the deposition of moisture.

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[1] It is difficult to avoid entirely the sublimation of silica on the windows in the process of sealing them into the tube. The slight irregularities resulting from this became invisible on the inner surfaces when the cell was filled with liquid ammonia. The fog on the outside of the windows was made to disappear by cementing thin cover glasses to them with Canada balsam.

After the cell had been dried by evacuation, pieces of capillary tube of known length and diameter, full of sodium, were introduced through *C* (Fig. 4). The cell was then filled as described in paragraph IV (*c*). The amount of sodium introduced into the cell was more than sufficient for the highest concentration desired. After mixing thoroughly the position of the meniscus of the solution in the tube *B* was measured. A portion of the solution was then transferred to the cell *H*, and the position of the meniscus in *B* was again measured. By cooling the cell the remainder of the solvent was distilled from *B* to *H*. This process was repeated until the desired dilution was obtained. The volumes corresponding to the various positions of the meniscus in *B* were determined after the tube had been opened at the end of the experiment.

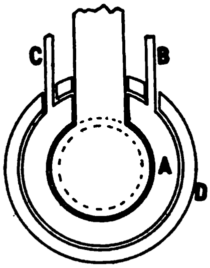

Fig. 6.

The method of reading the spectrophotometer and the method of averaging the readings were essentially the same as those of Grünbaum and Martens.[1] Grünbaum and Martens compare the cell containing the colored solution with a cell of the same dimensions containing the pure solvent. This method would have necessitated the construction of a second quartz cell, and a duplication of the cooling apparatus. To avoid this, the colored solution and the pure solvent were compared in the same cell at different times with a glass cell containing water. The water cell was used to compensate approximately for the reflections from the windows of the quartz cell. Under these conditions the sensibility of the readings is improved.

A complete determination at any wave-length involves the determination of four angles:

1. The angle α_1, with the quartz cell containing liquid ammonia in position *R* (Fig. 3) and the glass water cell in position *L*.

2. The angle α_2, with the quartz cell containing liquid ammonia in position *L* and the glass water cell in position *R*.

3. The angle α_3, with the quartz cell containing sodium solution in position *R* and the glass water cell in position *L*.

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[1] Loc. cit.

4. The angle α_4, with the quartz cell containing sodium solution in position L and the glass water cell in position R.

The glass water cell might be replaced by an imaginary quartz cell excactly similar to the cell containing the ammonia solution but filled with a hypothetical liquid of extinction coefficient such that the fraction of the incident light absorbed by the hypothetical cell would be exactly equal to the fraction absorbed by the actual glass water cell. The difference between the extinction coefficient ϵ_{NH_3} of the pure liquid ammonia and the hypothetical extinction coefficient ϵ_h is then given by the relation

$$\epsilon_{NH_3} - \epsilon_h = \frac{\log \tan \alpha_2 - \log \tan \alpha_1}{d},$$

where d is the length of the column of liquid traversed by the light in the quartz cell.

The difference between the extinction coefficient ϵ_{Na} of the blue solution and ϵ_h is

$$\epsilon_{Na} - \epsilon_h = \frac{\log \tan \alpha_4 - \log \tan \alpha_3}{d}.$$

Hence the required difference between the extinction coefficients of the blue solution and the pure solvent (liquid ammonia) is

$$\epsilon_{Na} - \epsilon_{NH_3} = \frac{\log \tan \alpha_4 - \log \tan \alpha_3 - \log \tan \alpha_2 + \log \tan \alpha_1}{d}.$$

Even at − 33.5° C. the fading of the blue color due to the amide reaction is sufficiently rapid to necessitate correction. The angles α_3 and α_4 for each wave-length were therefore determined at measured intervals and plotted against the time of reading. The values of α_3 and α_4 at the time of the first reading of the series were obtained by graphical extrapolation. The largest correction applied to any reading amounted approximately to 4 per cent. of the extinction coefficient. The error in the graphical extrapolation is certainly not greater than 25 per cent. of this correction, so that the resultant error in the extinction coefficient due to this cause cannot exceed 1 per cent. This maximum error applies only to reading (6), series III., in the table below. The uncertainty in the fading correction is much less for all other readings (see Column VIII., Table II.).

IV (e). MEASUREMENTS WITH MAGNESIUM.

Cottrell[1] had observed that solutions of magnesium in ammonia were extraordinarily stable even at room temperature. Magnesium was

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[1] J. Phys. Chem., *18*, 85 (1914).

TABLE II.

Temperature = − 33·5° C. Length of Column d = 1.523 cm.

	I. Reading No.	II. Wave-Length $\mu\mu$.	III. $\epsilon_{NH_3}-\epsilon_h$ (See Table IV.).	IV. Reading a_3.	V. Time of Reading (Minutes).	VI. Reading a_4.	VII. Time of Reading (Minutes).	VIII. Corrected Reading. a_3	a_4	IX. $\epsilon_{Na}-\epsilon_{NH_3}$	X. $\frac{a^2}{\epsilon}$	XI. $1/\lambda^2 \times 10^{-8}$ (cm^{-2})
Series (1)......	(1)	529	.116	87.09°	0	1.87°	6	87.1°	1.75°	1.73	17.2	3.54
Conc. of Na =	(2)	502	.123	85.13	14	2.81	18	85.4	2.5	1.48	20.0	3.94
2.8 × 10^{-6} mols	(3)	480	.133	83.23	36	3.58	30	84.0	3.0	1.35	22.0	4.33
per cc.	(4)	529		86.14	44	2.58	47					
Series (2)......	(1)	608	.098	79.48	0	6.84	3	79.7	6.7	0.99	29.8	2.70
Conc. of Na =	(2)	585	.107	76.88	8	8.32	5	77.4	8.0	.880	33.8	2.92
1.3 × 10^{-6} mols	(3)	564	.112	74.62	9	9.69	11	75.3	9.0	.745	37.5	3.13
per cc.	(4)	546	.114	72.66	15	10.87	13	73.7	10.1	.728	40.9	3.34
	(5)	529	.116	70.76	18	12.00	20	72.0	10.6	.682	43.6	3.54
	(6)	515	.119	69.38	25	12.87	22	71.0	11.6	.636	46.7	3.74
	(7)	502	.123	68.32	27	14.00	28	70.1	12.5	.596	49.9	3.94
	(8)	608		77.54	34	7.93	30					
	(9)	608		77.37	41	8.08	44					
	(10)	529		69.17	48	12.99	46					
	(11)	608		76.74	50	9.30	53					
	(12)	529		68.34	58	13.34	56					
	(13)	608		75.22	64	9.09	67					
	(14)	529		67.04	72	13.92	69					
	(15)	608		73.84	88	11.92	91					
	(16)	529		65.75	98	15.63	94					
Series (3)......	(1)	608	.098	74.31	0	10.18	3	74.3	10.2	.753	39.5	2.70
	(2)	635	.084	75.60	13	9.34	7	76.6	8.7	.860	34.9	2.48
	(3)	608	.098	72.83	16	11.45	18					
	(4)	529	.116	65.13	25	16.09	22	66.6	15.1	.496	60.0	3.54
	(5)	502	.123	63.25	32	17.09	39	65.0	15.3	.464	64.1	3.94
	(6)	480	.133	59.28	59	18.13	48	62.0	15.7	.409	72.8	4.33
	(7)	608		68.98	63	13.88	68					
	(8)	608		65.90	102	15.96	99					
	(9)	564		62.24	106	18.65	109					
	(10)	529		59.64	117	20.59	113					
	(11)	502		58.20	121	20.71	127					
	(12)	480		55.31	140	21.08	132					
	(13)	608		62.07	150	19.33	154					
	(14)	564		58.68	159	21.35	158					
	(15)	529		57.00	162	22.44	163					
	(16)	502		55.70	168	23.03	165					
	(17)	491		54.55	171	23.52	177					
	(18)	608		59.70	183	20.85	181					

therefore chosen for our earlier measurements in order to avoid the difficulties involved in cooling the cell. The vapor pressure of liquid ammonia is about eight atmospheres at room temperature, and two cells exploded under this pressure before a measurement was obtained. The strength of the cell was then increased by diminishing its size considerably, but the apparatus with which the determination was finally obtained exploded just as the series of measurements had been completed. The cell, however, was intact, and the explosion may have been caused by the pressure of a metal clamp which was used to mount the apparatus in the spectrophotometer.

As the construction of the quartz cells is somewhat troublesome, our subsequent experiments with sodium were performed at the boiling point of ammonia as described in paragraph IV (*d*), in order to avoid the risk of explosion.

Except that the cell was not cooled, the method of performing the measurements was the same for magnesium as for sodium. The magnesium was introduced in the form of freshly cut turnings. It was found necessary to leave the magnesium in contact with the liquid ammonia for at least a week before the blue color appeared. This delay in the appearance of the color is probably due to the formation of a protecting layer of hydroxide on the surface of the metal.

The fading during the course of the reading was very small. In 100 minutes ϵ for wave-length 6,080 changed from 0.942 to 0.914. This amounted to 0.03 per cent. per minute. The values for the fading correction given in Column IX., Table VII., were calculated from this value.

The explosion of the cell prevented the direct determination of the value $\epsilon_{NH_3} - \epsilon_h$. To obtain this value the following measurements were made:

(1) The "magnesium" cell filled with water compared with the glass water cell, $\epsilon_w - \epsilon_h$ (Table III.).

TABLE III.

Water in Mg Cell—Glass Water Cell.

λ $\mu\mu$	Cell R	Cell L	$\epsilon_w - \epsilon_h$
664	39.53	36.54	.0306
608	29.29	35.81	.0358
564	39.09	35.23	.0401
529			.0494
502	38.41	33.70	.0553
480			.0600

TABLE V.

Water in Na Cell—Glass Water Cell

λ $\mu\mu$	Cell R	Cell L	$\epsilon_w' - \epsilon_h$
664	45.72	32.83	.131
608	44.68	32.68	
564	45.02	33.23	
529	45.66	32.34	
502	46.09	32.16	

(2) The "sodium" cell filled with NH_3 compared with the glass water cell, $\epsilon_{NH_3} - \epsilon_h$ (Table IV.).

(3) The "sodium" cell filled with water compared with the glass water cell, $\epsilon_w' - \epsilon_h$ (Table V.).

By subtracting (3) from (2) we obtained $\epsilon_{NH_3} - \epsilon_w$. Adding to this the value (1), $\epsilon_w - \epsilon_h$, we obtained the required value $\epsilon_{NH_3} - \epsilon_h$ (Table VI.).

TABLE IV.

NH_3 in Na Cell—Glass Water Cell.

λ $\mu\mu$	Cell *R*	Cell *L*	$\epsilon_{NH_3} - \epsilon_h$
664			.072
635			.084
608	43.37	33.80	.098
585			.107
564	44.03	33.15	.112
546			.114
529	44.27	32.97	.116
515			.119
502	44.35	32.44	.123
491			.128
480			.133

TABLE VI.

NH_3 in Mg Cell—Glass Water Cell (calc.).

λ $\mu\mu$	$\epsilon_{NH_3} - \epsilon_h$
664	−.028
608	+.003
564	+.021
529	+.034
502	+.047
480	+.062

TABLE VII.

Magnesium at Room Temperature (about 16° C.).

Exp. No.	λ $\mu\mu$	Reading.		$\epsilon_{Mg} - \epsilon_h$	$\epsilon_{NH_3} - \epsilon_h$	$\epsilon_{Mg} - \epsilon_{NH_3}$	Time in Min.	Per Cent Correction.	$\epsilon_{Mg} - \epsilon_{NH_3}$ (Corrected).	a^2/ϵ	$\frac{1}{\lambda^2} \times 10^{-8}$ (Cm.$^{-2}$).
		a_3	a_4								
1	664	80.50	4.90	1.212	−.028	1.240	0	0	1.241	24.0	2.26
2	608	74.70	7.65	0.942	+.003	0.940	23	0.8	1.014	29.4	2.70
3	564	70.06	10.21	0.779	+.021	0.758	43	1.3	0.769	38.8	3.13
4	529	66.90	11.18	0.705	+.034	0.671	57	1.7	0.684	43.6	3.54
5	502	64.35	13.25	0.622	+.047	0.575	68	2.0	0.587	50.8	3.94
6	480	62.04	13.92	0.579	+.062	0.517	84	2.5	0.529	56.3	4.33
7	608	74.87	8.51				100				

The difference between the extinction coefficients of the blue magnesium solution and of liquid ammonia was then obtained by subtracting the value $\epsilon_{NH_3} - \epsilon_h$ from the value $\epsilon_{Mg} - \epsilon_h$. The measurements made with magnesium are summarized in Table VII., and are plotted in Fig. 2.

SUMMARY.

1. Kraus has shown that the electrical properties of solutions of metals in liquid ammonia can be explained on the assumption that the atoms of

the metal dissociate into electrons partly combined with the solvent and into the cations present in solutions of its salts in ammonia.

In order to test the assumption that the blue color of these solutions is due to the unsolvated portion of the electrons resulting from this dissociation, the absorption spectra of dilute solutions of sodium and magnesium in liquid ammonia were determined in the visible spectrum.

2. A solution of magnesium and a solution of sodium of the same intensity of color were found to have the same absorption spectrum. This is strong evidence in favor of the assumption that the coloring principle is the same in both cases.

3. Approximate measurements of the concentrations of the sodium solutions make it probable that Beer's law holds for the total dissolved sodium.

4. Qualitatively the evidence is in favor of the assumption that the absorption is due to the unsolvated electrons, but our measurements cannot be interpreted quantitatively in terms of Drude's theory of metallic dispersion.

CHEMICAL LABORATORY OF THE UNIVERSITY OF CALIFORNIA,
July 15, 1915.

THE FREE IONS IN THE ATMOSPHERE OF THE TROPICS.

By J. R. Wright and O. F. Smith.

A STUDY of atmospheric electricity involves, among its several phases, the determination of the number of free ions per unit volume in the earth's atmosphere together with the variation of this number due to altitude, location, and meteorological conditions.

The first attempt to measure the ionization of the free atmosphere was made by Elster and Geitel[1] while investigating the causes of the discharge of electrified bodies surrounded by air in closed vessels. They found that when a piece of wire gauze, connected to an electroscope, was exposed to the open air both positive and negative electricity was discharged at a much more rapid rate than when placed in a closed vessel, also that the rate of discharge of positive and negative was generally different. The rate of discharge was taken as a measure of the ionization, but, since by this method they had no means of determining the volume of the air tested, quantitative measurements were impossible and only comparative values were obtained.

A short time later an instrument was devised by Ebert[2] which made the determination of the number of positive or negative ions present per cubic centimeter in the atmosphere comparatively easy. With this apparatus determinations have been made at various points on the earths' surface by different observers, and although the individual results vary greatly the general conclusions that have been drawn are fairly uniform.

During the past two years we have been investigating atmospheric-electric phenomena at Manila and vicinity and as a part of that work have made a large number of determinations of the free ions present. These observations were taken by an improved form of Ebert's apparatus made by Gunther and Tegetmeyer of Braunschweig, Germany. It has a vertical rather than a horizontal collecting cylinder, and a Wulf[3] electrometer in the place of the Elster and Geitel electroscope. The theory of the instrument is well known, but a brief discussion seems of value here for the sake of completeness. A certain volume of air, measured by an air meter, is drawn between concentric cylinders by means of an air turbine which is operated by clock work. The inner cylinder, which is carefully insulated, is charged to a known potential and connected directly to the quartz fibers in the Wulf electrometer. The voltage discharge is therefore easily measured. Knowing the capacity of the instrument, the number of ions per cubic centimeter is given by the equation $N = CE/eV$, where N represents the number of ions per cubic centimeter, V the volume of air drawn through, C the capacity of the instrument, E the fall in potential (corrected for the natural leak), and e the ionic charge.

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[1] Phys. Zeit. (1900), 2, 116; (1901), 2, 560.

[2] Phys. Zeit. (1901), 2, 662.

[3] Phys. Zeit. (1909), 10, 152.

The capacity of our apparatus was 16.25 electrostatic units, the free area between the concentric cylinders being 6.87 square centimeters, and the inner rod 40 centimeters long. Since the rate of flow of air was on the average 1.25 liters per second, the time required for an ion to pass through the instrument was approximately 0.22 second. The range of voltage used was from 175 to 165, thus all ions having a mobility greater than 0.033 centimeter per second per volt centimeter were caught by the apparatus.

Through the work of Langevin,[1] Mache and von Schweidler,[2] Pollock,[3] and others, the atmospheric ions have been classified according to their mobilities into the three following types:

Type of Ions.	Mean Value of Their Mobility.
Small	1.02 to 1.8
Intermediate	0.1 to 0.006
Large	0.0003 to 0.0008

It is evident, therefore, that under the conditions of the experiment, the instrument should record all the small ions and a portion of the intermediate ions. However, since we were always careful to charge as nearly as possible to the same potential, the proportion of intermediate ions caught ought to be nearly constant.

The limited time at our disposal for the present work, and the large number of factors that may influence the ionization of the atmosphere, made an exhaustive study impossible. Consequently, the work was confined to the following points:

1. The effect on the ionization of the character of the light in which the determinations are made.
2. The determination of the number of positive and negative ions per cubic centimeter in the atmosphere and the variation of this number with altitude, location, and relative humidity.
3. The ratio of the positive to the negative ions.

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[1] Comptes Rendus (1905), 140, 232.

[2] Phys. Zeit. (1905), 6, 71.

[3] Science (June, 1909).

THE EFFECT ON THE IONIZATION OF THE CHARACTER OF THE LIGHT IN WHICH THE DETERMINATIONS ARE MADE.

The effect of tropical sunlight on the atmosphere is an unsettled question and has been the subject of several articles appearing in the Philippine Journal of Science. In an article by R. F. Bacon[1] two conclusions are drawn, neither of which seems to be substantiated by the data quoted. The author first states that tropical sunlight ionizes the air, and concludes that the ionization of the atmosphere in the tropics is greater than in temperate or colder regions. To support these claims, observations were taken on the rate of discharge of an electroscope when air was drawn through it at a known rate, first in the dark, then in the diffuse light of the laboratory, and finally in the bright sunlight.

The results recorded are very consistent and show a mean rate of discharge of 6.5 volts per hour for darkness, 15.6 volts for diffuse light, and 35.7 volts for sunlight. However, these results can scarcely be taken as a measure of the ionization produced by the tropical sun, and it is impossible to explain just why such consistent data were obtained, since no diagram or description of the apparatus is given. It was also impossible to learn whether the observations in the darkness or the diffuse light were made on the confined air of the laboratory or otherwise.

We have also been unable to decide how Bacon arrived at the conclusion that the ionization is greater in the tropics than in temperate regions. His results are expressed in volts per hour, no attempt being made to reduce the readings to ions per cubic centimeter. Moreover, since the electrical capacity of his testing apparatus is not given, sufficient data is not at hand to permit the reader to make the necessary calculations. A mere statement of the leak per hour expressed in volts, is certainly insufficient to permit a comparison with readings taken with different instruments and under different conditions.

From the nature of the experiments it seems that what was really attempted was to measure the effect on the ionization of the character of the light in which the determinations were made. The author himself seems to be in some doubt for he says:

"However, I incline to the view that the ionization observed may not be a direct effect of the tropical sunlight on the air, but may be a secondary phenomenon connected with the large amount of moisture in the air of tropical islands and with the luxuriant vegetation of this region."

This view seems to carry with it the idea that the ionization would be the same, both on the outside and inside of a building, unless the air in the latter case was stagnant or contaminated with some ionizing agent.

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[1] Phil. Jour. Science, Sec. A (1910), 5, 267.

Further on in discussing the work of Elster and Geitel,[1] the author states in substance:

"It is a curious coincidence, if nothing more, that days when the sky is about half overcast are days . . . when the activity of the sun is greatest . . . on the ionization of the air." This statement seems entirely contrary to the conclusion drawn directly from his data.

In view of this article it was deemed advisable to make some determinations to ascertain whether or not the ionization readings are a function of the character of the light in which the apparatus is placed. Consequently observations were made with the Ebert's ion-counter, in the dark, in the diffuse light of the laboratory, and in the bright sunlight.

For the inside readings at Manila, the instrument was always placed by an open window, and a good circulation of air maintained by means of an electric fan. Readings were never taken until the room had been open to the outside for some time. For the outside readings the apparatus was placed on a stand in the open just outside the physics building. The results are given in Table I. A few observations taken on Mount Pauai, Luzon, Philippine Islands, are also included in the table.

The mean values for Manila show no appreciable difference for the different kinds of light. At Mount Pauai the mean value for darkness is much greater than for sunlight, but no importance is attached to this fact, since the high value at Pauai is probably due to the relative humidity, a point which will be discussed later. Furthermore, the observations in the sunlight at Mount Pauai were taken at a time when the ionization was very low, as shown by the reading in the diffuse light taken immediately following those in the sunlight, which gave a value of 432 ions per cubic centimeter.

Our results are in good agreement with those of Schuster,[2] who obtained the same ionic density both in the open air and inside the physical laboratory. McClellan and Kennedy[3] also found no apparent difference between night and day readings.

THE DETERMINATION OF THE NUMBER OF POSITIVE AND NEGATIVE IONS PER CUBIC CENTIMETER IN THE ATMOSPHERE AND THE VARIATION OF THIS NUMBER WITH ALTITUDE, LOCATION, AND RELATIVE HUMIDITY.

The mean values obtained for the specific numbers of both positive and negative ions at Manila, on Mount Pauai, and at Baguio, are given in Tables II., III., and IV., respectively. The determinations were taken

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[1] Ann. der Phys. (1900), 2, 425.

[2] Proc. Man. Phil. Soc. (1903–4), 48, No. 12, 1.

[3] Proc. Roy. Irish Acad., Sec. A, 30, No. 5, 72.

TABLE. I.

Effect of Light on the Ionization of the Atmosphere.

Place.	Date.	Time.		Ions per c.c.	Character of Light.	Remarks.
	1914	a.m.	p.m.			
Manila........	March 30		7.15	346	Darkness	All readings are on the positive ions.
Do.	Do.		8.15	562	Do.	
Do.	Do.		9.15	383	Do.	
Do.	Do.		10.15	390	Do.	
Do.	March 31	12.24		591	Do.	
Do.	Do.	1.30		405	Do.	
Do.	Do.	2.30		1,360	Do.	
Do.	Do.	3.30		1,154	Do.	
Do.	Do.	4.00		888	Do.	
Do.	Do.	5.15		324	Do.	
			Mean.	640		
Manila........	June 10	9.15		655	Diffuse	
Do.	Do.	9.45		971	Do.	
Do.	Do.	10.45		1,302	Do.	
Do.	(*a*)	....		597	Do.	Mean of 91 observations.
			Mean.	609		
Manila........	June 10	11.00		471	Bright sun	
Do.	Do.	11.30		719	Do.	
Do.	Do.	12.00 noon		532	Do.	
Do.	June 11	9.30		791	Do.	
Do.	Do.	10.00		590	Do.	
Do.	Do.	11.00		554	Do.	
Do.	Do.	11.30		640	Do.	
Do.	Do.	12.00 noon		504	Do.	
			Mean.	596		
Mount Pauai..	(*b*)	....		2,634	Darkness	Mean of 17 observations.
Do.	(*c*)	....		1,254	Diffuse	Mean of 50 observations.
Do.	April 30	(*d*)		468	Bright sun	Mean of 3 observations.
Do.	Do.	12.00 noon		432	Diffuse	Mean of 2 observations.

(*a*) From December, 1913, to March, 1914.

(*b*) From April 23, to 29, 1914.

(*c*) From April 20 to 30, 1914.

(*d*) From 9.00 a.m. to 10.00 a.m.

in every case at a place which was comparatively far from any source of dust or smoke, so that the air tested was of a high degree of physical purity. This was especially true for Mount Pauai and it is an extremely important factor in a comparative study of our results, since Eve[1] has shown that the presence of dust, smoke, or other nuclei may cause a variation of more than 50 per cent. in the ionization as recorded by

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[1] Phil. Mag. (1913), 19, 657.

Ebert's instrument. The natural leak was always determined just before and immediately after a reading and the observations corrected accordingly.

TABLE II.

Mean Values of Ionic Density at Manila; Elevation 3 Meters.

Date.	Positive.			Negative.		
	Mean.	Maximum.	Minimum.	Mean.	Maximum.	Minimum.
1913						
December 8	579	1,110	253	1,152	1,621	402
9	487	519	446	677	777	576
15	504	569	382	557	843	369
23	2,782	3,047	2,516	23,327	45,860	1,815
24	1,116	2,555	324	446	447	445
29	561	656	489	564	708	447
1914						
January 1	400	565	202	408	565	291
5	351	454	232	353	526	258
6	487	512	461	473	663	282
12	609	664	582	497	548	418
13	420	512	324	317	338	296
14	400	433	366	267	267	267
19	363	512	274	378	483	288
20	346	353	339	275	275	275
26	322	360	260	332	353	311
27	156	195	116	311	311	311
28	412	412	412	.....	.....	.....
29	402	462	296	325	412	275
February 2	982	2,080	383	997	1,560	434
3	613	731	383	673	710	636
9	713	900	390	436	436	436
10	601	841	282	695	993	397
16	418	569	274	432	576	295
17	418	598	295	423	562	188
24	324	324	324	.....	.....	.....
March 30	627	2,101	324	.....	.....	.....
Mean value of the positive ions for 91 observations = 597.				Mean value of the negative ions for 50 observations = 523.		

From the tables it is seen that the values for Manila are consistently lower than those for the higher altitudes, and in fact somewhat lower than the average values obtained by most observers in other parts of the world. The number of each kind, as usually given, being between 1,000 and 2,000 per cubic centimeter. Apparently, the high values for high elevations are not due to altitude alone for the values obtained at Baguio, elevation about 1,500 meters, are uniformly higher than those on Mount Pauai, elevation 2,460 meters.

TABLE III.

Mean Values of Ionic Density at Mount Pauai; Elevation 2,460 Meters.

Date.	Positive.			Negative.		
	Mean.	**Maximum.**	**Minimum.**	**Mean.**	**Maximum.**	**Minimum.**
1914						
April 20	724	929	476	....	....	....
21	1,548	2,980	489	....	....	....
22	914	1,679	482	1,239	1,740	737
23	2,142	5,086	849	....	....	....
24	2,712	4,225	1,109	....	....	....
25	843	1,242	649	989	1,387	598
28	2,102	4,521	303	....	....	....
29	2,083	3,079	1,118	....	....	....
30	663	1,478	234	....	....	....
Mean value of the positive ions for 66 observations = 1,694.				Mean value of the negative ions for 5 observations = 1,089.		

TABLE IV.

Mean Values of Ionic Density at Baguio; Elevation 1,500 Meters.

Date.	Positive.			Negative.		
	Mean.	**Maximum.**	**Minimum.**	**Mean.**	**Maximum.**	**Minimum.**
1914						
May 7	4,389	7,295	2,648	....	....	....
8	3,427	7,998	942	....	....	....
9	7,265	13,560	1,342	6,283	12,620	1,110
11	3,524	9,508	1,108	4,975	9,180	1,442
12	2,396	4,480	1,136	....	....	....
Mean value of the positive ions for 30 observations = 3,845.				Mean value of the negative ions for 7 observations = 5,526.		

To explain the higher values for the ionization at high altitudes, there are several factors that must be considered. Among these are; the increased ionizing effect of the ultra-violet rays of the sun, the accumulation of ions due to the concentration of the earth's electrical charge at points, and the variations in the relative humidity. The only factor that we were able to investigate in the present work was the relative humidity.

In Table V. is given a summary of the mean values of the relative humidity and the ionization at different altitudes, together with the mean of the daily means. It is interesting to note here that although the relative humidity is higher at the higher altitudes, the absolute humidity is almost invariably lower.

TABLE V.

Mean Values of Relative Humidity and Ionic Density.

Place.	Elevation, Meters.	Relative Humidity.		Ionic Density.	
		Mean, Per Cent.	Mean of Daily Means, Per Cent.	Mean.	Mean of Daily Means.
Manila	3	66.0	65.6	597	591
Baguio	1,500	78.9	79.1	3,845	4,200
Mount Pauai	2,460	81.1	77.4	1,694	1,525

The mean of the daily means gives the most reliable data, as the average of all observations would be materially influenced by grouping the determinations during a period of either high or low relative humidity. Throughout our observations a condition of fog, mist, or rain, invariably gave a high value for the ionization. This is well demonstrated in Table II. by the observations of December 23 when the negative ions increased from 1,815 to 45,860 in a period of about two hours, the weather changing from partly cloudy to a heavy shower.

The effect of the humidity on the ionization is also well shown by Figs. 1 and 2 in which the relative humidity and the ionization for 24-hour periods at Manila and on Mount Pauai are plotted against the time of day.

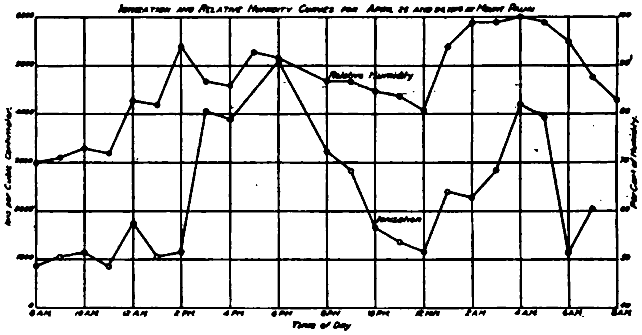

Fig. 1.

Ionization and relative humidity curves for April 23 and 24, 1914, at Mount Pauai.

The ionization curve lags slightly behind the humidity curve as might be expected, but on the whole the two curves show too great a uniformity to be purely accidental. In all our observations there was no apparent relation between the maximum and minimum values and the time of day, except in so far as the time of day influenced the value of the relative humidity. This probably accounts for the high values obtained for darkness at Mount Pauai, as shown in Table I., since the relative humidity was nearly always greater at night.

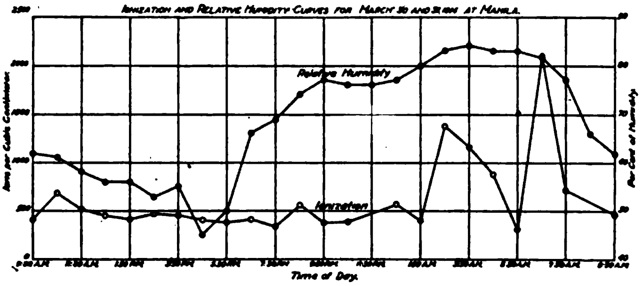

Fig. 2.

Ionization and relative humidity for March 30 and 31, 1914, at Manila.

No attempt has been made to measure the large ions in the atmosphere, the number of which, as shown by the work of Langevin,[1] Pollock,[2] McClelland and Kennedy,[3] and others is much larger than that of the small ions. The mean values given in Table VI. are for the ionization as recorded by Ebert's apparatus.

For the sake of comparison a brief summary of the results of some other observers is given in Table VII.

Table VI.

Mean Values of Small Ions Recorded by Ebert's Apparatus.

Place.	Elevation, Meters.	n_+	n_-
Manila	3.0	587	523
Mount Pauai	2,460.0	1,694	1,089
Baguio	1,500.0	3,845	5,536

From a general survey of the literature on the subject of atmospheric ionization and the published results of investigations, it is evident that a large amount of experimental evidence must yet be obtained before any definite conclusions may be drawn. The work thus far shows that the ionization varies between wide limits, not only from day to day but from hour to hour. The limits of this variation, also, seem to be independent of the location in which the observations are made. Even the determinations taken over the ocean do not vary greatly from those over the land.

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[1] Loc. cit.

[2] Loc. cit.

[3] Loc. cit.

Table VII.

Mean Values of Small Ions as Recorded by Other Observers.

Observer.	Place.	n_+	n_-	Reference.
Boltzmann.....	Over Ocean...........	1,150	800	Phys. Zeit., 1904.
Simpson.......	Lapland..............	1,120	972	Phil. Trans., 1905.
Eve..........	Over Ocean...........	975	783	Phil. Mag., 1907.
Eve..........	Montreal and Quebec...	323	372	Do.
Eve..........	Gulf of St. Lawrence....	761	743	Do.
Schuster......	Manchester...........	3,000		Proc. Man. Phil. Soc., 1904.

From our results it seems quite evident that the number of small ions in the atmosphere of the tropics is of the same order of magnitude as the number observed in other parts of the world, also that the number of ions per cubic centimeter increases with altitude and with relative humidity. The increase in ionization with altitude is contrary to the radio-active theory of atmospheric ionization, which as pointed out by Eve,[1] should decrease rapidly as we ascend from the surface of the earth. It is true that observations on a mountain peak do not give a true altitude effect, and yet some evidence of a decrease ought to be manifest, since the authors[2] in previous work found that the radium emanation content of the air at high altitudes is much lower than at sea level. It seems therefore, that there must be some other causes influencing the ionization which are as yet not completely determined.

The Ratio of the Positive to the Negative Ions.

The fact that the higher atmosphere carries a positive charge would necessitate that the positive exceed the negative ions in number. The mean value obtained for the ratio of positive to negative ions (1.17) however, is much larger than is necessary to explain the positive charge of the atmosphere. Eve[3] has pointed out that this apparent excess, as

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[1] Phil. Mag. (1911), 21, 26.

[2] Phys. Zeit. (1914), 15, 31.

[3] Phil. Mag. (1910), 19, 657.

Table VIII.

Ratio of Positive to Negative Ions at Manila.

Date.	Interval.				n_+	n_-	$\frac{n_+}{n_-}$	Weather Remarks.
	From —.		To —.					
1913	a.m.	p.m.	a.m.	p.m.				
Dec. 8	11.33			12.41	314	402	0.782	Following light shower.
8		2.36		3.46	1,110	1,527	0.727	
9	10.16		10.59		446	576	0.773	Variable, threatening.
9		2.32		3.42	508	777	0.654	Do.
15	11.06			12.24	475	461	1.032	Fair.
15		2.12		3.30	562	606	0.926	Do.
23	10.31		11.46		2,781	1,815	1.535	Variable.
24	10.44		11.56		568	447	1.046	Fair.
29	10.48		11.57		572	537	1.065	Do.
29		2.40		3.50	537	577	0.931	Do.
1914								
Jan. 2	11.07			12.14	498	565	0.883	Fair.
2		2.46		3.47	202	334	0.605	Do.
5	10.53		11.57		300	258	1.162	Do.
5		2.32		3.11	454	526	0.733	Partly Cloudy.
6		1.29		2.31	487	663	1.186	Fair.
12	10.45		11.52		663	526	1.206	Do.
12		1.56		3.05	582	483	1.206	Do.
13	10.10		10.59		375	338	1.110	Do.
13		3.08		3.49	512	296	1.730	Do.
14	11.20			12.02	366	267	1.371	Do.
19	10.44		11.46		289	362	0.799	Do.
19		3.28		4.31	512	386	1.329	Do.
20	9.12		10.19		346	275	1.258	Do.
26	11.30			12.08	346	353	0.981	Do.
26		3.00		4.06	310	310	1.000	Do.
27	9.00		10.07		156	311	0.500	Do.
29	9.06		10.17		373	289	1.290	Do.
29		1.43		2.48	462	344	1.345	Do.
Feb. 2	10.41		11.45		434	434	1.000	Do.
2		2.42		3.26	2,080	1,560	1.332	Cloudy with mist.
3	10.06		11.31		513	636	0.860	Do.
3		2.26		3.32	713	710	1.003	Do.
9		2.07		3.09	645	436	1.481	Fair.
10	10.09		11.10		667	993	0.673	Do.
10		1.54		3.00	536	397	1.351	Partly cloudy.
16	10.33		11.37		490	576	0.850	Fair.
16		3.11		4.15	274	360	0.760	Do.
17	10.00		11.02		328	188	1.748	Do.
17		2.29		3.33	598	540	1.110	Partly cloudy.

Mean value of ratio $\frac{n_+}{n_-} = 1.051$

measured by the Ebert's instrument, may be explained as due to the following causes:

1. A real excess.
2. More rapid diffusion of the negative ions to parts of the instrument, due to their greater mobility.
3. More rapid diffusion to centers; dust, smoke, mist, etc.
4. Apparent loading due to water vapor.

Consequently, the ratio as given by Ebert's ion-counter is probably in excess of the true value; furthermore, it is to be remembered that this instrument detects only the small and a portion of the intermediate ions.

In our determinations of this ratio three readings were taken in every case, so that any sudden change in the number of positive or negative ions would be detected. The results are given in Tables VIII. and IX.

Table IX.

Ratio of Positive and Negative Ions at Baguio and Mount Pauai.

Date.	Interval.				n_+	n_-	$\frac{n_+}{n_-}$	Weather Remarks.
	From		To					
1914	l.m.	p.m.	a.m.	p.m.				
May 9....	11.40			12.47	4,117	5,120	0.804	Cloudy.
9....		2.05		2.38	13,560	12,620	1.074	Rain.
11....	8.35		9.08		2,201	1,442	1.540	Cloudy.
11....	11.05		11.43		2,604	4,638	0.567	Do.
11....		3.35		4.13	9,508	9,180	1.037	Do.
	Mean value of ratio $\frac{n_+}{n_-}$ for Baguio =						1.004	
April 22....	8.56		9.54		685	737	0.930	Slightly hazy.
22....		2.44		4.48	1,142	1,740	0.656	Cloudy.
25....	8.48		9.49		649	598	1.086	Fair.
25....		2.48		3.48	1,242	1,184	1.050	Cloudy.
30....	9.02		10.12		585	234	2,500	Bright.
	Mean value of ratio $\frac{n_+}{n_-}$ for Mount Pauai =						1.244	

Our mean value for Manila is 1.051; for Baguio, 1.004. These are both lower than the average given above, but our mean for Mount Pauai, 1.244, is decidedly higher. It is interesting to note that many of our observations give a value less than unity for this ratio. This means that the gradient of the electric intensity becomes negative. It is to be regretted that we were unable to take any observations on the potential gradient, for simultaneous readings on these two factors would be of interest.

Summary.

1. The number of positive and negative ions per cubic centimeter in the earth's atmosphere has been determined at Manila, Baguio, and on Mount Pauai. The ratio of the positive to the negative ions for the above mentioned places has also been investigated. These determinations were made with an Ebert's ion-counter.

2. The mean value obtained for the ionic density does not vary greatly from the value obtained by observers in other regions. The number increased with altitude and showed a fairly uniform tendency to follow the variations in relative humidity.

3. The ratio of the positive to the negative ions at Manila, and at Baguio, is less than the mean of earlier observers, while the value at Mount Pauai is greater. Many of the observations gave values less than unity, which shows that at times the gradient of the electric intensity has a negative sign.

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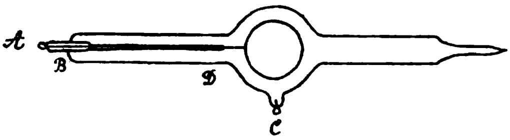
ON THE CONSTRUCTION OF SENSITIVE PHOTOELECTRIC CELLS.

BY JAKOB KUNZ AND JOEL STEBBINS.

THE high sensitiveness of photoelectric cells of alkalihydrides has been discovered by Elster and Geitel. For several years we have tried to apply this cell in stellar photometry. J. G. Kemp[1] and W. F. Schulz[2] have shown that it is possible and advantageous to replace the selenium in stellar photometry by the photoelectric cell. Practically at the same time corresponding measurements have been made in Germany, especially in the observatory of Berlin.

One of us has reported on an astronomical discovery made by the photoelectric cell in the Evanston meeting of the American Astronomical Society in September, 1914. In the last two years we have tried to improve the photometric properties of the cell and we have arrived at a form which seems to be satisfactory with respect to sensitiveness, constancy, absence of the dark current, etc. The final form is indicated by Fig. 1, which is drawn full size. The glass bulb is 3.4 cm. in diameter.



Fg. 1.

It contains a small platinum cathode C, a platinum ring of 1.8 cm. in diameter as an anode A, which passes through a platinum cylinder B; this cylinder was found to be very necessary in order to lead surface and electrolytic currents of the glass to earth. Strips of tinfoil were occasionally wrapped around the glass cylinder at D and the cathode C, in order that dark currents might be suppressed. The tubes are connected to the mercury pump and heated two to three hours to 330° C. to drive off the remaining gases. A small quantity of the pure alkali metal is distilled on the silver mirror of the cell, which is kept cool by cold water or ice, at the same time the end AD of the cell is heated from 160 to 240°, according to the alkali metal, by means of a heating coil.

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[1] J. G. Kemp, PHYS. REV., Vol. I., p. 274, 1913.

[2] W. F. Schulz, Astrophysical Journal, Vol. XXXVIII, p. 187.

The most sensitive cells have been obtained when the metal was deposited in a thin uniform layer. Pure hydrogen from palladium was then admitted and its pressure so adjusted that a potential difference of 280 to 400 volts between the electrodes produced a uniform glow discharge. Often a spark or arc appears instead of the bright uniform glow, and the spark is apt to destroy the sensitive layer. By experience one finds the best conditions for the glow to appear. The sensitiveness will be tested during the formation of the hydride. As a rule the formation requires only one to three seconds for the maximum sensitiveness; if continued, the colors of the compound change and the deflection in the galvanometer decreases. During the formation the electrode *C* is negative and *A* positive. But in certain gases like ammonia and ethane a sensitive layer is also formed if the current is reversed. After the formation the gas is carefully pumped out and replaced by an inert gas, helium, argon, or neon. The pressure is so chosen as to get a maximum sensitiveness.

Experiments have been made with the object of finding out the influence of the size and shape of the cell on the sensitiveness. The diameter varied from 5 to 2.5 cm. and the sensitiveness rather increased with decreasing diameter. The silver mirror was sometimes deposited on a flat or conical bottom, so that the incident light should be reflected and its action increased; but very little increase in the deflection of the galvanometer was observed, so that the ordinary spherical shape was chosen.

Efforts have been made to replace the hydrogen by other gases, for instance ethane, ammonia and acetylene. With ethane and the current reversed a very dark violet-blue color was obtained of a high sensitiveness, and of a beautiful metallic luster, but unfortunately the sensitiveness proved not to be constant. When dry ammonia vapor was used instead of hydrogen for the formation, a bright blue layer was obtained of high sensitiveness which however decayed also in the course of time. Acetylene finally formed a black layer with potassium under the influence of the electric field, but it was very little sensitive. So far hydrogen seems to give the most sensitive and the most constant cells.

Four alkali metals have been used, viz., sodium, potassium, rubidium and cæsium. The best results have been obtained with rubidium and neon. The metal was distilled in the cell while the silver mirror was cooled with ice. A potential difference of 280 volts produced a glow in the hydrogen and a very beautiful violet reddish sensitive layer with a bright metallic luster. The hydrogen was then replaced by helium, argon or neon. The neon was received from the Bureau of Standards.

The three curves *A*, *B* and *C*, of Fig. 2 show the relative sensitiveness of the rubidium cells filled with these three gases. Helium gives the smallest, neon the best sensitiveness. Nevertheless it is possible that the helium and argon cells are better than the neon cell because the curve for the neon rises much quicker than the other curves, in other words the neon cell is more sensitive to small changes of the potential difference acting between the electrodes than the helium and argon cell. It is very important to use perfectly pure gases.

The sensitiveness of some cells decays slightly during the first few days after the formation and then becomes constant. Some distinct white spots appeared on the surface of some of the very bright violet rubidium metals, and in one or two instances such a spot became wider in the course of time and covered finally the whole surface, which then appeared bright bluish, and whose sensitiveness was considerably less than that of the original violet surface. When the cells were of a larger size, these bright violet-red surfaces on rubidium were never obtained, but rather sky-blue and blue-green colors which exhibit very beautiful iridescence. The potassium cells were formed with a potential difference of 360 volts. The glow discharge gives almost instantly rise to a most beautiful golden rose color, which is exceedingly sensitive, but not very stable. When the formation is continued for a second or two, a deeper violet-red appears which remains practically constant, but the golden hue gradually fades away. The sensitiveness of the cell when filled with the different gases is shown by the curves of Fig. 3. Neon again

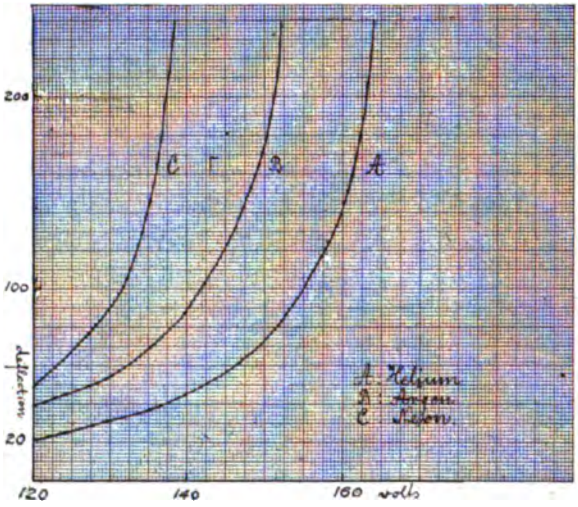

Fig. 2.

shows the greatest sensitiveness, hydrogen the smallest, argon seems to give a curve which lies between *A* and *B*, but this question is not quite settled. A comparison of Figs. 2 and 3 shows that for rubidium we find the same photoelectric current with a potential difference about 40 volts smaller than for potassium. Very striking iridescence can be obtained by the potassium.

Cells have also been formed with sodium and cæsium. The former metal gives very sensitive cells, but their construction is more difficult than that of the potassium and rubidium cells, the sodium seems to act somewhat on the silver mirror, so that the distilled metal does not seem so bright on the silver as on the glass. If however, the metal is distilled on the glass bulb directly, then the contact with the electrode is unsatisfactory. The pure metal seems to give a very sensitive golden layer. The cæsium finally is liquid at 28° and can therefore not be used directly. A solid amalgam of this alkali metal has been formed which was, however, of a rather weak sensitiveness.

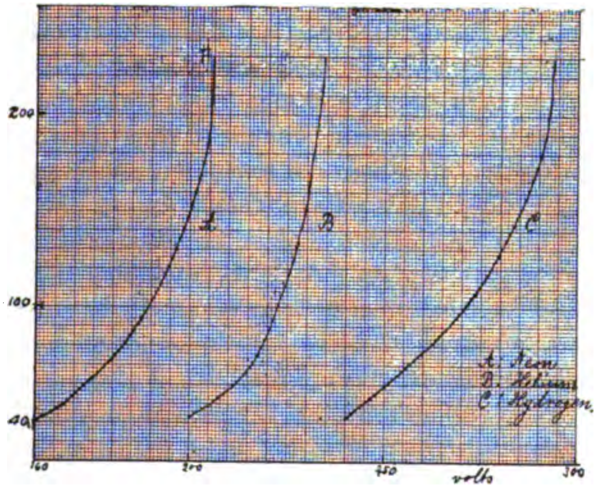

Fig. 3.

The cells described in this article show a very small dark current; if it exists at all, it can be compensated by the application of a convenient small potential at the platinum cylinder between the two electrodes. As far as our present measurements indicate, there is an accurate proportionality between the intensity of the incident light and the photoelectric current. The cells are used in stellar photometry.

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University of Illinois,
August 12, 1915.

REFLECTION FROM, AND TRANSMISSION THROUGH, ROUGH SURFACES.

By A. F. Gorton.

It is a matter of common knowledge that matt surfaces, such as ground glass, smoked glass, paper and plaster-of-Paris, which scatter in all directions the light that falls upon them, behave like polished surfaces when viewed at grazing incidence. According to the theory usually advanced in explaining this action, if the light scattered in a given direction by the elevated portions of the surface differs in phase from that returned in the same direction by the hollows or pits in the surface by a quantity small in comparison to $\lambda/2$, the light may be regarded as regularly reflected. The light thus reflected will be diminished in intensity, depending on the magnitude of the phase difference. Lord Rayleigh[1] showed this experimentally by sending light from a Welsbach lamp upon a thermo-couple after reflection from two parallel plates of silvered ground glass. A third ground and silvered plate was found to reflect as much energy as a polished silvered mirror. This experiment is in a sense qualitative, for it does not tell with what intensity the individual wave-lengths are reflected.

A study of rough surface reflection has been made recently by Meyer,[2] but his experiment also may be looked upon as qualitative in the sense referred to above. He employed a rather interesting scheme for showing that rough surfaces reflect like mirrors for sufficiently long waves. Light scattered by the matt surface was received by a concave mirror and focused on the slit of a fluorite prism spectrometer, which threw approximately monochromatic light on a thermo-couple. A galvanometer in series with the thermo-couple measured the amount of energy received from the matt surface. By rotating the concave mirror, the blurred image could be gradually thrown off the slit, first in one direction, then in the other. With the spectrometer set for wave-length $8\,\mu$, measurements taken with the rough surface in question showed a sharp decrease in intensity as the image was shifted off the slit,—this decrease being very nearly as sudden as that obtained when a polished mirror was substituted for the rough surface. As the wave-length was decreased, this drop in intensity became more and more gradual. In other words, waves longer than $8\,\mu$ are regularly reflected and therefore sharply focused on the slit; while the shorter waves are scattered, and form a broad, diffuse image.

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[1] Nature, 64, p. 385, 1901.

[2] T. J. Meyer, Verh. Deutsch. Phys. Ges., p. 126, Feb., 1914.

Though Meyer's experiments tell us how a surface of a certain degree of roughness behaves towards waves of different lengths, the method is not directly designed to show the reflecting power of such a surface throughout the spectrum. The experiments described in the following pages were undertaken with the idea of ascertaining the reflecting power, for the range of wave-lengths $.6\,\mu$ to $14\,\mu$, of matt surfaces which can be easily and quickly prepared, and thereby showing how well such surfaces are fitted to act as screens for cutting off the short waves. Not only reflection, but also transmission experiments (using roughened plates of rock-salt) were performed. It is not necessary to remark that a reflection "screen" is preferable to a transmission cell because it is good for an unlimited range of wave-lengths in the infra-red, whereas rock-salt and other so-called "transparent" substances possess regions of opacity. Besides these experiments on rough surfaces, some work was done on the reflecting power of polished glass, quartz and Iceland spar, in the regions of metallic reflection, as a function of the angle of incidence. The curves obtained have been included in this paper, together with a brief discussion of their bearing on the reflection of plane-polarized light.

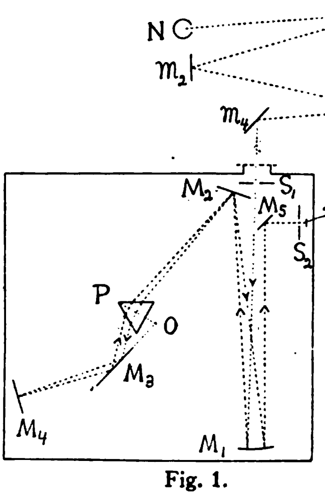

Fig. 1.

Apparatus.—The arrangement of apparatus is shown in Fig. 1. The source of light was a Nernst glower *N*, suitably protected from air currents by a metal cover. The glower was operated on a 110-volt storage battery, and the constancy of its radiation was found to be ample for the short period of time (5 minutes or less) required for taking measurements at a definite wave-length. The light was focused by a concave silvered mirror m_1 (7.5 cm. in diameter and 26 cm. focal length) upon the rough surface m_2, whence the light was received by m_3 (a duplicate of m_1) and focused on the first slit S_1 of the spectrometer. Although the diagram shows the arrangement for throwing a converging beam of light on the rough surface, all the earlier experiments were performed using strictly parallel light. In both cases the spectrometer receives that proportion of the light which is scattered by the rough surface at the same angle (angle of incidence = angle of reflection) as that at which light is regularly reflected by a polished mirror. The matt surface was pressed firmly by a spring against three screws held in a suitable framework—a device which ensured the exact replacement of one mirror by another. In addition, this framework, together with m_1 and N, rested upon a brass plate which rotated about a vertical axis passing through the middle point of the surface m_2, so that it was possible to alter the angle of reflection from 0° to 90° without changing the relative positions of N, m_1 and m_2 or of m_2, m_3 and m_4.

The spectrometer was of the usual type, consisting of a rock-salt prism P (area of faces 5×9 cm.), mounted with the plane silver mirror M_3 according to Wadsworth's arrangement for minimum deviation on a prism table whose axis of rotation passed through O. The table was turned by means of a device used by Mendenhall—a micrometer arrangement (not shown in the drawing) by which angular displacements of 1/10 second could be accurately read. The concave mirror M_1 (the only one used in the spectrometer) was of speculum metal, of 10 cm. aperture and 58 cm. focal length, figured by Brashear. The parallel beam of light reflected by M_1 reached the prism P after striking the plane silvered mirror M_2, which was introduced in order to decrease the astigmatism and thus improve the definition of the image. With the idea of doubling the resolving power, it was decided to return the light through the prism, so that the same effect was obtained as from two prisms in series. This was accomplished by inserting the plane silvered mirror M_4, which received the light from M_3 and returned it over approximately the same path, so as to reach the thermo-couple by way of M_1 and M_5. The edge of M_5 was beveled, so that the angle between the incident and return beams was as small as 2°. This slight departure from the strict conditions for minimum deviation caused no appreciable shift in the spectrum, as shown by the observed positions of the CO_2 emission bands ($2.7\,\mu$ and $4.4\,\mu$), the bands of metallic reflection of quartz ($8.5\,\mu$ and $9\,\mu$), and the Iceland spar band ($6.7\,\mu$) agreeing with the calculated positions. The thermo-couple T was a 3-junction compensated instrument, of bismuth-tin alloy, with a receiving surface 12 mm. long and 1 mm. wide; and the galvanometer was of the D'Arsonval type with a silver wire suspension. Both instruments were constructed by Dr. A. H. Pfund.[1] The thermo-couple-galvanometer system had a sensibility of 150 cm. scale deflection for a candle at 1 meter, the scale distance being 4 meters. Throughout the work but little trouble was experienced with the galvanometer, which was not magnetically shielded. Over periods of time as long as 15 minutes the zero was constant to within 0.1 mm., and at all times readings could be repeated to within 0.3 mm., though such accuracy was not needed in view of the larger experimental errors inherent in the method employed. This steadiness is thought to be due in part to the care taken to protect the thermo-couple from stray radiation. An air-tight cover of galvanized iron, wrapped with a thick layer of felt, and provided with a small window for admitting the incident light, fitted securely over the whole spectrometer. After the initial adjustment for minimum deviation, this top was screwed down and the spectrometer permanently closed, save for the short intervals of time (30 seconds) when the window was opened for the purpose of taking readings. Dishes of sulphuric acid and calcium chloride were placed near the prism to remove moisture.

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[1] Phys. Zeit., 13, 1912, p. 870.

The arrangement described above for returning the light through the prism—a combination Wadsworth-Littrow mounting—possesses several advantages over the ordinary type of infra-red spectrometer: it yields double the resolving power afforded by a single passage of the light through the prism, and, for a prism of given size, delivers four times as much energy as the ordinary type of spectrometer with double the focal length; it involves the use of only one concave mirror, which, however, must be carefully figured; and it permits of an extremely compact spectrometer. As an illustration of the resolving power of the present instrument, a curve showing the transmission of pure ethyl alcohol is given below (Fig. 2). This curve is taken from some unfinished work of the writer's on the absorption spectra of organic substances during chemical reaction. Fig. 4, which shows the transmission of a film of alcohol 0.01 mm. thick, is copied from Coblentz's "Infra-red Absorption Spectra." (The percentage transmission is plotted as ordinates, and wave-lengths in μ as abscissæ.) It will be observed (see Fig. 2) that there are three small absorption bands between 2 μ and 3 μ, and perhaps five from 4 μ to 6 μ, while the great band at 3.5 μ is plainly double, one component being at 3.3 μ and the other at 3.7 μ, with a slight intermediate maximum at 3.5 μ (see magnified view in Fig. 3).

Experimental Procedure.—In the work on the reflection from rough surfaces, the procedure adopted was as follows. With the spectrometer set at a particular wave-length, the galvanometer deflection obtained with the rough surface in position (m_2, Fig. 1) was noted, together with the deflection observed when the matt surface was replaced by a polished plane mirror of the same substance. The ratio of the two deflections may be termed the "relative" reflecting power of the matt surface in question. Proceeding in this manner throughout the spectrum, a curve was drawn with wave-lengths as abscissæ and "relative" reflecting powers as ordinates. Then the angle of incidence was changed (but at all times the angle of incidence equalled the angle of reflection) and a new curve drawn. Having obtained a number of curves at different angles of incidence, a rougher (or smoother) surface was substituted and the same procedure repeated. In this way the effects of the two factors—angle of incidence and degree of roughness—could be analyzed separately, and the most suitable method of roughening the surface ascertained. In view of the fact that the object of the experiments was to develop an infra-red screen, it may seem more logical to get the absolute reflecting power of the matt surfaces, either by a direct method, or by comparison with a plane silvered mirror. This was not done for several reasons. In the first place, it was found convenient to use ordinary plate glass in grinding most of the surfaces. Though some of the surfaces were afterwards silvered, the result was not a success, because the customary polishing with rouge could not be resorted to. Since the absolute reflection curve of glass is not only very low (4 per cent.) in the transparent region, but is complicated by a curious maximum near 9 μ, it is clear that the absolute reflection curves of rough glass, taken at various angles, would not be very illuminating. Moreover, it was deemed advisable to ascertain if the character of the substance used in making the surface had any influence on the shape of the reflection curves. This could be done only by the method first described.

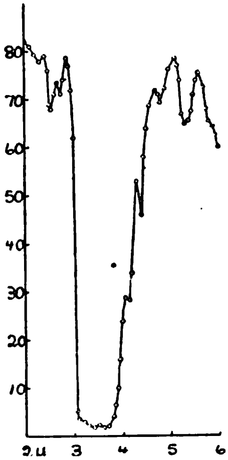

Fig. 2.

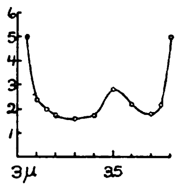

Fig. 3.

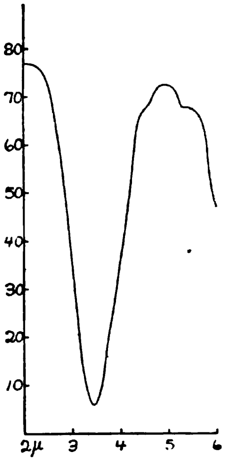

Fig. 4.

In the experiments on transmission through rough surfaces, a slight change was made in the disposition of apparatus. The framework used for holding the matt surface m_2 (Fig. 1) was removed, and the brass plate supporting N and m_1 was rotated so that the light traveled directly from m_1 to m_2, being brought to a focus midway between the two mirrors. At this focal point was introduced the roughened plate of rock-salt whose transmission was desired. The ratio of the deflection obtained with the rock-salt plate in position to that observed when the plate was removed gave the percentage transmission.

In the supplementary work on the reflecting power of polished glass, quartz and Iceland spar as a function of the angle of incidence, comparison was made with a plane silvered mirror, whose reflecting power was found later by a direct method, and the reflecting powers relative to silver were thus reduced to absolute values.

Results.—The simple theory given above tells little about the behavior of ordinary rough surfaces towards waves of different lengths, for the reason that such surfaces are extremely irregular in structure. This fact is brought out by examination with the microscope, and is shown also by the absence of decided interference minima in the reflection curves. Such minima are exhibited by a surface of regular topography—for instance, a plane reflection grating placed with its grooves parallel to the plane of the incident and reflected beams of light. The writer used a small speculum grating, of 7,000 lines to the inch, ruled by Dr. J. A. Anderson. The grating was placed with its lines horizontal and its reflecting power, at 23°, compared with that of polished speculum. The result is shown in Fig. 5. (Wave-lengths in μ are plotted as abscissæ, "relative" reflecting power in per cent. as ordinates.) A deep minimum is noticed in the curve at 0.71 μ, where the relative reflecting power is

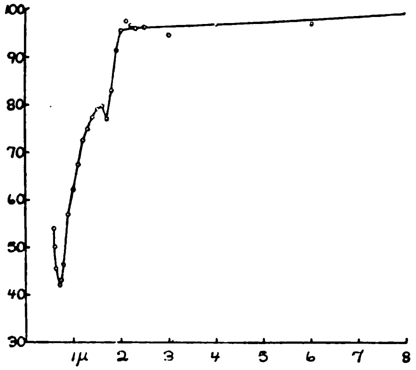

Fig. 5.

42 per cent., and a second one, less marked, at 1.7 μ. If we assume for the sake of simplicity that the grooves are all shaped like the letter V, it is a simple matter to calculate their probable depth. The condition for the principal interference minimum is $2H \cos i = \lambda$, where H = depth of groove, i = angle of incidence. Substituting the values $\lambda = .71\ \mu$, $i = 23°$, H is found to be .38 μ, or 1/66,000 inch.[1]

Rough Surface Reflection.—Various processes of preparing matt surfaces were tried, but only one—that of grinding with the finest emery and water—was found satisfactory. Glass plates which had been roughened with the sand-blast and others which had been etched were tested, but some were found to be too coarse and all were lacking in uniformity. The attempt was made to plot the reflection curves of smoked glass at various angles of incidence, in order to draw a comparison between this type of matt surface and the ordinary ground surface, the former being topographically the reverse of the latter. Uniform films of smoke of any desired thickness are easily produced, but, as one might expect, their reflecting power, even at large angles of incidence and for waves as long as 10 μ, is very small. Some observations were made on smoke films which had been gold-plated cathodically, but they proved disappointing, perhaps because the deposit of gold was too thin to increase the reflecting power appreciably. The surfaces whose curves appear in Figs. 6, 7, 8 were all prepared by the method first described. Two pieces of ordinary plate glass $\frac{1}{4}$ inch thick were ground together, using the finest grade of Bausch & Lomb emery and enough water to prevent sticking. A few minutes' grinding yielded a fairly fine and very uniform surface. Much smoother surfaces were obtained by washing off the emery and continuing the grinding with water alone, but great care had to be taken to avoid scratching. In order to increase the reflecting power and to see if the material of the surface played a rôle, some of the plates were coated cathodically with gold, platinum and silver. Some typical curves are shown in Fig. 6. Curve I. gives the "relative" reflecting power at 23° of a comparatively rough plate which had been silvered; Curve II. was taken at 70°. Curve III. shows the relative reflecting power of a much finer surface, also silver-plated, at 45°. (In all the reflection curves values of the relative reflecting power, *i. e.*, the reflecting power of the rough surface divided by that of a polished mirror of the same material, are plotted as ordinates, and wave-lengths in μ as abscissæ.) In the case of both surfaces, the comparison mirror was silvered simultaneously with the matt surface, to ensure films of equal thickness. One notices that increasing the angle of incidence from 23° to 70°, in the case of the rougher surface (Curves I. and II.), increases the steepness of the curve and eliminates the horizontal portion at the short-wave end, but causes no considerable increase in reflecting power beyond 10 μ. It is also plain that the curve for a smoother surface

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[1] *Note.*—It is to be observed that the curve (Fig. 5) in reality gives the energy distribution in the principal image of the grating.

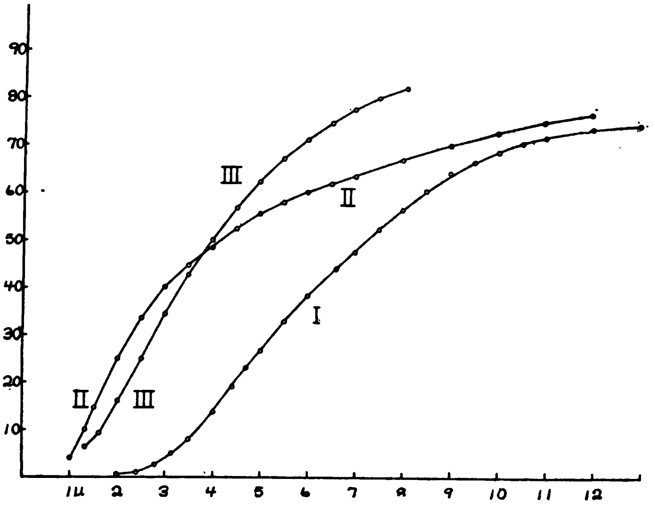

Fig. 6.

(Curve III.) is steeper than that for a rougher one even at a greater angle of incidence. These and other facts are brought out better by Figs. 7 and 8. Fig. 7 gives the relative reflecting power of a surface not quite

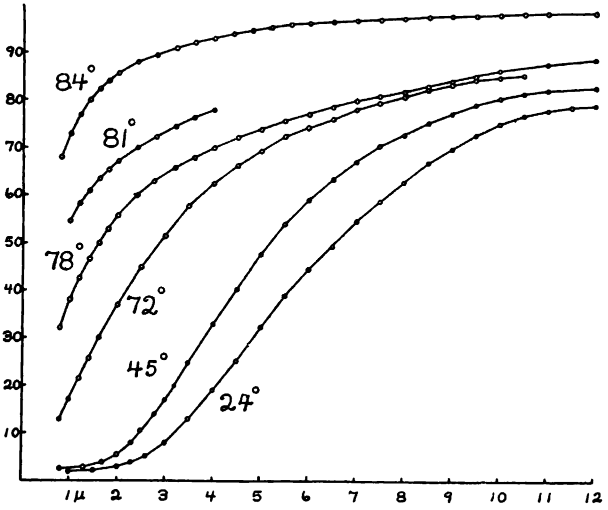

Fig 7.

as coarse as the first one described above, which was platinized cathodically, and Fig. 8 shows the results obtained with an exceedingly fine surface of ground glass which was not covered with any metallic deposit. The former was compared with a polished mirror platinized simultaneously with the rough surface, and the latter was compared simply with a polished plate of glass. The angle of incidence is marked on each curve. In order to show, for a given rough surface, how the relative reflecting power for a fixed wave-length varies with the angle of incidence, some points were taken from the curves in Fig. 8 and plotted as shown in Fig. 9. A glance shows that the relative reflecting power increases most rapidly at 1 μ for angles between 75° and 80°.

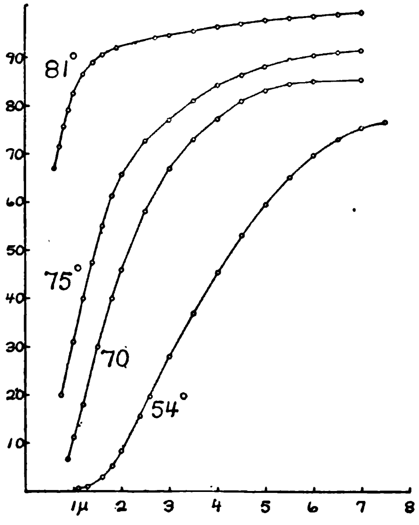

Fig. 8.

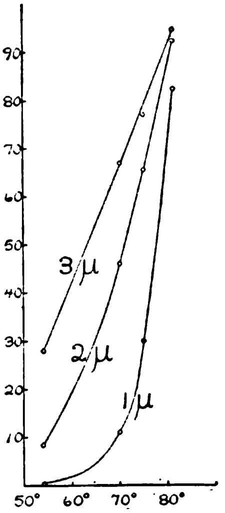

Fig. 9.

Analysis of all the curves brings out the following points:

1. For small angles of incidence the curves are not steep and reach a limiting value considerably below the theoretical value of 100 per cent. This is attributed to the presence in the surface of pits much deeper than the average. For the same angle, the curve for the finer surface is steeper and attains higher values.

2. Increasing the angle of incidence steepens the curve, and the smoother the surface, the greater the increase in steepness.

3. For the purpose of serving as a screen for cutting off the short waves, a rough surface at a large angle is inferior to a smoother surface at a smaller angle.

4. The steepest curves of all are obtained with the finest surfaces used at large angles of incidence.

5. The material of the surface does not affect the shape of the curves for the "relative" reflecting power. Hence if the ground glass surface mentioned above were silver-plated, the curves in Fig. 8 would give absolute values of its reflecting power.

Transmission through Rough Surfaces.—Fig. 10 gives the results of some experiments on the transmission of rough rock-salt crystals. Since both faces of each crystal were roughened, and therefore the light had to pass through two matt surfaces, the curves are steeper than they would be if only one face were rough. Curve I. was taken with a crystal ground with emery, whose surfaces were very rough. Curve II. shows the transmission of a finer plate, which was transparent to red light. Curve III. is for a crystal which was roughened simply by breathing on it for a few seconds. On account of the steepness of the curve, this plate makes a satisfactory transmission screen for eliminating wave-lengths shorter than 1 μ. A curious feature of this particular crystal was that, when held before the Nernst filament, it transmitted only the blue-green,—a striking phenomenon of interference exhibited beautifully by films consisting of minute bubbles,[1] and by gratings whose central images are colored. Viewed under the microscope the surface showed a regular structure resembling that of a honeycomb.

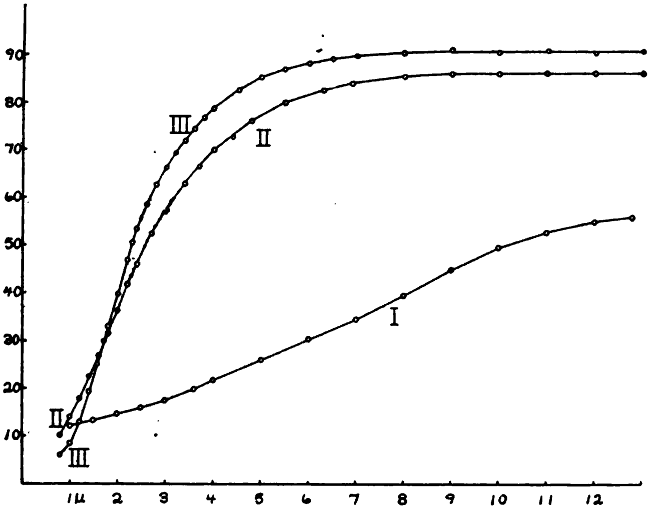

Fig. 10.

Reflecting Power of Polished Glass, Quartz and Iceland Spar as a Function of the Angle of Incidence.—In mapping the "relative" reflecting power of ground glass, it was thought advisable to plot the reflection curves of polished glass at various angles. The results are given in Fig. 11, ordinates being absolute values of the reflecting power, and abscissæ wave-lengths in μ. The curves were taken at the following angles of incidence: I. at 23°, II. at 54°, III. at 60°, IV. at 63°, V. at 70°, VI. at 81°, VII. at 84°. The reflecting power at 23° is low in the visible (4 per cent.), drops to a fraction of 1 per cent. at 8 μ, and rises to a maximum at 9.4 μ, with a slight bulge in the curve near 8.8 μ. Increasing the angle of incidence has the effect of doubling this maximum, and the minimum at 8 μ is accentuated, the rise from 8 μ to 8.3 μ being enormously steepened. In the region of transparency (*i. e.*, below 7 μ), the increase in reflecting power follows the law for the reflection of unpolarized light,

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[1] Wood's Optics, p. 253, new edition.

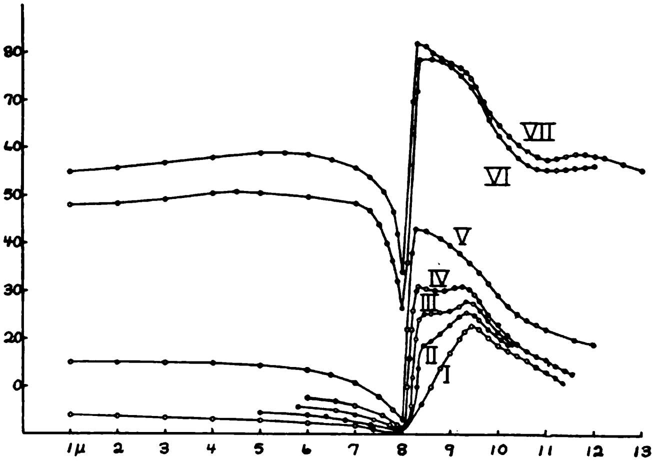

Fig. 11.

$$J = a^2\left\{\frac{\sin^2(i-r)}{\sin^2(i+r)} + \frac{\tan^2(i-r)}{\tan^2(i+r)}\right\}.$$

In the region of absorption (beyond 7 μ), the changes in the index of refraction and the extinction coefficient would appear to be rather complex. That these curious effects are due in part to polarization seems probable for several reasons: (1) The doubling of the maximum begins to be marked in the vicinity of the angle of maximum polarization (56°). (2) Nyswander[1] mapped the reflection curves for the crystals calcite and aragonite, using light plane-polarized in different azimuths, and showed that certain maxima of reflection are due to one component of the incident light but not to the other, while other maxima are due to both.

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[1] Nyswander, Phys. Rev., 28, 1909, p. 291.

Nyswander's results seem to have a direct bearing on the curves for glass given above, and also on those for quartz and Iceland spar, which are given below in Figs. 12 and 13 respectively. At large angles of incidence, the minima on both sides of the maxima become very striking. In the case of quartz (Fig. 12) the minimum at 7.5 μ becomes sharply defined at 78° and is very deep at 84°, while that at 9.6 μ is absent in all but the 84° curve. At 84° the maximum at 8.6 μ is apparently double, and a curious minimum appears at 12.7 μ. In the case of Iceland spar

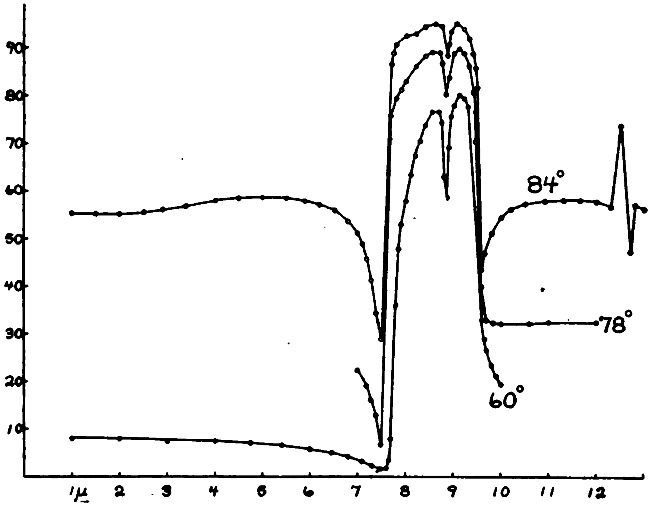

Fig. 12.

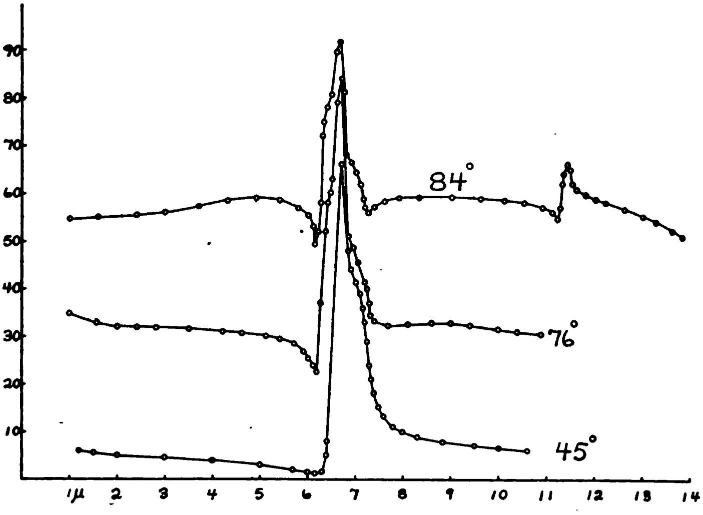

Fig. 13.

(Fig. 13) the maximum at 6.7 μ shows signs of being double at 45°, and is apparently triple at 84°. These distortions of the reflection curves may be traced to the elliptical polarization[1] which is known to take place at the surface of the crystal. Results of theoretical importance in regard to a possible connection between the angle of incidence and the optical constants of the crystal, in the region of metallic reflection, might be hoped for if these experiments were repeated, using plane-polarized light. This the writer expects to do in the near future.

SUMMARY.

1. A study of the reflecting power of matt surfaces at various angles of incidence and of the transmission of roughened plates of rock-salt, in the region 0.6 μ to 13 μ, has been made for the purpose of finding a suitable screen for cutting off the short waves.

2. The best results, in the case of reflection, were obtained with the finest surfaces used at rather great angles of incidence. Experiments showed that a surface of plate glass, which had been ground uniformly with the finest emery and then silvered, when used at an angle of 75°, reflected 90 per cent. at 4 μ, approaching 100 per cent. for longer wave-lengths, and only 10 per cent. at 1 μ, less than 5 per cent. in the visible red, approaching zero for the shorter waves. Very similar results were obtained for the transmission of a plate of rock-salt which had been roughened merely by breathing gently upon it. In both cases, the finer the surface, the more suddenly does it cut off the short waves.

3. Increasing the angle of incidence was observed to effect a profound and curious change in the reflection curves of polished glass, quartz and Iceland spar, which may best be described as a quasi-resolution of the bands of metallic reflection. This is thought to be due to a change in polarization at the reflecting surface (*i. e.*, a change probably from plane-polarized to elliptically-polarized light).

4. A description is given of an infra-red spectrometer which is believed to possess better definition and greater resolving power than is usually furnished by instruments of similar type.

In conclusion, the writer wishes to thank all those who have aided him in this investigation. He is under obligations to Professor J. S. Ames, for advice and criticism, and to Dr. J. A. Anderson, for the loan of gratings and mirrors. The mechanical excellence of the spectrometer is due to the skill of Mr. C. M. Childs, one of the mechanicians in this laboratory. To Dr. A. H. Pfund, who suggested and directed the work, the writer is very grateful for his ingenious suggestions, inspiring criticism, and constant encouragement.

JOHNS HOPKINS UNIVERSITY, June, 1915.

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[1] A. H. Pfund, Astrophysical Journal, XXIV., 1, 1906, p. 29.

THE DISTANCE BETWEEN TWO PARALLEL TRANSPARENT PLATES.[1]

By C. Barus.

1. *Introductory.*—The problem of finding the distance apart of two parallel glass disks, as well as their degree of parallelism, is frequently one of practical importance. Thus in my work on the repulsion of such disks, it would enter fundamentally and it has long been my intention to repeat that work with two half-silvered glass disks, for comparison of the case with metallic disks. It occurred to me since, that the method devised by my son, Mr. Maxwell Barus, and myself,[2] would probably be ideal for the purpose, both for very small distances (within .1 mm.) as well as for distances ten or even several hundred times larger. This method admits of the use of the film grating. There are three types of interferences of successive orders of fineness, the first virtually involving the colors of thin plates, resolved spectroscopically, the other two being dependent on diffraction. To measure the thickness of the air space, it would be necessary to count the number of fringes between two definite Fraunhofer lines, only, supposing the constants of the grating to be given.

2. *Apparatus.*—The apparatus has been designed for transmitted light, in preference, though the case of reflection is also available.

MM, Fig. 1, is the base of a Fraunhofer micrometer, firmly attached below to a massive tripod (not shown). *SS* is its raised slide, and *E* the head of the micrometer screw, reading to 10^{-4} cm. The open case *A* is screwed to the slide *SS* and contains the glass plate *H* half silvered on the right. *H* is attached to a plate on the plane-dot-slot principle, and may therefore be rotated around the vertical and horizontal axis by aid of a rearward spring mechanism (not shown) and the adjustment screws *abb'* (the last not visible). The grating *G*, with a ruled face on the left, is similarly carried by the open rectangular case *B*, screwed down to the base *M* of the micrometer. Thus *B* is stationary, while *A* moves. Three adjustment screws *cdd'* (not shown) and a spring pulling to the right, suffice to rotate *G* around the vertical and the horizontal axis. The thickness of the efficient air film is thus *e* and *H* and *G* may be brought to touch, or to recede from each other several cms. *L* is the collimator (slit and lens) furnishing intense white sunlight (or arc light) and the beam, after traversing the system, is viewed by the telescope at *T* (direct beams, Fig. 1*A*) or at *D* (diffracted beams).

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[1] Abridged from a forthcoming Report to the Carnegie Institution of Washington, D. C.

[2] Carnegie Publications, No. 149, Part I., Chap. II., III. Cf. Phys. Rev., XXXI., p. 591–8, 1910.

The plate *H* is half silvered, but the grating *G* has been left clear. In this case, however, only the fine fringes are seen strongly, on transmission.

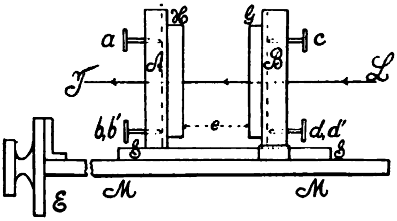

Fig. 1.

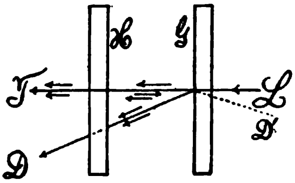

Fig. 1*A*.

The others appear on reflection, at *D'*, preferably in the second order of spectra. Fine fringes are not well reflected, but the medium and coarse fringes are very strong and clear, and the first observations were made by means of them.

Thereafter the ruled face of the grating was half silvered. This largely destroys the reflected field *D'*, except the fine fringes; but the transmitted field is now strong, particularly in the second order of spectra, for all the three sets of fringes in question. Mr. Ives's direct vision prism grating shows the fine fringes well in the direct beam *T*. The lines are always rigorously straight, so far as they can be observed, *i. e.*, it is impossible to bring *H* and *G* quite in contact, not only because of deposits of dust, but since the grating (at least) is not optical plate. The fine fringes may always be found in the principal focal plane of a telescope, but the medium and coarse fringes usually lie in focal planes differing from each other. By placing the ocular, it is thus possible to eliminate any of the interferences or to show a single set in the field only.

To find the fringes, the direct white slit images are made to coincide throughout their extent or the same may be done with a pair of spectrum lines in the superimposed spectra. The proper *e* is then to be sought. In case of imperfect plane parallel plates, it may be necessary to correct this by the adjustment screws on the mirror, until sharp strong fringes are seen in the corresponding focal plane.

3. *Equations.*—The equations for the three useful interferences in question are in case of $r < \theta_m$

$$n\lambda = 2e\mu \cos r, \tag{1}$$

$$n\lambda = 2e\mu \cos \theta_m', \tag{2}$$

$$n\lambda = 2e\mu (\cos r - \cos \theta_m'), \tag{3}$$

and a similar group for $r > \theta_m$, where λ is the wave-length of the color used, n the order of the interference, e the thickness of the sheet to be measured and μ index of refraction. If I is the angle of incidence the white light on the grating, r the angle of refraction in the plate (μ) and θ_m' the angle of diffraction of the mth order of spectrum therein. If the sheet is an air space, these equations become simplified since $\mu = 1$ and r is replaced by i, θ_m' by θ_m, the angle of diffraction in air. Thus, since positive values are in question,

$$n\lambda = 2e \cos i, \tag{4}$$

$$n\lambda = 2e \cos \theta_m, \tag{5}$$

$$n\lambda = 2e(\cos i - \cos \theta_m). \tag{6}$$

In the present apparatus I have made $i = r = 0$, a more convenient method of testing the method, though not necessary and in fact, often inconvenient in practice. The equations are then finally

$$n\lambda = 2e, \tag{7}$$

$$n\lambda = 2e \cos \theta_m = 2e\sqrt{1 - \left(\frac{m\lambda}{D}\right)^2}, \tag{8}$$

$$n\lambda = 2e(1 - \cos \theta_m) = 2e\left(1 - \sqrt{1 - \left(\frac{m\lambda}{D}\right)^2}\right) \tag{9}$$

if D is the grating space, and the interference in question is due to the grating spectrum of the mth order.

The meaning of the equations 7, 8, 9, is given in Fig. 2. The case of equation 7 may be seen in the direct white ray, Fig. 2*a*, provided the light of the focused slit image is resolved by the direct vision spectroscope. For this purpose Mr. Ives's grating with attached (direct vision) prism, may conveniently be placed in front of the telescope T, Fig. 1, focused on the slit. After adjustment these fringes appear strong. Of course H and G must be parallel and all but touch. Under the same conditions these fringes may be seen laterally in any order of spectrum, as in Fig. 2, *b*. Fig. 2, *c*, illustrates equation 8 and Fig. 2, *d*, equation 9.

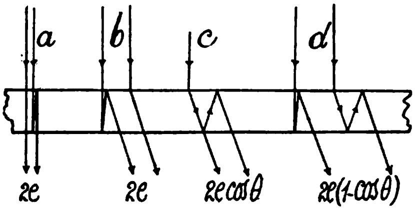

Fig. 2.

The first and second order of spectrum are alone intense enough to produce marked effects. In case of $i = 0$, a double diffraction of the first order (θ') reinforces a single diffraction of the second order θ_2, since

$$\frac{\lambda}{D} = (\sin\theta' - \sin\theta) = \left(\sin\theta' - \frac{\lambda}{D}\right) = \sin\frac{\theta_2}{2} = \frac{(2\lambda/D)}{2}$$

or

$$\sin\theta' = \frac{2\lambda}{D}.$$

Probably for this reason they are visible. The general case equations 4, 5, 6 is illustrated in Fig. 3 the rays I, I', I'' being incident, R reflected and D diffracted. The retardations are ef and df, respectively. If these diffractions differ by a whole number of wave-lengths, the total diffraction is obtained. One would be tempted to resolve the problem by aid of a wave from ab, in which case the equation would be different; but they do not reproduce the phenomena.

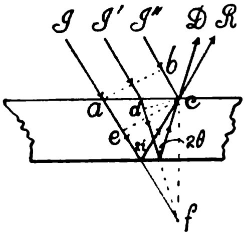

Fig. 3.

4. *Method.*—Suppose now that two Fraunhofer lines λ and λ' of the spectrum are selected, as the rays between which interference fringes are to be counted. Then in case of equation 7 if n is the number of interference lines between λ and λ'

$$n\lambda = (n + n')\lambda' = 2e, \tag{10}$$

$$n = \frac{n'\lambda'}{(\lambda - \lambda')}, \tag{11}$$

$$2e = \frac{n'\lambda\lambda'}{(\lambda - \lambda')} = C_1 n'. \tag{12}$$

In order to measure e therefore, it is necessary to count the number of fringes n' between λ and λ' and e varies directly as n'.

If the mean D and (magnesium) b lines be taken as limiting the range $10^6\lambda = 58.93$ cm., $10^6\lambda' = 51.75$ cm., $C_1 = 10^{-4} \times 4.25$ cm. and

$n = 1$, $10^3 e = .2$ cm.
10, 2.1
100, 21.2

As it will not be convenient to count more than 100 lines ordinarily, the method is thus limited to air spaces below .2 mm. and becomes more available as this film is thinner. Of course in case of plates which contain specks of dust, or lint, or are not optically flat on their near surfaces, it is extremely difficult to get e down below .002 cm., so that less than ten fringes between D and b would require very careful preparation.

If equation 8 is taken, λ is to be increased to

$$L = \frac{\lambda}{\cos \theta_m} = \frac{\lambda}{\sqrt{1 - \left(\frac{m\lambda}{D}\right)^2}},$$

where m is the order of the grating spectrum whose rays interfere. Thus equations 11 and 12 now become, since $nL = (n + n')L' = 2e$,

$$n = \frac{n'L}{(L - L')}, \tag{13}$$

$$2e = \frac{n'(LL')}{(L - L')} = C_2n'. \tag{14}$$

If first order of diffractions are in question, $m = 1$, $10^6L = 59.11$, $10^6L' = 52.33$, $C_2' = 10^{-4} \times 4.20$, scarcely differing from the preceding case, so that one would not know in which series one is working.

If the diffractions occur in the second order, $m = 2$, $L_2 = 10^{-6} \times 62.56$, $L_2' = 10^{-6} \times 54.15$, $C_2'' = 10^{-4} \times 4.03$, thus again differing but slightly from the above.

If we inquire into the conditions of coincidence and opposition of these fringes, it follows easily, if C is the new constant, that

$$C = \frac{C_1C_2}{(C_1 - C_2)} = 10^{-2} \times 4.05.$$

This would place the fringes beyond the coarse group below, but naturally C is enormously dependent on small errors in C_1 and C_2.

Finally, if equation 9 be taken, the λ is to be increased to

$$M = \frac{\lambda}{(1 - \cos \theta_m)} = \frac{\lambda}{1 - \sqrt{1 - \left(\frac{m\lambda}{D}\right)^2}}$$

in order that equations similar to the above may apply for $2e = n'M$. Thus

$$n = \frac{n'M}{(M' - M)}, 2e = \frac{n'MM'}{(M' - M)} = C_3n'.$$

If the diffractions are of the first order of spectrum, $m = 1$ and

$$10^3 M = 4.150; \; 10^3 M' = 4.747,$$

whence

$$C_3' = .0330.$$

These are the coarse order of fringes, so that

$n =$	$e =$
1,	.0165 cm.
10	.165
100	1.65

Fringes are thus still strongly available even if the distance apart of the plates is over 2 cm.

If the diffractions are of the second order of spectrum, $m = 2$, $10^3 M = 1.06$, $10^3 M' = 1.165$, $C_3'' = 10^{-3} \times 7.85$. These fringes are therefore of intermediate size, since

$n =$	$10^3 e =$
1,	3.97 cm.
10	39.75
100	397.5

They are enhanced since they cooperate with the double diffractions of the first order.

5. *Observations. Preliminary Work.*—The following work was done merely with a view to testing the equations and with no attempt at accuracy. The grating was left unsilvered, so that the ruled face confronted the half silvered surface on ordinary plate glass. Consequently the fine fringes were observed by transmitted light behind, and the medium and coarse fringes by reflected light, in front. The micrometer was a good instrument for general purposes, but hardly equal to the present work, where the slightest rocking of the slide introduces annoyances.

To the count the number of fringes between D and b, since the fringes were not generally seen in the principal focal plane of the telescope, it was considered sufficient to rotate the cross hair into an oblique position, until its ends terminated in the D and b lines, respectively, and then to count the number of fringes on running the eye down the hair, from end to end. When there are many fringes, 25 to 50, the eye is apt to tire before reaching its destination, so that several counts must be made and the mean taken.

The results are given as a whole in the chart, Fig. 4. In this, the distance between plates, measured in cms. on the micrometer, beginning at an approximate zero, is laid off horizontally and the number of fringes vertically, in case of each of the three series. The computed line $e = Cn'/2$ is drawn in full and the observations laid off with regard to it. The zeros do not quite correspond, as very small distances here are significant. With the fine fringes I did not spend much time. They are virtually colors of thin plates seen by diffraction. The chief difficulty with these small distances is this that when the plates touch, a complete readjustment is necessary. After contact, the micrometer acts like a forcing screw and its reading is therefore too low. This is the meaning of the displaced data in the curves *a* and *a′* (the latter with its horizontal scale magnified ten times). The object of this series is chiefly to locate the position of the line in relation to the other lines.

The observations for medium and fine fringes were made together, so that a single micrometer reading suffices. Beginning with very small distances apart (called zero), this was rapidly increased to nearly 1 cm. The fine fringes soon vanish; later the medium fringes follow; finally, when *e* is several cm., the coarse fringes also vanish, the three together, therefore, covering with appropriate accuracy, a relatively enormous range for work of this kind.

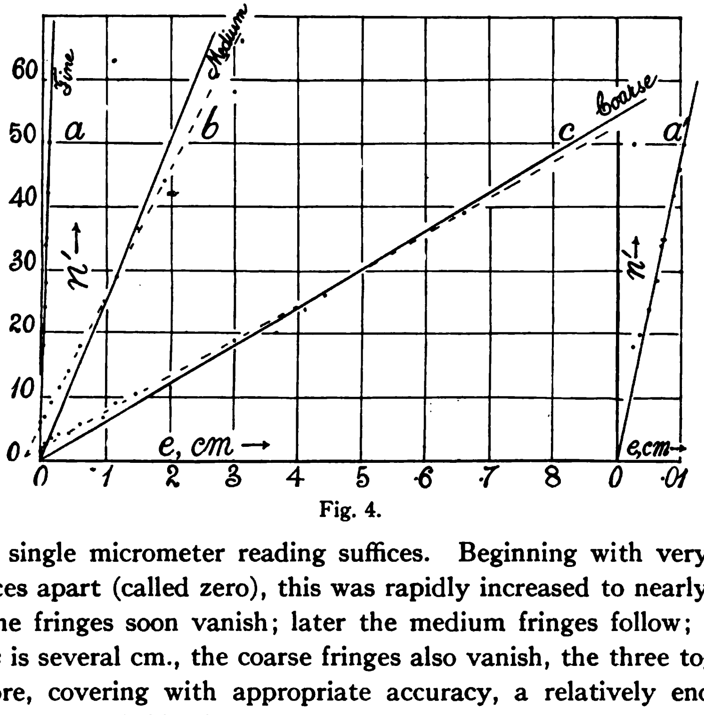

Fig. 4.

The curves *b* and *c* show that the observations do not completely correspond to the computed loci. Mean lines drawn through the observations must in the first place indicate that the zeros disagree; for *b* and *c*, sufficiently to locate the common zero. This discrepancy is inevitable, since the micrometer begins to count at a small distance, which is otherwise arbitrary. In fact it should be noticed as an accessory property of the present method of interferometry, that the two lines for finding the zeros determine the absolute reading of the micrometer, mutually and the latter readings are here .22 mm. too large. But even if the zeros were horizontally to coincide, the observations would not adequately reproduce the computed loci. All that can be affirmed is that the angle between the observed and computed lines is about the same.

The main reason for the divergence is referable to the fact that the air space is not quite plane parallel, but slightly wedge shaped, so that the effect of the angle of the wedge is superimposed on the interferences. Any slight unsteadiness of the micrometer slide, for instance, would already introduce the wedge discrepancy, without necessarily interfering with the sharpness of visibility; while any attempt to readjust would destroy the continuity of measurement. There will also be further reasons for divergence, as for instance the three separate focal planes in which the fringes lie; the fact that the glass plates, which limit the air space, are themselves wedge-shaped. If the half silvered surface were optically plane parallel plate, then the adjustment for visibility of fringes would automatically correct for the wedge-shaped air space. Other, but fainter, fringes are marching through the spectrum, such cases of coincidence and opposition, for instance, as were pointed out above. However, the real cause of discrepancy is the assumption $i = 0$, which was probably not true.

To give the subject further study, I have since half silvered the grating as specified, so that all the fringes may be seen by transmitted light and preferably in the second order, since there is an abundance of light available. The apparatus in such a case is in good shape and is convenient for manipulation. But these details will have to be given at some other time and it is the chief purpose of this paper to exhibit the phenomenon as a whole.

In conclusion I may recall, that if we regard one hundred fringes between the *D* and *b* lines still available for counting under proper facilities, the successive ranges of measurements will be roughly as follows:

	$e = .021$	$e = .392$	$e = 1.65$ cm.
Fine fringes	$n' = 100$	———	———
Medium fringes	5.4	$n' = 100$	———
Coarse fringes	1.3	24	$n' = 100$

The transition from fine fringes to medium is a little abrupt. Otherwise, in cases where manual interference is not permissible, all thicknesses of air films from a fraction of a wave-length of light, to nearly 2 cm., may be adequately measured in this way, to advantage. It is probable moreover that it would be advisable to observe the fine fringes by transmitted light but to leave the grating (which may be a film grating) clear, and to observe the medium and coarse fringes by reflected light. A concave mirror and lens (reflecting telescope) should be used for this purpose, as it will put the observer behind the plates in all cases.

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COMPARISON OF THE FALL OF A DROPLET IN A LIQUID AND IN A GAS.

By O. W. Silvey.

Introduction.

In his early work on the determination of the elementary electrical charge by the oil drop method, Professor Millikan[1] found that the values of the charge computed from droplets of different radii indicated a breakdown in Stokes's[2] formula by which the radii were determined. Since this formula was deduced on the assumption of a continuous medium and no slipping at the surface, a breakdown in the law was to be expected when these conditions were not fulfilled, and various attempts have been made to modify the formula so as to make it apply to cases where slipping occurs and where the medium becomes discontinuous. Among the formulæ making such corrections are those of Basset,[3] Cunningham,[4] and Hadamard.[5] Basset treated the phenomenon as a case of slipping at the surface; Cunningham made use of the kinetic theory point of view and assumed that the medium becomes discontinuous when the ratio of the mean free path of the molecules of the medium to the radius of the drop becomes appreciable, and that the departure from the law is further modified by the fraction of the whole number of impacts which are elastic. Hadamard deduced a correction for the case of liquid drops moving in a fluid, assuming a vortex motion of the liquid within the drop due to its motion through the fluid. He wrote his correction in terms of the coefficient of viscosity of the liquid and that of the fluid through which it moves.

To account for the apparent variation of the value e, Millikan wrote empirically the correction to Stokes's law as a function of the ratio of the mean free path to the radius of the drop, which when higher powers of $1/a$ than the first are neglected, reduces to the approximate formula:

$$v = 2/9\, g \frac{a^2(\sigma - \rho)}{u} (1 + A\ 1/a),$$

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[1] Millikan, Phys. Rev., Vol. 32, 1911, p. 350.

[2] G. G. Stokes, Mathematical and Physical Papers, III, p. 59.

[3] Basset, Hydrodynamics, Vol. II., p. 270.

[4] Cunningham, Proc. of the Royal Soc. of London, Series A, Vol. 83, 1909–1910, p. 357.

[5] Hadamard, Comp. Rendus, March, 1911.

where v is the velocity of fall, a the radius of the spherical droplet, σ the density of the falling substance, ρ the density of the surrounding medium, u its viscosity, 1 the mean free path of the molecules of the medium, and A an empirical constant.

Coincident with the determination of the absolute value of the elementary charge, by plotting the apparent value of $e^{2/3}$ as ordinates and $1/a$ as abscissas, he obtained the value of the constant A from the slope of the curve, the intercept of which on the axis of ordinates gave the absolute value of e. The value of A given in the original paper for oil drops in air was .874, which in accordance with Maxwell's analysis indicates that most, though not all, of the impacts are non-elastic. Assuming that the same value held for mercury, he computed the value of the charge on droplets of this metal which were found to be in good agreement with those computed from oil drops. Experiments with substances, other than oil and air, have given slightly different values. Lee[1] obtained for shellac drops in air, $A = 1.067$, while Millikan[2] obtained $A = .820$ for oil and very pure hydrogen.

Recently an attempt was made by Roux[3] to determine the constant A for mercury and sulphur drops by allowing them to fall in air, then in a liquid, and measuring their velocity in the two media. He assumed that Stokes's law holds for the fall in the liquid and that from the ratio of the velocities the value of A could be determined.

Roux's method appears capable of yielding good results but since the correction in the formula depends upon a 5.1 per cent. increase in the velocity in air for the smallest drop used by him (time of fall in air, 5.2 sec.), and this time of fall was measured by means of a stopwatch, which permitted an error of 0.2 second on both stop and start, he has attempted to measure a quantity which depends on 5.1 per cent. error in the time of fall by an instrument which permitted an error of 7.7 per cent. of the quantity measured.

I.

GENERAL METHOD.

It was, therefore, proposed to repeat the work of Roux under conditions which permitted a more accurate measurement. Instead of a stopwatch, the Hipp chronoscope, used by Millikan, was employed, by which time intervals could be measured to 1/1000th of a second. To aid in overcoming the personal error, an apparatus was so arranged that a charged droplet could be raised by an electrostatic field and allowed to fall a number of times. About 40 falls usually were measured for each droplet. It was then allowed to fall into the liquid and its fall measured through as many one or two millimeter intervals as was possible. From these measurements and the necessary constants the value of the radius a, and of the constant A, were calculated.

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[1] Lee, PHYS. REV., Vol. IV., Second Series, Nov., 1914, p. 420.

[2] Millikan, Barber and Ishida, PHYS. REV., Vol. V., Second Series, 1915, p. 334.

[3] Roux, Annales de Chimie, Vol. 29, p. 69. 1913.

Choice of Liquid.

No better liquid was available than xylol, the one used by Roux. In order that droplets of the size required enter the liquid it was necessary that the liquid have little or no surface film; in order that the ratio of the two velocities be small, the viscosity should be low; and in order that there be as little evaporation as possible the boiling point should be high. Xylol has the first two of these properties, but in spite of its high boiling point it is quite volatile. To prevent changes of viscosity and density due to evaporation, the xylol was distilled, the middle third being reserved for use. This had a boiling point of about 140° C. and measurements of density and viscosity made a month apart on liquid that had stood in the apparatus during that interval showed no appreciable change.

Measurement of Density and Viscosity.

The density was measured by means of a pyknometer, and the viscosity by improved Ostwald viscosimeters, which had been used a number of years both here and at the Boston School of Technology. These[1] tubes were kindly loaned me by Professor Harkins, of the Kent Chemical Laboratory. Measurements were also made by a longer tube of the same kind which was made for the Ryerson laboratory. The measurements made by these three tubes (16.5, 21.5 and 25 cm. in length) agreed within 0.2 per cent. For comparison, water was used, the absolute value of the viscosity being taken as the mean of the various values given in the tables. A linear relation was assumed between the values at 20.°0 and 25.°0 C. for water and between 23° and 25° for xylol when determining the viscosity within these ranges. The values obtained for density and viscosity are

	23°.00.	25°.00.
Density	.8504	.8487
Viscosity	.006226	.006107

For the viscosity of air at 23° the value, .0001824, was used.

Deduction of the Value of A.

To deduce the formula used in solving for A, the corrected Stokes's equation was written for the velocity in air

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[1] A description of a similar type of tube is given by Washburn, Journal American Chemical Society, p. 737, 1913.

$$v_1 = 2/9\, ga^2 \frac{(\sigma - \rho_1)}{u_1}(1 + A\, l/a) \qquad (1)$$

and the ordinary equation for the fall in the liquid,

$$v_2 = 2/9\, ga^2 \frac{(\sigma - \rho_2)}{u_2}, \qquad (2)$$

where the subscripts 1 and 2 refer to the constants of the air and the liquid respectively.

Dividing (1) by (2) we get

$$\frac{v_1}{v_2} = \frac{\dfrac{\sigma - \rho_1}{u_1}(1 + Al/a)}{\dfrac{\sigma - \rho_2}{u_2}}.$$

Whence

$$A = \left\{ \frac{v_1 u_1 (\sigma - \rho_2)}{v_2 u_2 (\sigma - \rho_1)} - 1 \right\} \frac{a}{l}.$$

If now, v be replaced by $1/T$ and $u_1/u_2(\sigma - \rho_2)/(\sigma - \rho_1)$ by α, where T is the time required for the droplet to fall unit distance and α a constant for the particular temperature of the medium at the time of fall, we obtain

$$A = \left(\frac{T_2}{T_1}\alpha - 1 \right) \frac{a}{l}.$$

The radius a was obtained by solving equation (2). For this we get

$$a = \sqrt{\frac{9}{2} \frac{u_2}{g(\sigma - \rho_2) T_2}} = \beta \sqrt{\frac{1}{T_2}},$$

where

$$\sqrt{\frac{9}{2} \frac{u_2}{g(\sigma - \rho_2)}} = \beta$$

and

$$T_2 = \frac{1}{v_2}.$$

The Apparatus.

The apparatus used (Fig. 1) consisted of a pair of condenser plates PP_1, of the Millikan type each 22 cm. in diameter. At the center of the upper plate were three holes about 0.7 mm. in diameter through which droplets of mercury might fall. Over the center of the plate was a sheet-iron disc D_1 which by means of a lever could be moved aside to allow the falling droplets to reach the holes. At one edge a small cup filled with mercury provided an electrical contact with the wire leading through the ebonite plug E. The lower plate, shown in horizontal section in Fig. 1*A* was made of sheet iron about 0.36 mm. thick. It was fastened by means of screws to a short wooden cylinder of the same diameter and about 5 cm. deep. On it rested three glass posts, *T*, *T*, *T*, 15.875 mm. high, all cut from the same piece of plate glass. These supported the upper plate. In the center of this thin plate was cut a circular hole 6.4 mm. in diameter and to the lower side was cemented a rectangular glass vessel *V*, the inner dimensions of which were 4.9 × 4.9 × 2.1 cm. The wooden cylinder was cut away at the center to receive the vessel *V*, and was furrowed along the diameter L_1L_2 *O*, as shown by the dotted lines to allow light to reach the vessel. A second furrow along the radius *CO*, allowed the illuminated vessel to be seen. The vessel, except on the upper side was, therefore, almost completely surrounded by wood. A brass cylindrical post screwed into the center of the lower side of the wooden cylinder supported the whole condenser firmly with its axis coincident with that of an enclosing cylindrical sheet

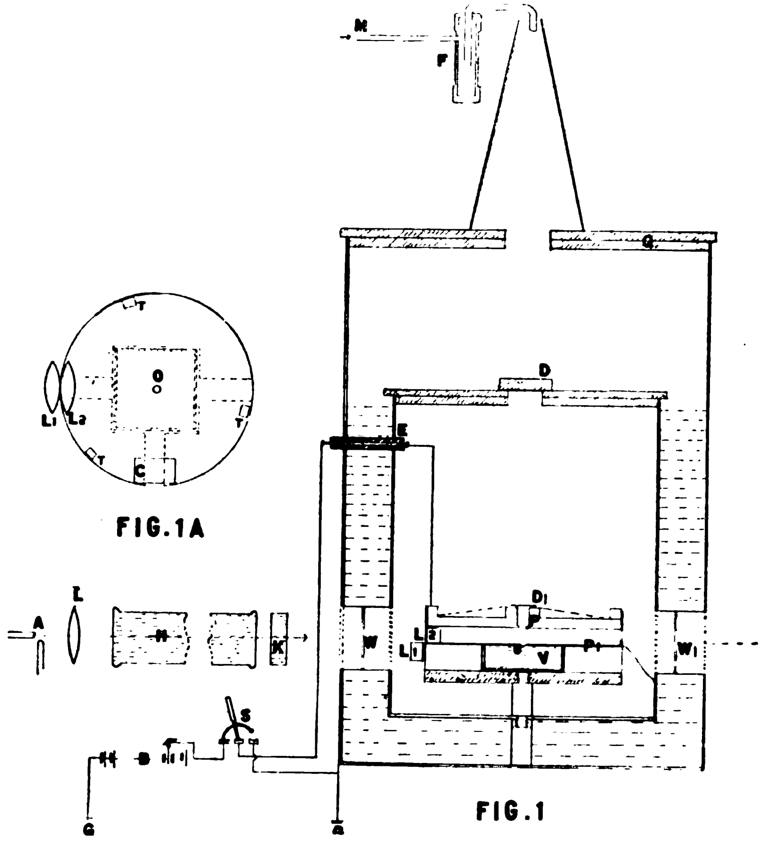

FIG. 1A

FIG. 1

iron tank, 30.5 cm. in diameter and 30.5 cm. high, which in turn was enclosed in a larger tank of the same material, 45.7 cm. in diameter and 67.5 cm. high. The two tanks were held rigidly in their relative position by the central brass post, which supported the condenser and by four pieces of telescope tubing. One of these, 2.5 cm. in diameter, held the insulating ebonite plug *E*, the other three were 7 cm. in diameter. Two of these in the position *WW* allowed light to pass through the vessel *V* and between the condenser plates, the other at right angles to the line WW_1 permitted the observer to see the vessel. Each of the 7 cm. tubes contained a glass window. The lower plate of the condenser was electrically connected to the inner tank and therefore, to earth by grounding the outer one. Both tanks were provided with wooden covers and to prevent rapid changes of temperature the outer was covered on top and sides with asbestos about 6 mm. thick. The space between was filled with oil.

In his original work on mercury drops, Millikan had formed his drops both by atomizing liquid mercury and by condensing the vapor rising from boiling mercury. Roux had used the latter method and it was followed here. The droplets were formed by blowing a sudden strong pulse of air through the iron vessel, *F*, which contained a small amount of boiling mercury. To prevent, as much as possible, the heat from the bunsen burner and the boiler *F* reaching the condenser, they were placed about 50 cm. above the top of the outer tank. The mercury vapor carried by the air blast condensed into droplets after leaving the boiler and fell freely to the condenser when the slide covering the inner tank and the one covering the holes in the condenser were held aside. A conical sheet iron shield supported by the cover of the outer tank prevented the mercury from being blown over the room.

The system was illuminated by a right-angle arc lamp, *A*, the crater of which was focused by lens *L* in the position of the window *W*, and was further focused by cylindrical lenses L_1 and L_2 into a vertical band of about 8 mm. wide and 3.5 cm. long at the axis of the condenser. Lens L_2 was of short focal length and it was necessary to place it between the condenser plates. Lens L_1 was of longer focal length and was placed just outside the wooden cylinder. To eliminate non-luminous radiation, the light was filtered through a 52.5 cm. column of water *H* and through *K*, a 12 per cent. solution of cupric chloride in alcohol 1 cm. thick. This has been shown by work at the Ryerson Laboratory more efficient than a solution of cupric chloride in water. It allows practically no radiation except that of the visible spectrum beyond the yellow to reach the condenser.

The upper plate was electrically connected to a series of storage cells *B* by closing switch *S*, and the two plates were short circuited and earthed by the opposite throw of the switch. The potential of the plate could be varied by using a greater or less number of cells, the maximum potential being about 4,000 volts.

To prevent air currents, due to irregularities in the density of the air in the tank, the condenser was surrounded by a transparent celluloid screen in which windows were cut at positions L_1L_2 and at *C* (Fig. 1*A*). Lenses L_1 and L_2 closed the window at this position.

The droplets which fell through the holes in the upper condenser plate were viewed in the direction *CO* through a short-focus telescope of constant length which had an objective 2.5 cm. in diameter and a magnifying power of about 23. A carefully ruled scale in the eyepiece provided a means of measuring in millimeters the distance traversed by the droplets. The telescope could be raised by rack and pinion so as to view the droplet in the air, then lowered to see it in the liquid which except for the curved meniscus at *O* completely filled the vessel *V*. To insure finding the droplet in the liquid a piece of optical glass *C* was placed between the condenser plates which served the double purpose of closing the window in the surrounding shield and of compensating for the change of optical path when the droplet entered the liquid. The thickness of xylol for which the glass *C* would compensate was determined by experiment before cementing *V* to the condenser plate.

Manipulation.

With the condenser plates earthed, slide *D* open, and D_1 closed, a cloud of droplets was blown. Slide D_1 was now opened, allowing them to fall through the small holes of the condenser, switch *S* was then closed and the droplets of the correct sign were drawn up by the electric field. One of these was chosen for observation, slide D_1 again closed to prevent any more entering the condenser, then by letting the chosen droplet rise and fall a number of times, all others were cleared from the field. This droplet was now drawn to the upper part of the field and allowed to fall under gravity through one centimeter distance, the time of fall being measured each time by the chronoscope. This was repeated a number of times, usually about 40. During this time all other droplets which had entered the liquid fell to the bottom. The droplet was now allowed to fall into the liquid, the telescope was quickly lowered and the droplet was seen to come through the meniscus and slowly fall. The telescope was now shifted until the droplet was accurately in focus and the time of fall measured through as many millimeter or two millimeter distances as was possible. The average of these millimeter readings multiplied by ten was taken as the time of fall of 1 cm. in the liquid. Irregularities in the speed at various parts of the path made it necessary to record the time of fall of separate millimeter distances rather than over the whole course.

The electrical field was not uniform at the center because of the central hole in the lower plate. This non-uniform field proved an advantage over a more uniform one obtained by means of a wire gauze employed in preliminary experiments, because, if the droplet was not originally well over the hole, it rose following along the lines of force but fell vertically. Therefore, after a few journeys it was well over the hole and within the band of concentrated light. Its chance of falling into the liquid instead of striking the plate was thereby assured.

Sources of Error.

Simple as the experimental arrangement seems, some difficulties were met in carrying out the measurements. These were of two types, one the faint illumination of the isolated droplet in the liquid, and the other the convection currents in both the air and the liquid. The first was met by using the cylindrical lenses previously discussed and by eliminating all possible reflecting surfaces of glass. This was the reason for closing the windows in the shield by the lenses and optical glass *C*. The part of the window at *C* below the lower condenser plate was left open. A further reason for eliminating these glass surfaces was that the xylol sometimes condensed on them, producing a hazy, or double image of the droplet.

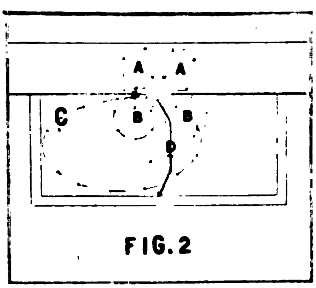

FIG. 2

The most difficult problem was the elimination of convection. It was found that nearly all irregularities in readings were finally traced to this cause and that a very weak convection current in either air or liquid caused an enormous error in the measurements. After all convection due to any mechanical disturbance had died away, which usually was within an hour, a set of currents gradually increasing in strength, shown by *AA* and *BB*, Fig. 2, appeared. These were due to the evaporation and consequent cooling of the xylol at the hole *O*. The mercury droplet, having a comparatively large inertia gave no information regarding these currents except that in air it appeared to fall with different speeds in the upper and in the lower part of the path. In the liquid if streams of particles were allowed to fall from all three holes, during the first five millimeters, the three streams, which were originally about 2 mm. apart, were drawn very nearly together, the droplets moving very rapidly. After falling this distance, the three streams again separated, fell vertically, and more slowly for a few millimeters, to be again slightly accelerated at the bottom of the vessel. The cause of this behavior was discovered by introducing dust particles which were sufficiently light to take on the motion of the medium. The complete path of the lighter particles was then seen to be as shown by *BB* in Fig. 2.

The acceleration at the bottom of the vessel which for a long time was thought to be only apparent and due to a defect in the optical system was caused by a weaker convection current which probably took the course indicated by *C*, Fig. 2. This drift was not discovered until *A* and *B* were eliminated and then only by the introduction of lighter particles. The particles introduced were obtained by leaving the holes in the upper plate uncovered when the cloud of droplets was blown. The shower of droplets which fell under these conditions was sometimes preceded by a cloud of slowly falling particles, especially if the mercury in *F* had been boiled very vigorously for about a minute before blowing. Some of these were found later to be extremely small mercury droplets, too small to be seen in the liquid, others were probably compounds of mercury, and others, no doubt, were dust particles of the air. When one of these particles entered the liquid, it was seen to drift out of the field of view in the direction of the light propagation at the top of the vessel. A six-second[1] mercury droplet drifted sometimes as much as two millimeters in the direction of the light during the first four or five millimeters of fall in the liquid, then fell vertically with a uniform velocity through from 5 to 10 millimeters, then drifted in the opposite direction with an accelerated speed during the last few millimeters of fall. The path of the droplet was as shown by *D*, Fig. 2. Mercury droplets less than four seconds showed the change of speed but no appreciable drift.

Since this type of convection current was found to increase with time of observation, and was in the direction of the light at the top and oppositely directed at the bottom of the vessel, it was probably due to a very slight heating of the liquid on the side of the incident light. The fact that no convection appeared at the center was probably due to the particular size of vessel used.

When taking observations it was necessary to know to what extent these convection currents existed. The presence of continually visible, slowly falling particles was undesirable and was avoided as much as possible, but it was learned early in the investigation that a few minute, irregular particles were always present in xylol. These were invisible except when a reflecting side was turned to the light, but, due to Brownian movements, they would be turned to show scintillation for a second or two, which would fade away to sparkle again in approximately the same place a few seconds later if the liquid was perfectly at rest. This test for convection was used through all the latter part of the work, and the fact that the convection was not observed at the top and bottom was, as stated above, because the change of speed at this position was thought to be apparent and due to an optical defect, and only the middle half of the liquid was tested for convection. After it was proven that the change of speed was real and that no optical defect existed, a careful study of the scintillations plainly showed the drift.

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[1] It is convenient to refer to the size of the droplets in terms of the time required for them to fall one centimeter in air. Hence a six-second droplet is one that falls one centimeter in six seconds. The smallest droplet upon which measurements were made was a 6.19 second drop, and the largest was a 1.41 second drop.

Since the illumination could not be further decreased without diminishing the brightness of the droplet, no attempt was made to eliminate the convection of type *C* further than to keep the light screened from the liquid except during the time of observation of the droplet in it. With this precaution observations could be taken on five to eight droplets before it became too troublesome. In the computation, only the minimum speed was used, for it was with this minimum speed that the droplet was falling through this middle portion of the field where the liquid was at rest and Stokes's law was applicable.

The convection currents *AA*, *BB*, were eliminated by partially saturating the air in the tank with xylol vapor. The space could not be completely saturated because the xylol then condensed on the glass surfaces, thus producing a blurred image of the droplet and also condensed on the droplet itself, thus decreasing its speed in the air. To produce this saturation, the inner tank was lined at top and sides with cotton which was held in place by iron wire gauze. This cotton was saturated with xylol and about 200 c.c. of xylol was kept at the bottom of the tank. The tank was left closed for at least twelve hours before observations were taken, then if all other conditions were favorable, no evaporation took place at *O*, and no convection currents were present. Before beginning observations the tanks were uncovered and the saturated air removed by an electric fan. The cover of the inner tank was as a rule not replaced during observation and since the cover to the outer one was not tight-fitting, the air in the tank was not completely saturated. Under these conditions, a small droplet sometimes showed a gradual increase in time of fall, and in time of rise under the electric field, showing that they were slowly collecting liquid. In the computation, only the average of the first ten falls was used for droplets showing this condensation.

Mercury droplets as a rule evaporate in dry air, the evaporation being plainly shown by their gradual increase in time of fall and their gradual decrease in time of rise under the field. This has been observed here by all who have attempted to work with mercury droplets and has been observed and well described by A. Schidlof and A. Karpowicz.[1] However, with our experimental arrangement, not all droplets showed this evaporation, and it was found to decrease with time and finally cease. In the presence of xylol vapor even far below saturation, this evaporation did not occur or was so slight that it was not detected.

Besides the convection currents just described, even a small change of room temperature, extending over a period of as much as three hours, produced unequal heating in the liquid, thus setting up convection, which did not die away until the room temperature had been held constant several hours. Therefore, for the latter part of the work, a thermostat and an electrical-heating apparatus was set up, by which the temperature of the room was maintained constant within 0.°4 C. except during the time of observation. A Beckmann thermometer placed in the oil between the tanks showed no greater change than 0.°01 C. in the temperature when the arc was not in use.

During an observation period, the extra heating of the room due to the arc, and the blasts of hot air used in blowing the droplets into the chamber gradually caused convection currents which first began to be detectable after two or three hours of observing. Care was taken that the observations recorded below are free from error due to this source.

A further troublesome feature was a film forming on the surface of the xylol when it stood undisturbed for a few hours. As a result of this, the droplets which fell into it at the beginning of a period gave no agreement among themselves for the value of the constant A. It seemed that the film retained the smaller particles, and a droplet with sufficient weight to break the film carried with it part of the film together with the neighboring small particles. Very frequently a droplet would carry a sufficient amount with it to glow brighter in the liquid than in the air. To eliminate this difficulty, at the beginning of each period, the surface film was, therefore, destroyed by allowing several showers of large particles to fall through it.

It has already been stated that not all the particles which fell were pure mercury. In the choice of a droplet, one was not always able to select pure mercury. The dust particles could be easily avoided because they either fell much more slowly, or were very much larger and brighter. Sometimes the droplet selected seemed to be a small dust particle upon which mercury had condensed. These on entering the liquid fell with a greater or less speed than if they had been pure mercury, depending upon whether they lost or retained the dust particle at the surface. A few of this type which were selected showed clearly a difference in brightness on rising with the field, from that shown when falling. After carrying out observations on the first one of these, any others which showed this effect were dropped as undesirable.

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[1] Schidlof and A. Karpowicz, Phys. Zeitschrift, 15, No. 4, Feb., 1915, p. 42. Also Comp. Rend. 158, p. 1992, 1914.

Observations.

A set of observations on a single drop are as follows:

April 14. Temperature: 25°.18.

Time of Fall, Sec. per Cm.	Time of Fall, Sec. per Cm.	Time of Fall, Sec. per Cm.	Time of Fall, Sec. per Cm.
2.940	2.995	2.861	2.865
2.953	2.901	2.919	2.853
2.977	2.899	2.919	2.924
2.909	2.928	2.897	2.865
2.897	2.879	2.880	2.845
2.927	2.884	2.821	2.872
2.978	2.881	2.922	2.898
2.920	2.886	2.855	2.851
2.839	2.912	2.916	2.890
2.864	2.902	2.910	2.911
		2.881	

Fall in Liquid.		Time of Fall, Sec. per Cm.
Distance.		
2 mm.	21699	108.49
2 mm.	21639	108.19
1.8 mm.	19442	108.01
2 mm.	19960	
2 mm.	20060	

Average time of fall in air, 1 cm. = 2.851 sec.
Average time of fall in liquid, 1 cm. = 108.23 sec.
a = .000143.
A = 1.050.

Since the chronoscope ran only seventy seconds without rewinding, no readings were taken, in general, until the droplet had fallen five millimeters in the liquid. In this case, three readings were taken before it ran down. This corresponded to a fall of 7 mm. Time required to rewind allowed the droplet to fall into a liquid which accelerated its speed.

Observations on a smaller drop are as follows:

April 21. Temperature: 25°.00.

Time of Fall in Air.	Sec. Per Cm.	Fall in Liquid, One and 2 Mm. Distances.
6.119	6.268	23.976
6.144	6.246	23.702
6.106	6.254	48.981
6.083	6.206	46.495
6.105	6.284	21.563
Average................	6.181	

Average time of fall, 1 cm. in air = 6.181.
Average time of fall, 1 cm. in xylol = 241.6 sec.
$a = .0000956.$
$A = 1.039.$

In all, forty-three readings were taken on this droplet in air but the constant increase in time of rise under the field and fall under gravity showed clearly that xylol was condensing on it. It was so small that it drifted where the liquid was not at rest. It fell vertically while the first three readings were taken, which corresponded to about 5 mm. During the fourth reading it drifted about 1 mm. Therefore, only the first ten readings in air and the first three in the liquid were used in computation.

	Date of Observation.	No. of Falls in Air.	Temperature.	Average Time of Fall, 1 Cm. in Air, Sec.	Max. Time of Fall in Xylol, Sec. per Cm.	$\frac{Tl}{Ta}$	$a \times 10^4$	A
1	Jan. 29	44	22.82	1.421	54.78	38.5	2.036	1.07
2	Jan. 27	43	22.82	1.443	55.40	38.38	2.025	.95
3	Jan. 27	39	22.82	1.532	59.06	38.50	1.961	1.01
4	Jan. 29	43	22.82	1.642	62.88	38.8	1.900	.84
5	Jan. 29	42	22.82	1.721	66.32	38.5	1.850	.95
6	Jan. 29	42	22.82	1.782	68.00	38.2	1.828	.74
7	Jan. 27	43	22.82	1.792	68.52	38.3	1.821	.78
8	Jan. 29	42	22.82	1.862	72.38	38.8	1.772	1.07
9	Jan. 27	42	22.82	2.041	79.14	38.8	1.693	1.02
10	Jan. 27	39	22.82	2.308	89.55	38.8	1.589	.93
11	Feb. 27	43	22.82	2.439	94.57	39.2	1.549	.95
12	Jan. 27	43	22.82	2.548	100.7	39.5	1.502	1.11
13	Feb. 27	41	22.82	4.020	159.39	39.7	1.194	.96
14	Mar. 6	43	22.82	4.435	175.83	39.7	1.138	.92
								.95

For this temperature $\alpha = .027219.$
and $\beta = .0015072.$

Before installing the heating system, regulated by the thermostat, an attempt was made to maintain the temperature constant at 23.°0 (calibration of the thermometer showed this to be 22.°82), and on some days conditions were sufficiently good to take observations. The temperature was not accurately known. It was measured by a thermometer placed beside the condenser and read through the window W_1. The temperature recorded was probably as much as 0.°2 C. in error in some cases. A summary of the readings taken under these conditions is as follows:

In the early spring the increase of the outside temperature and the fact that work was being carried on in an adjoining room at higher temperature than 23.°00 made it necessary to work at a higher temperature if the room remained constant. The remainder of the work was, therefore, done at 25.°00.

The following observations were made:

	Date of Observation.	No. of Falls in Air.	Temperature.	Average Time of Fall, 1 Cm. in Air, Sec.	Max. Time of Fall in Xylol Sec. Per Cm.	$\frac{T_x}{T_a}$	$a \times 10^4$	A
15	Apr. 14	43	25.2	1.668	62.49	37.4	1.878	1.13
16	Apr. 17	20	25.2	1.707	63.59	37.4	1.740	.95
17	Apr. 16	43	25.3	1.987	74.40	37.4	1.720	1.05
18	Apr. 19	43	25.0	2.059	77.52	37.7	1.692	1.06
19	Apr. 21	42	25.0	2.114	79.69	37.8	1.665	1.07
20	Apr. 21	43	25.0	2.145	80.69	37.7	1.732	1.03
21	Mar. 23	43	25.3	2.257	84.95	37.7	1.610	1.08
22	Apr. 16	43	25.3	2.866	108.56	38.0	1.424	1.06
23	Apr. 14	42	25.2	2.849	108.23	38.0	1.427	1.05
24	Apr. 21	43	25.0	2.924	110.94	37.8	1.415	1.01
25	Apr. 19	43	25.0	3.074	116.33	38.0	1.377	.98
26	Mar. 22	43	25.2	3.754	143.30	38.2	1.241	.99
27	Apr. 14	42	25.2	4.852	185.00	38.2	1.091	.86
28	Apr. 14	43	25.2	5.189	200.59	38.75	1.048	1.00
29	Apr. 21	10	25.0	5.343	208.44	38.75	1.029	1.06
30	Apr. 21	10	25.0	6.182	241.64	39.10	0.9563	1.04
31	Apr. 3	40	25.0	6.190	244.00	39.5	0.952	1.08
								1.03

Discussion of Results.

The temperatures recorded here were taken by placing the bulb of a thermometer in the vessel of xylol after completing the observations on the last droplet of the period. Where heating occurred during the period, the use in the formula of the coefficient of viscosity at the final temperature would make the computed value of A for early droplets of the period a little too high. Further, since only the maximum values of the time of fall in the liquid were used in the computation, the value of A can not be greater than the average of the results given, if the simple Stokes's formula holds for the fall in the liquid.

Comparison of the chronoscope with a standard laboratory clock showed it to have an error of 0.4 per cent. when the period was long enough that the personal error could be neglected. With the short periods used here, the personal error was the greater. Comparisons with the clock on different days showed this to be sometimes in one direction, sometimes in the other, the greatest error being 1.4 per cent. on a one-second interval. An error of 1.4 per cent. in the fall of the largest drop of the second series, 1.668 sec., would produce 28 per cent. in the computed value of A. For intervals greater than 4 seconds, the maximum error was 0.7 per cent., which would produce an error of at least 6 per cent. on the smallest drop of the second series.

The application of the correction of Hadamard to the fall in the liquid yields a value of A about 140 per cent. too high. With this correction, the modified Stokes's formula for the velocity in the liquid becomes

$$v_2 = 2/9\, ga^2 \frac{\sigma - \rho_2}{u_2} \left\{ \frac{u^1 + u_2}{u^1 + \frac{2}{3}u_2} \right\},$$

where u^1 is the viscosity of mercury.

The viscosity of mercury given in the tables for 25° is .0124; that of xylol at 25° is .00628. Then the correction becomes

$$\frac{.0124 + .00628}{.0124 + .00419} = 1.125.$$

The equation for the radius of the drop becomes

$$a = \beta \sqrt{\frac{1.125}{T_2}} = 1.0606\beta \sqrt{\frac{1}{T_2}}$$

and that for A becomes

$$A = \left\{ 1.125\alpha \frac{T_2}{T_1} - 1 \right\} \frac{a}{1}.$$

Solving for a and A in these modified formulæ, we get for the 6.19 second drop

$$a = 1.00979 \times 10^4 \text{ instead of } 0.952 \times 10^4.$$

and

$$A = 2.635 \qquad \text{instead of } 1.0808.$$

It was pointed out by Nordlund[1] that the Hadamard correction did not apply for mercury droplets in water, and the above data indicates that it does not apply to mercury droplets in xylol.

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[1] Nordlund, Zeitschrift für Physikalische Chemie, 87, 1914, page 40, also Science abstracts, 180, Vol. 17, 1914.

II.

DETERMINATION OF A FROM THE APPARENT VALUE OF THE ELEMENTARY CHARGE.

The lower plate and wooden cylinder were now replaced by an iron condenser plate of the same diameter and an attempt was made to determine roughly the value of A from the apparent value of the elementary charge. With the installation of an X-ray outfit, the apparatus thus altered became the same as that originally used by Millikan for measuring e.

With the apparatus two experimental conditions rendered an accurate measurement impossible. (1) The evaporation of the droplet made the work progress very slowly and produced great uncertainty when it did not entirely cease. Occasionally a droplet could be selected which did not evaporate, whose elementary charge determination showed that it was mercury. As a rule, however, the more slowly falling particles indicating no evaporation, gave values for the ultimate charge much lower than if they had been pure mercury droplets with the same speed under gravity, although the change in charge was an exact multiple of the charge originally on the droplet. These were regarded as impure mercury particles, or dust particles. It was found better to select a droplet which showed evaporation and retain it until the evaporation either ceased, or took place very slowly. In general it was necessary to hold a droplet about an hour before any readings could be taken, and in many cases it faded from view before the evaporation ceased. Why it should cease in any case was not learned. When the plates were coated with oil to catch dust particles it seemed to cease more quickly, although the vapor pressure of the oil used was very low. If the pressure of the oil vapor caused the evaporation to cease it must have been due to a change of surface of the droplet, and we should expect the value of A to agree with the value for oil drops. (2) A second source of error in the measurement was the piling up of the mercury on the lower plate. Unlike the oil which spread over the surface, the mercury accumulated in small globules which rendered the electrical field non-uniform.

The first fall of this droplet was in twenty seconds. It was held until the evaporation ceased, before the above readings were taken. The time required to obtain the above data was fifty minutes.

The values found for $e_1^{\frac{2}{3}}$ and $1/a$ are plotted as ordinates and abscissæ respectively, in order to obtain the value of A.

The curve was drawn to give the greatest weight to the droplets whose fall under gravity was greater than 20 seconds per centimeter, because measurements on droplets of this size were more reliable. The

The following are the observations on one of the smaller droplets:

Drop No. 13.

T_g	T_f	$\frac{1}{T}$	n'	$\frac{1}{n'}\left(\frac{1}{T_f'}-\frac{1}{T_f}\right)$	n	$\frac{1}{n}\left(\frac{1}{T_g}+\frac{1}{T_f}\right)$
55.6	43.8	.02304			1	.04083
56.6	43.0					
55.8	9.635	.10446				
56.8	9.640		2	.04083	3	.04075
55.8	9.443					
56.2	9.572					
56.2	42.4	.02353	2	.04046		
56.4	42.6				1	.04131
	15.464	.06437	1	.04084		
	15.487				2	.04108
	15.651		1	.04078		
	42.4	.02359			1	.04137
56.4	9.427	.10514	2	.04078		
	9.594				3	.04098
	43.0	.02320	2	.04097		
	43.2				1	.04099
	15.674	.06377	1	.04047		
	15.423				2	.04073
	15.994		1	.04008		
56.4	42.4	.02359			1	.04137
56.22				.040684		.041023

$V_1 = 3410.$ $V_f = 3385.$

$t = 24.8$ C. $a = .00003001.$

$$\frac{1}{a} = 3203,$$

$$e_1 = 6.585 \times 10^{-10},$$

$$e_1^{\frac{2}{3}} = 75.69 \times 10^{-8}.$$

Observations and computed values on other droplets are recorded in the following table:

Drop.	Tg.	$a \times 10^6$	$e_1 \times 10^{10}$	$1/a \times 10^2$	$e_1^{2/3} \times 10^8$
1	8.066	8.469	5.143	11.36	64.19
2	8.327	8.463	4.967	11.23	63.74
3	11.054	7.175	5.350	13.24	66.11
4	13.668	6.447	5.345	14.72	65.86
5	19.340	5.332	5.572	17.82	67.72
6	23.261	4.812	5.762	19.74	69.24
7	25.910	4.536	5.906	20.95	70.39
8	26.437	4.533	5.720	20.96	68.91
9	27.970	4.442	5.979	21.88	70.97
10	36.150	3.761	6.305	25.26	73.53
11	40.140	3.575	6.228	26.57	73.49
12	45.000	3.399	6.266	27.95	73.22
13	56.22	3.001	6.585	32.03	75.69
14	62.15	2.791	6.645	34.04	76.14

slope of this curve gives .886 for the value of A.[1] The low value of the intercept (59.3 instead of 61.05) shows the imperfection of the apparatus.

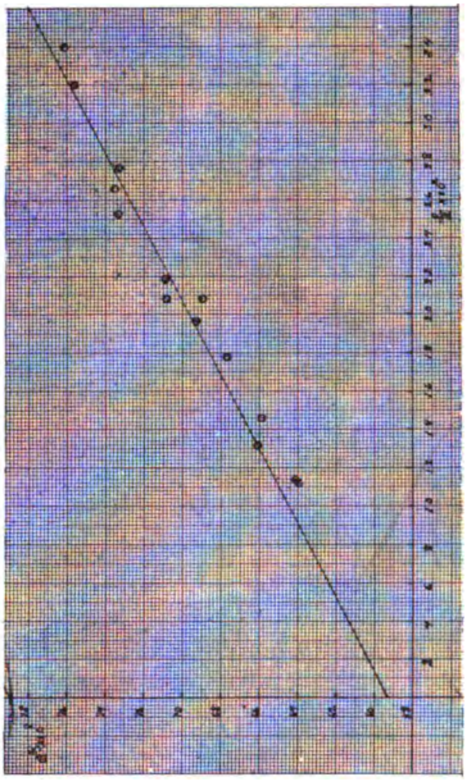

Fig. 3.

Summary.

I. By a direct measurement of the velocity of fall of a droplet of mercury in air and in xylol, and assuming that Stokes's law is applicable to the motion in a liquid, the value 1.03 was found for the empirical constant A of the modified formula for the fall in air.

II. These measurements show that the Hadamard correction does not hold for mercury droplets in xylol.

III. A rough measurement of A from the apparent value of the elementary electrical charge on mercury droplets gave a value in the neighborhood of that obtained by use of oil drops.

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[1] Strictly Millikan's value of the intercept should be used in the calculation of A. If his value, 61.05, is used the value of A is found to be .824. This low value of the slope of the curve is in accordance with the assumption that oil vapor condensed on the droplets for a film on a smaller droplet would have a greater effect than one on a large droplet thus decreasing the slope of the curve.

I wish to express my thanks to Professor R. A. Millikan, at whose suggestion the above problem was undertaken, and whose helpful suggestion and kindly counsel have been an inspiration throughout the investigation. My thanks are also due to Messrs. R. S. Buddenburg, P. I. Pierson, S. A. Rowland, and J. B. Derieux, who, in turn, assisted me in making the observations.

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THE FALL OF MERCURY DROPLETS IN A VISCOUS MEDIUM.

BY O. W. SILVEY.

INTRODUCTION.

IN a recent comparison[1] of the fall of a mercury droplet in air and in xylol, it was found that if the simple Stokes's[2] equation were assumed to hold for the fall in the liquid a correction of the formula could be obtained for the fall in air, which was in fairly good agreement with the value found by Professor Millikan[3] by another method. Brownian movement measurements by Nordlund[4] on mercury particles, and a consequent determination of the Avagadro constant which agreed well with the value given by Professor Millikan showed that the simple Stokes's law was applicable to these minute particles. Nordlund[5] to verify his results, carried out an investigation of the fall of mercury drops in a mixture of glycerin and water and in 105 experiments found a variation from Stokes's law of only .076 per cent.

Although the study of the fall of solid spheres both in air and in liquids has been very carefully studied experimentally by Allen,[6] Ladenburg,[7] Zeleny,[8] Arnold[9] and others, the fall of liquid spheres has been neglected presumably because of difficulty met in measuring the radius of the drop.

Stokes[10] in his Memoirs discusses the case of liquid spheres falling in a gas and concludes that although the relative motion of the particles within the globule should be taken into account, if the globule is assumed to be preserved in a strictly spherical shape by capillary attraction, the motion will be the same as that of a solid sphere in the same medium.

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[1] Loc. cit.

[2] Stokes, Mathematical and Physical Papers, Vol. III., p. 55–61.

[3] Millikan, PHYS. REV., 4, Apr., 1911, p. 349; also 2, Aug., 1913, p. 109.

[4] Nordlund, Zeitschrift fur Physikalische Chemie, 87, 1914, p. 40.

[5] Nordlund, Ark. für Mat. Astron. Och. Fysik, Stockholm, 9, No. 13, p. 1–18, 1913. Science Abstracts, 180, Vol. 17, 1914.

[6] Allen, Phil. Mag., 1900, p. 323.

[7] Ladenburg, Ann. der Physik, 32, p. 287 (1907) and 33, p. 447, 1907.

[8] Zeleny and McKeehan, PHYS. REV., Vol. XXX., 5, May, 1910, p. 535.

[9] Arnold, Phil. Mag., 22, 6, p. 755.

[10] Stokes, Mathematical and Physical Papers, Vol. III., p. 55–61.

The first work on the fall of liquid spheres was that of O. Jones,[1] who attempted to make use of mercury globules to determine the viscosity of viscous liquids. He determined the radius of his drops by weighing a rather large drop, then dividing it into smaller parts, and assumed that the droplets thereby produced were aliquot parts of the whole. His verification of the Stokes's formula was no better than one should expect from such a method of measurement.

Arnold[2] made a few measurements of the motion of alcohol globules in olive oil, and because of continual dragging away of the drop obtained too large a value for the viscosity of the oil. He recorded no results on mercury droplets because of uncertainty in the measurement of the radii. Arnold's chief interest was not in the liquid spheres but in the behavior of the solid ones.

In addition to their work on spores and wax spheres Zeleny and McKeehan studied the fall of mercury spheres in air. For this work, extremely small spheres were necessary and they too were uncertain of their measurement of the radii.

In 1911, Rybczynski[3] and quite independently Hadamard[4] derived from theoretical consideration, a modified form of Stokes's equation for liquid drops falling in a viscous medium. They assumed a vortex motion of the particles within the sphere, due to the drag at the surface, and a consequently greater speed of fall. Their correction term involves the viscosity of the liquid within the drop and that of the medium through which it falls. The modified Stokes's equation is of the form:

$$(\sigma - \rho)g = \frac{9}{2} u \frac{v}{a^2} \left(\frac{u_1 + \frac{2}{3}u}{u_1 + u} \right),$$

where a, σ, v and u_1 are the radius, density, velocity and viscosity of the falling sphere, ρ and u the density and viscosity of the surrounding medium.

Besides the work of Nordlund, an attempt was made by Roux[5] to test the Hadamard equation by allowing mercury droplets to fall in castor-oil. Since the viscosity of castor-oil is about 300 times as great as that of mercury, this provided conditions such that the Hadamard correction should have very nearly its maximum value, unless other physical conditions are introduced which are not included in Hadamard's hypothesis. Roux used comparatively large droplets, 618 μ to 905 μ, and determined their radius by weighing the tube of oil before and after introducing the droplet. His results indicate a greater velocity than obtained by the Stokes equation, ranging from 6 per cent. greater for the smaller to 10 per cent. greater for the larger ones, although in no case did the speed attain a value necessary to agree with that calculated by the Hadamard equation.

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[1] Jones, Phil. Mag., 37, p. 451, 1894.

[2] See note 4 above.

[3] Rybczynski, Ac. R. des Sc. Cracovie, 9, 1911.

[4] Hadamard, Comptes Rendus, March, 1911.

[5] Roux, Annales de Chimie et De Physique, May, 1913, p. 69.

In view of the fact that Nordlund obtained such good agreement with Stokes's law, while Roux's results all show a divergence in the direction of the Hadamard values in spite of the fact that he does not mention having used the Ladenburg correction for the diameter and length of the fall tube, it seemed worth while to repeat his work. It was hoped that by using a sufficiently wide range of droplets that an experimental curve could be obtained, which would show the transition from the Stokes to the Hadamard curve.

THE APPARATUS.

The apparatus was very simple. It consisted of a glass tube 40 cm. long and 5.5 cm. in diameter, drawn at the two ends to join to short tubes about 2 cm. long and .8 cm. in diameter. On each of these end tubes was placed a short piece of rubber tubing, which was closed by a clamp. Near each end of the tube and at about one third its length, etched lines encircling the tube permitted one to read without parallax the initial and final instant of the period of fall. The tube was supported by a clamp at its middle, such that it could be inverted when the droplet had fallen the desired distance. Behind the tube at a distance of about 50 cm., was placed a white paper screen, which was illuminated from behind by three incandescent lamps. These lamps were sufficiently near the tube to change its temperature, if allowed to glow continuously, and were, therefore, not used more than two or three seconds at a time, when it was necessary to ascertain the position of the falling drop. Owing to the changeable summer temperature, the whole apparatus was set up in a basement room, where a thermostat and an electrical heating device maintained the temperature constant to 0.4° C.

MEASUREMENT OF THE DIAMETER OF THE DROPS.

The diameters of the drops were measured by means of a Zeiss micrometer ocular on a Leitz Wetzlar microscope, which had a magnification of about 60 diameters. Each turn of the screw corresponded to a shift of the cross hair of .00758 cm. The screw head bore fifty scale divisions and a setting of the cross hair tangent to the sphere could be duplicated with no greater variation than one half scale division. Allowing one half division for setting on each side, the measurement of the diameter of the smallest sphere used was correct to 1.4 per cent. The microscope was calibrated by comparison with a grating made for such measurements by Zeiss.

A preliminary study showed that the small mercury droplets could be readily picked upon the point of a steel needle and retained there if the needle were kept horizontal. If the droplet did not at first adhere to the needle, it was found to do so after dipping the needle into the oil to be used in the investigation, and the oil then removed by means of a dry cloth. Sufficient oil remained to cause the drop to adhere to the needle, yet not enough to cause any error in the measurement. To pick up the smaller drops, a very fine needle was used, while for the larger ones, it was necessary to use a larger needle. The microscope was turned to a horizontal position, and the needle (which for convenience in handling was held in the end of a small soft wooden cylinder about 10 cm. long), was placed in a suitable support which was held in position on the telescope table by the clamps designed for holding the slide. The microscope field was illuminated by an incandescent lamp behind a white paper screen about 30 cm. away.

When the microscope was focused on the drop, it was seen as a spherical object clinging to the lower side of the needle. Measurements along different diameters showed it to be spherical within the accuracy of the measuring device. If the droplet had recently been blown by an atomizer from clean mercury on to a clean piece of paper it was seen to be free from dust particles.

OBSERVATIONS.

After the measurement of the diameter, the needle was lifted by the wooden cylinder from the support and the droplet carried to the fall tube. If the droplet was not too small it fell into the oil when the needle was placed vertically. To free the smaller ones, it was necessary to dip the point of the needle into the oil and move it slightly. The released droplet fell along the axis of the tube, the interval of fall between two selected lines being measured by means of a stop watch. As soon as the droplet had crossed the lower line the tube was inverted and another fall measured. Six to ten falls were recorded for each drop. In no case was the droplet allowed to touch the walls of the tube until all measurements were made. After the last measurement, the droplet was allowed to fall to the end of the tube, the tube was inverted, quickly opened, and the droplet removed by means of a medicine dropper, and another put in.

The following data were obtained:

Radius in Cm.	Velocity in Cm. per Second.	Velocity Calculated from the Stokes-Ladenburg Equation.	Velocity Calculated from the Stokes-Hadamard Equation.
.00577	.0139	.0139	.0202
.00829	.0281	.0286	.0416
.00863	.0305	.0301	.0451
.00974	.0398	.0395	.0575
.01198	.0589	.0596	.0868
.01599	.1045	.1057	.1538
.01696	.1217	.1189	.1728
.01928	.1511	.1533	.2229
.02090	.1800	.1798	.2615
.02118	.1842	.1846	.2685
.02149	.1903	.1900	.2763
.02194	.2014	.1979	.2878
.02251	.2113	.2083	.3029
.02506	.2572	.2575	.3744
.02659	.2893	.2893	.4208
.02789	.3226	.3214	.4675
.02997	.3775	.3668	.5335
.03779	.5982	.6211	.9033

The first and second columns are respectively, the observed radii and corresponding velocities. The third column contains the velocity calculated by the Stokes-Ladenburg equation:

$$v = \frac{2}{9} \frac{a^2(\sigma - \rho)}{u\left(1 + 2.4\frac{a}{R}\right)\left(1 + 3.1\frac{a}{L}\right)},$$

where R and L are the respective radius and length of the tube. The values given in the fourth column were obtained by multiplying those of the third column by the Hadamard correction.

The density of the castor oil was found by the pyknometer method to be .95594 at 24.°8 C. The viscosity determined by a modified Ostwald viscosimeter was 6.5256. The viscosity of mercury was taken from the tables. These values of the viscosity gave for the Hadamard correction the value 1.4544.

DISCUSSION OF RESULTS.

It is seen that the observed values agree with those obtained by the Stokes-Ladenburg equation within the limit of experimental error for all drops observed. The agreement is shown more clearly by the curves, Fig. 1. The full line curves are drawn from values determined by the equations. The points are the actual observations. Curve S

represents the Stokes-Ladenburg equation, and Curve *H* the Hadamard values. The Hadamard correction, therefore, does not apply to the case of mercury droplets in castor-oil of this viscosity. The failure is probably due to some surface effect not included in the Hadamard hypothesis.

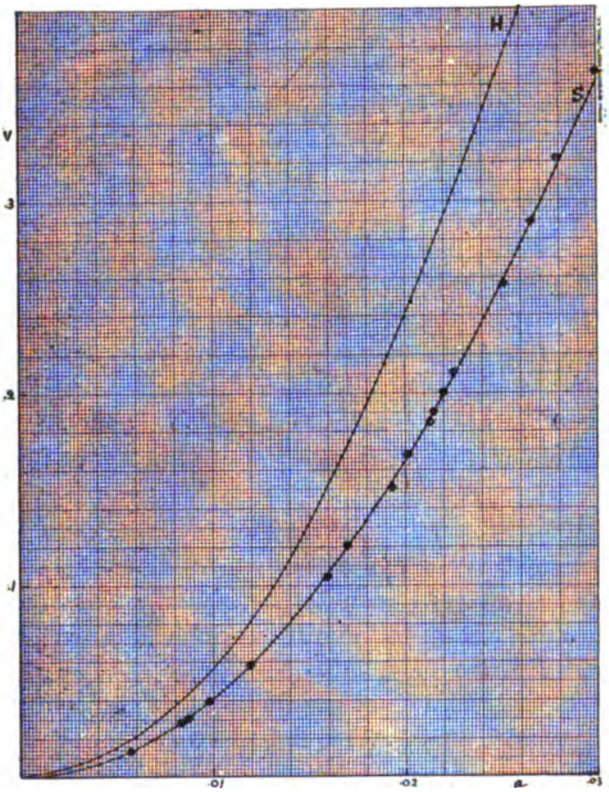

Fig. 1.

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A METHOD FOR THE MEASUREMENT OF SMALL CAPACITIES.

BY WILL C. BAKER.

THE method herein described permits capacities as small as one or two centimeters to be evaluated by comparison with the capacity of a parallel plate condenser of variable plate distance. As the air condenser is used at values from fifty to five hundred centimeters its capacity may be determined quite exactly from its dimensions. The extension of the method to the determination of the specific inductive capacity of small quantities of fluids and of solids of low melting point will be obvious.

I. THEORY.

Let C be the small system, of capacity x, to be measured, one of its "plates" being grounded. Let K_1 be the capacity of the parallel plate condenser when its plates (A and B) are d_1 cm. apart; A being at potential V and B being grounded. The quantity of electricity on A is given by K_1V; and if this be shared with C, the quantity on A falls to $K_1V[K_1/(K_1 + x)]$. Suppose now that the connection between A and C be broken, C discharged, and the connection again made. The quantity on A is now $K_1V[K_1/(K_1 + x)]^2$; and if this series of operations be carried out n times, the quantity on A is given by $K_1V[K_1/(K_1 + x)]^n$. Now let the plates A and B be drawn apart until the potential of A returns to V, and suppose that the new plate distance is d_2 and the corresponding capacity K_2. Thus we have the quantity on A before the separation given by $K_1V[K_1/(K_1 + x)]^n$ and after separation by K_2V. If there has been no leak during the operation we may write

$$K_1V\left(\frac{K_1}{K_1 + x}\right)^n = K_2V;$$

therefore

$$\frac{K_1}{K_1 + x} = \sqrt[n]{\frac{K_2}{K_1}}. \tag{1}$$

It will be at once evident that the value of the ratio on the left side of this equation may be very exactly obtained, owing to its being the nth root of the observed ratio K_2/K_1; also if the ratio on the left be thus exactly known, the value of x is known to the same accuracy as that of K_1. Under the conditions given below K_1 ranged from 80 to 700 which may be obtained quite accurately from the dimensions of the condenser. Therefore x should follow with an error nearly as small.

Again in the actual case K_1 is the sum of the capacity between the plates (K_{1p}); the edge correction (K_{1e}); and that of the lead wires and other constant capacities in the system (K_w): thus our ratio

$$\frac{K_2}{K_1} = \frac{K_{2p} + K_{2e} + K_w}{K_{1p} + K_{1e} + K_w},$$

but under the conditions quoted below K_{2e} is nearly equal to K_{1e}, and both, as well as K_w, are small in comparison with K_{1p} or with K_{2p}.[1] Thus the ignoring of these small terms is equivalent to the subtraction of a small constant from both numerator and denominator, leaving the ratio almost the same. Thus we may write

$$\frac{K_2}{K_1} = \frac{K_{2p}}{K_{1p}} = \frac{A/4\pi d_2}{A/4\pi d_1} = \frac{d_1}{d_2}$$

and equation (1) becomes

$$\frac{K_1 + x}{K_1} = \sqrt[n]{\frac{d_2}{d_1}}$$

and

$$K_1 + x = K_1 \sqrt[n]{\frac{d_2}{d_1}}. \tag{2}$$

Further, suppose a series of determinations to be made as follows: Start from K_1 as the initial capacity; make say 20 contacts with C as described, and restore the potential by lowering the capacity to K_2. Now with K_2 as the initial capacity make 20 more contacts and obtain a second value for x by restoring the potential in changing the capacity to K_3. With K_3 as the initial capacity obtain a third value for x, etc. Working in this way we obtain a series of determinations in which any error, either in setting the plates to restore potential (*i. e.*, in the reading of the electroscope) or in the reading of the plate distance (d) in a given observation, causes *an equal and opposite error* in the next succeeding observation. Thus the mean of such a series should be of a higher accuracy than the mean of an equal number of disconnected observations. See the example in section III.

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[1] In the readings given in sections III. and IV. below K_{2p}/K_{1p} is never greater than about 0.9 so even if $K_e + K_w$ were equal to half of the term K_p, *i. e.*, *even if it were of the order of a hundred centimeters*, the ratio K_{2p}/K_{1p} would differ from the ratio K_1/K_2 by only 3 per cent.

II. THE APPARATUS.

The particular electroscope used in these experiments consisted of a tin box (6 × 6 × 4 cm.) with glass sides. The leaf was an isosceles triangle (base 5 mm., altitude 25 mm.) of aluminium foil carrying at its apex a tiny bit of fine quartz fiber attached with the smallest possible touch of thin shellac. The position of the end of this very definite index was observed on the eyepiece scale of a reading microscope. In order to get a good "hinge" the leaf was attached to the staff of the instrument by means of a small bit of gummed paper armed (on the gummed side) at the lower edge with a small rectangle of thin copper. The copper served, first, to give a straight edge about which the foil could bend; and, second, to keep the puckering, where the foil meets the gum, well away from the "hinge" itself. The lowest part of the gummed paper holds the copper; just above this the foil is attached, while the upper part fixes the whole to the staff. A block of sulphur cast about the staff provides the insulation.

The parallel plate condenser was of the ordinary type—two circular discs mounted on carriages that slid on steel ways. The motion of the carriages was produced by screws, one of which was of 1 mm. pitch and had a divided head. After clamping one of the carriages in a fixed position the instrument was turned up so that the surfaces of the plates were horizontal. A block of wood between the lower plate and the table prevented any change in its position when the weight of the upper plate came upon it (see below). This plate was connected with the ground. The upper plate was insulated by a ring of cast sulphur that in turn was supported by three adjusting screws that bore on a brass ring cast into its lower surface. These adjusting screws were on the carriage controlled by the screw of one millimeter pitch. The moving plate itself carried on its upper surface two spirit levels set in soft wax. To secure parallelism, the upper plate was allowed to rest directly on the lower, quite free of the adjusting screws or of any other support. The bubbles of the levels were then brought to center by pinching the wax at one end or the other. The plates were then slowly separated and after the full weight of the upper plate was taken by the adjusting screws the bubbles were brought back to center by their means. On account of small irregularities in the motion of the carriage, the plate distances (d) were, in all later experiments, read by means of an optical lever, two legs of which rested on a brass bar fixed to the ways and the third (4 cm. from the line of the other two) on a piece of brass cemented to the sulphur ring. The telescope was 130 cm. away from the mirror. The zero reading was of course taken when the upper plate rested freely on the lower. This afforded a means of reading plate distances that was entirely free from "back lash" of any kind.

III. The Average Capacity of the Electroscope over a given Part of the Scale.

The capacity of an electroscope will in general be different at different deflections. The following section deals with the determination of the average capacity of the electroscope described above over that part of the scale lying between 2.50 and 3.50 (eyepiece scale). It is to be remembered that the capacity determined is that of the electroscope in the immediate vicinity of the charging wire; not that of the electroscope free from other objects. In order to prevent the hand of the observer from altering the capacity in charging, the lead wire from the condenser was carried on a sulphur post attached to the end of a lath (over a meter long). This lath could be bent by pulling on a string that led to the observer's seat; and in this motion the end of the wire was caused to dip into a tiny cup of calcium chloride solution in which the staff of the electroscope terminated. A plate of zinc soldered to a ground wire was used to discharge the electroscope. As this was introduced between the instrument and the charged wire it served also to shield the electroscope from charges induced by the wire.

In making a determination, the plates of the condenser were set at a known distance apart (d_1), and the system (condenser and electroscope) charged until the index stood at 3.50 on the scale. Connection was then broken; the electroscope discharged and a series of contacts, as described above, were made until the index had fallen as nearly as possible to the mark 2.50. (In the example quoted below this took nine contacts.) Leaving the electroscope in connection with the condenser the plates were separated until the index again stood at the mark 3.50 when the plate distance (d_2) was read. Then a second series of contacts (requiring this time only eight) to bring the index to 2.50 and a second separation of the plates (to distance d_3). Then another series of contacts and so on for eight or ten determinations. The last one in the example quoted took only two contacts. It will be easily seen from this that any error in the setting of the plates (judged by the position of the electroscope index) or in the reading of the plate distance will produce an equal and opposite error in the next determination. Strictly this applies to the readings, and only in a smaller degree to the reduced values.

The following table gives an example of such a set of readings. These were taken before the optical lever was adopted and so contain errors from the uneven motion of the carriage. The plate distances here were taken from the readings of the divided head of the screw that moved the carriage.

Capacity of Electroscope "B" for Range 3.50 to 2.50.

Area condenser plate.......................... 414.9 sq. cm.

Capacity (K_s) for plate distance d_s.............. $\frac{414.9}{4\pi d_s}$

$x = K_s \sqrt[n]{d_{s+1}/d_s} - K_s$..................(from equation 2)

d_s (Cm.)	n	K	x
0.1261	9	261.9	3.3
0.1409	8	234.4	2.9
0.1556	8	212.3	2.9
0.1737	7	190.1	2.8
0.1919	6	172.1	2.7
0.2108	6	156.7	3.0
0.2368	5	139.5	3.2
0.2655	5	124.4	2.9
0.2975	4	111.0	3.0
0.3314	4	99.7	3.2
0.3769	4	87.6	3.0
0.4314	3	76.6	3.3
0.4893	3	67.5	3.2
0.5618	3	58.8	3.2
0.6601	2	50.0	3.1
0.7476		Mean.......	3.04

In this table the first column gives the plate-distances in centimeters; the second, the number of contacts required to carry the index from 3.50 to 2.50; the third gives the initial capacity of the condenser; and the fourth column, the determined value of the capacity of the electroscope. It will be noticed that while the values of K_1 range from 261.9 to 50.0 and the number of contacts from 9 to 2 yet the values obtained for x are very nearly constant. Four successive determinations of the mean capacity of this instrument over the part of the scale from 3.50 to 2.50 (the one quoted above being the first) gave 3.04, 3.05, 3.05 and 3.04 cm. Similar determinations at other deflections showed that the capacity was nearly constant over the lower and middle parts of the scale (probably owing to the fact that the leaf here was passing the corner of the containing box) but that in the upper third of the scale it rose slowly.

IV. THE DETERMINATION OF THE CAPACITY OF SYSTEMS OTHER THAN THAT OF THE ELECTROSCOPE.

Several attempts were made to apply an extension of the method to systems the capacity of which could be calculated from their dimensions. The following section deals with the difficulties met with. The systems were all spheres but it must be noted that the capacity will not be that of a sphere far removed from other bodies. The presence of the charging wire will lower the capacity and any dielectric used as a support will raise it. These do not permit of easy calculation.

The lead wire from the condenser was soldered to the staff of the electroscope. It passed thence, on sulphur columns, along a lath about a meter long and finally rose (or fell) vertically for 30 or 40 cm. to the neighborhood of the sphere employed. The wire could be brought into contact with the sphere by bending the lath as described in the previous section. The discharge was accomplished as before by touching the sphere with a grounded plate interposed between it and the charging wire so as so shield the body from induced charges.

The first sphere was of brass (radius 3.86 cm.). It was originally supported on a sulphur column 40 cm. long. This was 1.2 cm. in diameter and it passed 2 cm. into an internal sleeve in the sphere. Six successive determinations on different days gave for the capacity of this system 4.05, 3.90, 3.98, 3.91, 4.00, and 3.99 cm. This is quite satisfactory as far as constancy is concerned but the case is not amenable to calculation. Obviously the lowering of capacity due to the lead wire is more than offset by the effect of the dielectric support.

Then a three-eighth inch bicycle ball (radius 0.95 cm.) was suspended by a quartz fiber 34 cm. long. The fiber was attached by sealing wax to a bit of leaden wire that just filled a hole in the ball, being flush at both ends. Here the effect of the support was negligible, the disturbing factor being the contact wire (though the metal bars of the support, about a meter away, would raise the capacity slightly). The three values obtained for this system were 0.85, 0.89, 0.87 cm. The individual values in each series were very nearly constant; those in the first case being as follows: 0.83, 0.83, 0.84, 0.85, 0.85, 0.87, 0.86, 0.89, 0.86.

Next taking a hint from the work of Hubbard and Stimson[1] the large brass ball was hung on a quartz fiber 34 cm. long and the hole in the end of the sleeve was stopped by a disc of zinc cut to fit. The nearest objects were the horizontal bar of the support 34 cm. away; the vertical of the standard, 44 cm.; and the head and body of the observer were about a meter from the sphere. Two charging wires were arranged to touch the sphere at opposite ends of a diameter. Using this system of double charging wires the values obtained were 3.46 and 3.53 (mean 3.49, difference of individual values from mean of about 1 per cent.). When one of the charging wires was bent back so that it lay beneath the lath carrying the lead to the condenser, the values 3.79 and 3.72 were found

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[1] Phys. Rev., I., p. 246, March, 1913.

(mean 3.75 with about the same difference as before). Therefore the lowering of capacity due to the second wire appears to be $3.75 - 3.49 = 0.26$. If we assume that the effect of the single wire is exactly half that of the double contact, the capacity of the sphere alone appears to be $3.75 + 0.26 = 4.01$ which is 0.16 or about 4 per cent. too high. A second set of measurements were made in which no object other than the contact wires and the supporting fiber were allowed to come within a meter of the sphere. The double contacts gave 3.19 and 3.12 and the single contacts 3.66 and 3.67. These on the assumption of a double effect for the double contact indicate for the sphere a capacity of 4.16. A disc was also tried in the same manner. It gave a result several per cent. too high but in this case there was evident trouble with the insulation. This indicates that the discrepancy is probably due to imperfect insulation, so it has been decided to postpone further tests until the dry winter season when the troubles from the summer moisture will be absent.

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August 3, 1915.

THE EFFECT OF MAGNETIZATION ON THE OPACITY OF IRON TO RÖNTGEN RAYS.

By A. H. Forman.

IN a previous paper[1] the results covering the investigation of the effect of magnetizing the iron in a plane perpendicular to the path of the Röntgen rays were published. The results were negative with a set up of apparatus sensitive enough to detect a change of one part in ten thousand under the most favorable conditions.

The work has been continued attempting to reach the same sensibility with the iron magnetized in a direction parallel to the path of the Röntgen rays. This proved rather difficult as it was necessary to magnetize a thin sheet of iron perpendicular to its plane and yet have a free path on both sides of the iron for the Röntgen rays. An electromagnet with hollow poles, as shown in Fig. 1, was designed and built. With this electromagnet it was possible to produce a field of 3,500 gauss perpendicular to the plane of the iron. However, the stray field from the electromagnet was quite strong in the vicinity of both the X-ray tube and the ionizing chambers. In the vicinity of the X-ray tube it caused trouble by deflecting the cathode stream. The stray field also acted on the secondary rays from the walls of the ionizing chamber first used and thus masked any effect there might be.

The deflection of the cathode stream was prevented by using neutralizing coils, as shown in Fig. 1. By their use the stray field was neutralized except for a small component of field parallel to the path of the cathode rays. The first ionizing chamber which was used is described in the previous paper. It was found that nearly all the ionization was produced by secondary rays from the inside walls of the chamber and this seemed to be the reason for the change in ionization due to the stray field. To prevent these secondary rays from reaching the walls, a conical chamber (Fig. 2) was built. Although the ionization seemed to be unaffected by the magnetic field, it was comparatively small, and so lowered the sensibility of the apparatus. With a view of overcoming the defects of these two ionizing chambers a third one, shown in Fig. 1, was built. It gave a large amount of ionization, and when placed quite a distance from the electromagnet made possible a sensibility of the same magnitude as in the first work and it was not affected by the stray magnetic field.

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[1] Phys. Rev., April, 1914, pp. 306–313.

Fig. 1 shows the final set up of the apparatus. The ionizing chambers consist of two lead cylinders which are grounded. Inside each are two brass posts which support sheets of aluminum leaf, alternating as shown. In one of the chambers one set of the aluminum leaves is charged to a p.d. of + 220 volts, while in the other chamber the corresponding leaves are charged to a p.d. of – 220 volts. The remaining set of leaves in each chamber are connected together to a grounding switch and to one of the quadrants of a Dolezalek electrometer.

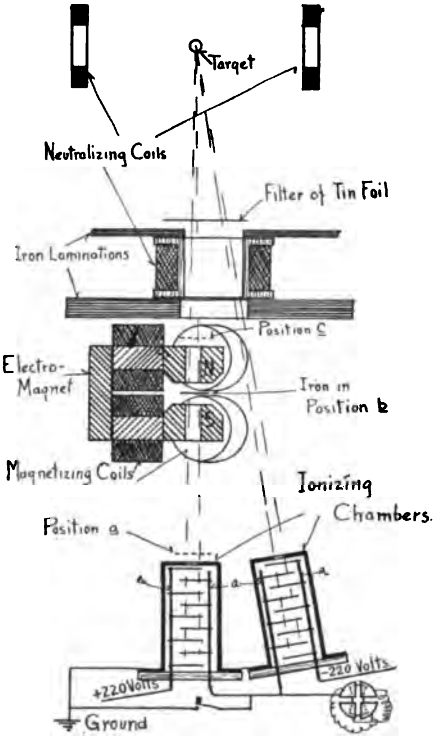

Fig. 1.

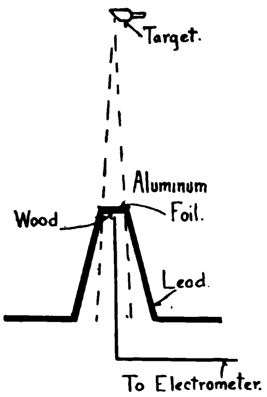

Fig. 2.

Perfect electrostatic screening of the system is effected by the use of a solid screening of tin-plate as in the earlier work.

The Röntgen rays pass through the top of the chambers and strike the aluminum, producing secondary rays which ionize the air between the leaves. The electric field due to a p.d. of + 220 volts causes a + charge to flow towards the electrometer from one chamber, while the p.d. of

− 220 volts causes a − charge to flow from the other chamber. When there is a perfect balance between the two chambers no charge accumulates and there is no deflection of the electrometer. A small change in the ionization of either chamber causes a charge to pile up and the electrometer deflection is proportional to the change in ionization and the time the grounding switch is open. The method of taking observations and calculating the sensibility are the same as explained in the first paper.

Observations made with this set up and using the same piece of iron that was used in the earlier work show an increase in the opacity of the iron when it is magnetized in a direction parallel to the Röntgen rays (perpendicular to the cathode stream of the *X*-ray tube). The effect seems to be about 5 parts in a thousand for a field of 3,500 gauss. Since soft iron is saturated at an induction of about 15,000, we have the iron molecules only slightly oriented in a field of 3,500 gauss, and so no doubt a greater effect would be found with a field of 15,000 or more gauss.

There was a feeling from past experience that the observed effect might be due to some secondary cause rather than the change in opacity of the iron. To check this the iron was put in position *a* (Fig. 1) or over the ionizing chamber where the field would be comparatively weak. In this position with the conditions the same as when the iron was in position *b* (Fig. 1) no effect was observed. Thinking that the effect when the iron was in position *b* might be due to the effect of the magnetic field on the secondary rays coming from the iron, the iron was placed in position *c*. Here it would be in a comparatively weak field, but the secondary rays from the iron would pass through the magnetic field of 3,500 gauss. With the iron placed in this position no effect was observed.

As a further check a filter of tinfoil was placed below the *X*-ray tube and above the iron. This filtered out the rays which were likely to produce secondary rays in the iron and gave more consistent readings by making the condition of balance between the two chambers more stable. Next the effect of stopping the secondary rays produced in the iron, before they reached the ionizing chambers, was tried by covering the top of each chamber with tinfoil. In all these cases the effect of magnetizing the iron was evident. Tables I., II., III. and IV. give the readings with the tinfoil filter. These data include only two voltages across the *X*-ray tube, viz., 21 and 81.5 K.V. Measurements were made without the tinfoil filter for voltages of 21, 27, 32.5, 45 and 51.8 K.V. and indicated an effect almost equal for all these voltages but slightly smaller in magnitude than with the tinfoil filter. The source of current was a high tension rectifying machine and the voltage was measured in terms of the effective voltage, so that the values above are less than the maximum or peak voltage across the *X*-ray tube.

TABLE I.

Filter of Tinfoil and Iron in Position b.

		Electrometer Deflection.				Electrometer Deflection.	
Mili-Amp.	K. V.	Mag. Current Off.	Mag. Current On.	Mili-Amp.	K. V.	Zero Setting of Lead Strip.	Setting Changed 2 Mm.
5	27	82					
5.1	26.4	55		5	27		121
5.05	26.7		70	5	27		124
4.9	27.3	73		5	27		124
4.95	27.2		71	5	27	94	
4.9	27.3		72	5	27	100	
4.95	27.2	59		5	27	94	
5	27	50		5	27		128
4.95	27.2		67				
4.8	27.6		79				
5	27	59					
5	27		67				

Mean deflection ...63 ± 3.3 71 ± 1.21

Difference......... 8 ± 3.55

Proportional change = 0.00388 ± 0.00163

Mean deflection..96 ± 1.35 124.2 ± 0.97

Difference........ 28.2 ± 1.66

Sensibility per scale division
= 0.000485 ± 0.000028

The experimental work indicates that the iron is less transparent to the more penetrating Röntgen rays when it is magnetized in a direction parallel to the path of the Röntgen rays.

Since the Röntgen ray is due to a disturbance created by an electron

TABLE II.

Filter of Tinfoil and Iron in Position b.

		Electrometer Deflection.				Electrometer Deflection.	
Mili-amp.	K. V.$_2$	Mag. Current Off.	Mag. Current On.	Mili-Amp.	K. V.	Zero Setting of Lead Strip.	Setting Changed 2 Mm.
5	81.5	−36		5	81.5	60	
5	81.5	−44		5	81.5	60	
5	81.5	−35		5	81.5		76
5.1	81.5		−26	5	81.5		69
4.9	81.5		−30	5	81.5		74
5.1	81.5		−31	5	81.5	55	
5	81.5	−37		4.9	81.5	60	
4.9	81.5		−32	5	81.5		76
5	81.5		−31				

Mean deflection. −38 ± 1.37 −30 ± 0.7

Difference...... 8 ± 1.54

Proportional change = 0.007 ± 0.00156

Mean deflection..58.7 ± 0.8 73.7 ± 1.11

Difference....... 15 ± 1.37

Sensibility per scale division
= 0.0009 ± 0.0000835

Table III.

Filter of Tinfoil with Iron in Position b and with Tinfoil Over Top of Each Ionizing Chamber.

Mili-amp.	K. V.	Electrometer Deflection.		Mili-amp.	K. V.	Electrometer Deflection.	
		Mag. Current Off.	Mag. Current On.			Zero Setting of Lead Strip.	Setting Changed 2 Mm.
5	81.5	−25		5.1	81.5	59	
4.9	81.5	−26		5	81.5	57	
5	81.5		−20	5	81.5	57	
5	81.5		−20	5.1	81.5		69
5	81.5	−25		5	81.5		74
5.1	81.5	−26		5	81.5		76
5.1	81.5		−19	5	81.5	59	
Mean deflection		−25.5 ± 0.19	−19.66 ± 0.27	Mean deflection...		58 ± 0.354	73 ± 1.4
Difference..			5.84 ± 0.33	Difference........			15 ± 1.44
Proportional change = 0.005 ± 0.000595				Sensibility per scale division = 0.0009 ± 0.000088			

whose motion is in the direction of the cathode stream of the X-ray tube, we would expect the absorption of energy from the Röntgen rays to be by those parts of the molecule free to vibrate in the same plane. And unless the molecule is symmetrical about all its axes, there is a plane through it where the absorption is a maximum. Magnetizing the iron will tend to align the molecules so that they will have their planes of maximum absorption parallel to one another. If the planes of maximum absorption are parallel to the cathode stream of the X-ray tube, then the iron will seem more opaque. The results of this work indicate that the plane of maximum absorption of the iron molecule is parallel to that of the electronic orbits which make up the elementary magnet.

Table IV.

Filter of Tinfoil with Iron in Position c.

Mili-amp.	K. V.	Electrometer Deflection.		Mili-amp.	K. V.	Electrometer Deflection.	
		Mag. Current Off.	Mag. Current On.			Mag. Current Off.	Mag. Current On.
5.1	26.5	62		4.9	81.5	42	
5	27	62		5	81.5	38	
5	27		61	5	81.5		42
5	27		62	5	81.5		40
5	27	57		5.1	81.5	37	
5.1	26.5	60		5.1	81.5		36
5	27		60				
Mean deflection		60.25 ± 0.79	61 ± 0.387	Mean deflection...		39 ± 1.40	39.3 ± 1.18
Difference......			0.75 ± 0.875	Difference........			0.3 ± 1.6
Proportional change = 0.0003 ± 0.000425				Proportional change = 0.00027 ± 0.0013			

It seems to the author that the failure of the Röntgen ray to ionize all the molecules of a gas through which it passes may be explained as due to the molecule having only one plane in which it can absorb sufficient energy from the Röntgen ray for ionization. As soon as time will permit, the effect of a magnetic field will be tried on the opacity of oxygen in an attempt to see if this is true.

In conclusion the author wishes to express his indebtedness to Prof. Shearer for the use of his X-ray laboratory and the generous loan of a Coolidge X-ray tube, without which this last work would have been impossible. He wishes also to thank Prof. Merritt for the encouragement received during the slow progress of the work.

September 20, 1915.

AN INVESTIGATION OF THE TRANSMISSION, REFLECTION AND ABSORPTION OF SOUND BY DIFFERENT MATERIALS.

By F. R. Watson.

THE experiments on the transmission of sound were performed with the following arrangement of apparatus.[1] The source of sound was an adjustable whistle blown by air from a constant pressure tank and mounted at the focus of a specially constructed parabolic reflector with a focal length of nine inches and an aperture of five feet. This was placed in front of an open doorway so that the sound, which proceeded in a large parallel bundle from the reflector, could pass through the doorway into another room. The receiver of sound, a Rayleigh resonator, was mounted in the other room in the path of the sound symmetrically opposite the reflector and doorway and measured the intensity of the transmitted sound.

The resonator used was a modification of Rayleigh's original design.[2] It consisted of a horizontal brass tube closed at one end by an adjustable piston. A mica disc was suspended by a quartz fiber at an angle of 45° with the axis of the tube. When the sound of the whistle reached the resonator it set up a back-and-forth surging of the air in the resonator and caused the mica disc, which was placed at a loop, to rotate. This action is in accordance with the general principle that any flat object in a current of air tends to set itself at right angles to the current. The amount of rotation was measured by means of a lamp and scale in connection with a mirror which was attached to the suspended system above the mica disc.

The readings on the scale are proportional, for small angles of rotation of the disc, to the intensity of the sound. This is shown as follows. The moment M of the couple turning the disc may be proven[3] to be

$$M = kW^2 \sin 2(\theta - \varphi),$$

where W is the velocity of the steam, θ is the angle of repose between the direction of the stream and the normal to the disc, φ is the angle of deflection, and k is a constant. In case the stream is not steady but alternating, as it would be in the case of the vibrating air in the resonator, W^2 may be replaced[1] by the mean value of W^2. The intensity, I, of the sound setting up the vibrations in the resonator is proportional to the square of the velocity,[2] so that

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[1] Phys. Rev., Vol. V., p. 342, 1915.

[2] Phil. Mag., Vol. 14, p. 186, 1882.

[3] W. König, Wied. Ann., Vol. 43, p. 51, 1891.

$$M = k_1 I \sin 2(\theta - \varphi).$$

Finally the turning couple M becomes equal for equilibrium to the restoring couple set up by the twisted quartz fiber and this latter couple is proportional for small angles of rotation to the angle of rotation φ, or

$$M = k_2\varphi.$$

Comparing the intensities of two sounds we get

$$\frac{I_1}{I_2} = \frac{\varphi_1}{\varphi_2} \cdot \frac{\sin 2(\theta - \varphi_2)}{\sin 2(\theta - \varphi_1)}.$$

In the experimental observations, φ was not measured directly; since the scale readings are proportional to tan 2φ, the scale being plane and the angle of deflection being doubled by the reflection of the spot of light from the mirror. Calculations for the data taken show that the ratios

$$\frac{\tan 2\varphi_1}{\tan 2\varphi_2}$$

and

$$\frac{\varphi_1 \sin 2(\theta - \varphi_2)}{\varphi_2 \sin 2(\theta - \varphi_1)}$$

differ about 2 per cent. for the maximum angle of deflection, 11°. Therefore, as stated, the readings on the scale may be taken as proportional to the intensity of the sound.

Measurements were taken, first, through the open doorway, then with one panel of material placed over the doorway, then two panels and finally three panels; the deflection of the resonator being noted for each case. Considerable trouble was experienced in getting steady deflections of the resonator. This was finally overcome to a great extent by arranging a delicate adjustment for keeping constant the flow of air to the whistle, and also by building a small house with a glass window for the observer. Any movement of articles or air in the room changed the deflection of the resonator so that rigid observance of immovability of objects was necessary. The samples to be tested were mounted on similar frames of one-inch cypress strips and fastened over the open doorway by two ropes. A strip of hairfelt was mounted around the woodwork of the doorway to prevent sound leaking through at the edges. Preliminary measurements were carried on for some time to get the apparatus and method of taking observations in satisfactory shape. On December 30, two complete sets of observations were taken, the average of these being used to obtain the comparative values of the transmission powers of the different materials. Table I. gives the results obtained.

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[1] Rayleigh, Th. of Sound, Vol. II., p. 44.

[2] Rayleigh, Th. of Sound, Vol. II., p. 16, and Zernov, Ann. der Phys., Vol. 26, p. 79, 1908.

Table I.

Transmission of Sound.

Material.	**Deflection of Resonator in Centimeters.**				**Average Deflection.**			
Thickness in Layers.	0	1	2	3	0	1	2	3
Open doorway	40.3 38.5				39.4			
½" hairfelt		23.0 22.3	15.3 15.5	10.8 9.9		22.6	15.4	10.4
¼" cork board		7.7 8.1	3.6 3.9	2.9 2.9		7.9	3.75	2.9
¾" cork board		1.1 1.2	2.1 2.0	1.0 0.7		1.15	2.05	0.85
¼" paper-lined hairfelt		5.7 4.3	22.3 21.1	4.4 3.2		5.0	21.7	3.8
¾" paper-lined hairfelt		7.1 5.9	2.0 1.9	0.5 0.3		6.5	1.95	0.4
¾" flax board		2.2 2.3	0.5 0.6	0.1 0.1		2.25	0.55	0.1
¼" pressed fiber		0.4 0.25				0.32		
¾" pressed fiber		0.2				0.2		

Table II. gives the calculated percentages of the sound transmitted and the sound stopped, it being assumed that the open doorway transmits 100 per cent. or, that it stops 0 per cent.

The data of Table II. are shown in the form of curves in Fig. 1. Inspection of these curves shows that ½ in. hairfelt stops less sound than the other materials, one layer stopping only 43 per cent. Next comes the ¼ in. cork board which stops 80 per cent. for one layer and 90.5 per cent. and 92.6 per cent. for two and three layers respectively. This is followed by the ¾ in. paper covered hairfelt, the ¾ in. flax board and finally the ¼ in. pressed fiber, one layer of which stops practically all the sound. These values do not tell the whole story concerning the acoustical efficiency of the materials, since other qualities must also be considered. The pressed fiber, for instance, is of little value acoustically because its sound absorbing power is very small.

TABLE II.

Transmission of Sound Through Different Materials.

Material.	**Percentage of Sound.**							
	Transmitted.				**Stopped.**			
Thickness in Layers	**0**	**1**	**2**	**3**	**0**	**1**	**2**	**3**
Open doorway	100				0			
½" hairfelt		57.0	39.0	26.0		43.0	61.0	74.0
¼" cork board		20.0	9.5	7.4		80.0	90.5	92.6
¾" cork board		2.9	5.2	2.2		97.1	94.8	97.8
¼" paper lined hairfelt		13.0	55.0	9.6		87.0	45.0	90.4
¾" paper lined hairfelt		1.7	0.5	0.1		98.3	99.5	99.9
¾" flax board		5.7	0.14	0.02		94.3	99.8	99.9
¼" pressed fiber		0.08				99.9		
¾" pressed fiber		0.05				99.9		

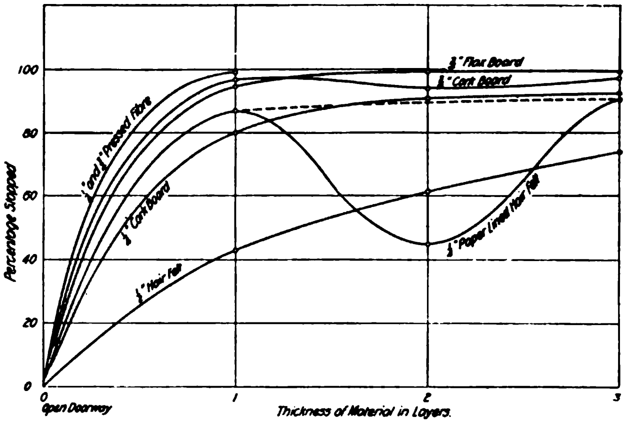

Fig. 1.

Percentage of sound stopped by materials.

Curves showing the percentage of sound stopped by different materials.

The curves for the ¼ in. paper-lined hairfelt show that this sample acts differently than the others. *Two* layers of this material stop *less* sound than *one* layer. Repeated measurements gave the same puzzling result. The ¾ in. cork board shows the same phenomenon, but to a less degree. After some consideration, it was decided to investigate other acoustical properties of the samples to see if additional data would explain this anomalous transmission.

If incident sound falls on a material, three things may happen. The sound may be partly reflected, partly absorbed and the rest transmitted. If these three fractions are added together, they must equal the incident sound, or

$$T + A + R = I = 100 \text{ per cent.}$$

Therefore to know what happens to the incident sound it is necessary to determine the amounts reflected, absorbed and transmitted. On considering the case of the paper-lined hairfelt in the light of this reasoning, it was decided to attempt to measure the reflection of sound.

Reflection of Sound.

By moving the parabolic reflector off to one side, the sound was sent obliquely toward the open doorway where it was reflected by the hairfelt and then passed to the Rayleigh resonator which had been moved into the same room with the reflector and placed so as to be directly in the path of the reflected sound. The observer, as in the transmission tests, stationed himself inside the small house and read the deflection of the resonator through the glass window. A small portion of sound was reflected from the sides of the doorway so that, even with no material over the open door space, the resonator gave a small deflection. This was taken as the zero deflection for the other readings. The deflection for 100 per cent. reflection was arbitrarily taken to be the largest deflection obtained, namely, the deflection given by one layer of ¾ in. cork board. This value is doubtless too small, but probably not much in error, especially when only *comparative* values are being considered.

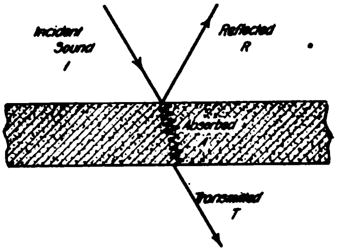

Fig. 2.

Action of a material in reflecting, absorbing and transmitting sound.

The average of two sets of observations on the reflection of sound from the materials is given in Table III. and in curves in Fig. 3. The interpretation of the results is best realized by combining curves from Figs. 1 and 3. Thus, for ½ in. hairfelt in Fig. 4, it is seen that the curve for the reflected sound follows very closely the curve for the stopped sound. Consideration of Fig. 2 shows that the sound which is stopped by the material is reflected and absorbed.

The amounts of reflected, absorbed and transmitted sound are thus

TABLE III.

Reflection of Sound by Different Materials.

Material.	**Deflection of Resonator in Centimeters.**				**Percentage of Sound Reflected.**			
Thickness in Layers.	0	1	2	3	0	1	2	3
Open doorway	3.9				0			
½″ hairfelt		4.9	6.6	10.5		19	25	40
¼″ cork board		15.7	22.0	22.6		61	85	87
¾″ cork board		25.9	21.2	22.1		100	82	85
¼″ paper lined hairfelt		20.7	5.9	10.0		80	23	39
¾″ paper lined hairfelt		10.4	6.6	9.3		40	25	36
¾″ flax board		22.5	20.0	20.0		87	77	77
¼″ pressed fiber		23.2				90		

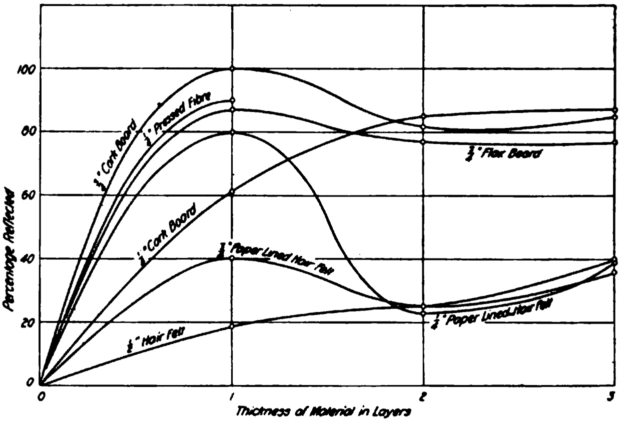

Fig 3.- Percentage of Sound Reflected by Materials

Fig. 3.

Curves showing the percentage of sound reflected by different materials.

easily shown in Fig. 4. The amounts of sound reflected and absorbed increase with the thickness, but the transmission decreases. It should be remembered that the absolute values for the absorption and reflection are doubtless in error but that the comparative values are in correct proportion.

The curves for the ¼ in. paper-lined hairfelt are shown in Fig. 5. The two curves of reflection and transmission follow each other closely. It is interesting to note how the absorption increases uniformly although the transmission and reflection both vary. The probable cause for the anomalous reflection and transmission, as will be discussed later, lies in the vibration of the material due to resonance. Certain thicknesses of the material vibrate vigorously under the action of the sound and thus create sound waves on the further side of the material. This explanation is advanced also by Weisbach[1] who made a similar test but with different apparatus and method.

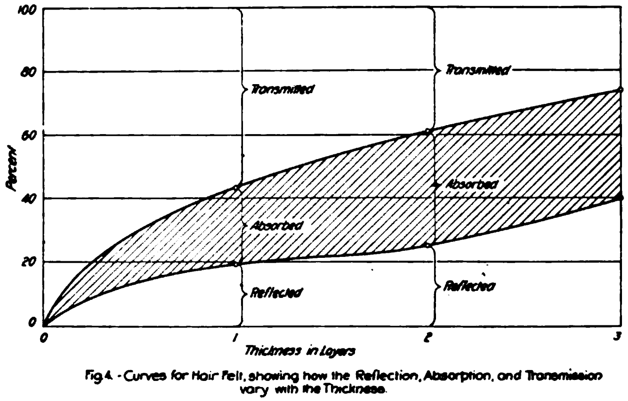

Fig. 4.

Curves for hairfelt, showing how the reflection, absorption and transmission vary with the thickness.

Discussion of Results.

The transmission of sound of constant pitch depends on at least three qualities of the transmitting material;—its porosity, density and elasticity. Porous bodies transmit sound in much the same proportion that

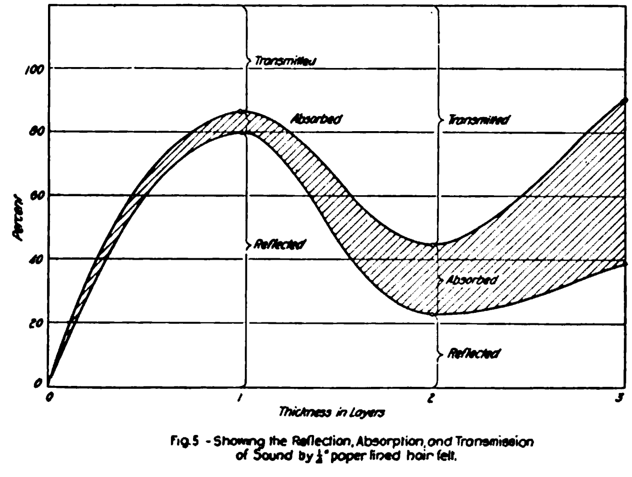

Fig. 5.

Showing the reflection, absorption and transmission of sound by ¼" paper-lined hairfelt.

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[1] "Versuche über Schalldurschlassigkeit, Schallreflexion und Schallabsorption," Annalen der Physik, Vol. 33, p. 763, 1910.

they transmit air.[1] This is why hairfelt transmits more sound than the other samples. Density also plays a part. Two samples stop sound in proportion to their densities, other conditions being equal.[2] Thus the pressed fiber stops more sound than the same thickness of cork because it is heavier. Finally, an elastic body may transmit sound if it happens to be in tune with the source of sound so as to vibrate. To make this clear, consider the material to form a wall in the path of the sound and imagine it to vibrate exactly as the air would if the material were not present. Under these circumstances, there would be no reflection but only transmission of sound. From the results obtained it seems probable that two layers of paper lined hairfelt approximate to such a vibration.

In case the pitch of the sound varies, porous walls and elastic walls reflect high pitched sounds in greater degree than low pitched ones.[3]

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[1] Tufts, Amer. Jour. of Science, Vol. 2, p. 357, 1901.

[2] Jäger, "Zur Theorie des Nachhalls," Sitzber. der Kaiserl. Akad. der Wissenschaften in Wien, Math-naturw. Klasse; Bd. CXX., Abt. II*a*, Mai, 1911.

[3] Jäger, loc. cit.

THE RADIOACTIVE DEPOSIT FROM THE ATMOSPHERE ON AN UNCHARGED WIRE.

By S. J. M. Allen.

IN the Physical Review for 1908 appeared an article by the author on the "Radioactivity of the Atmosphere," which stated that a considerable amount of active deposit could be obtained from a smoky atmosphere on an uncharged wire. This deposit was the same in nature as that obtained when the wire was charged to a high negative potential, having decay curves varying between the same wide limits.

Mr. Wilson, at Manchester, England,[1] published a paper in which he stated that he could get no appreciable deposit on an uncharged wire, though he got an effect when the wire was negatively charged. This seemed strange, as one would expect a considerable effect at Manchester, which by report is as smoky as Cincinnati. Granting that the conditions are favorable at Manchester for obtaining the active deposit, Mr. Wilson's negative result may be explained in several ways. With a wire of only 50 feet he could not expect to obtain enough deposit to measure unless he used a very sensitive apparatus. The active deposit on an uncharged wire is in general small compared with that on a charged wire. Careful correction, or elimination, of the "natural leak" of the apparatus would be necessary. The author used 360 feet of wire and a very sensitive balance method in which the natural leak was eliminated.

Mr. Harvey, at Denver, Colorado, states that he could obtain a very small effect on an uncharged wire in an atmosphere which was very clear as far as smoke was concerned, but contained a considerable amount of dust.

There does not seem to be much doubt that in an atmosphere containing small nuclei, such as dust, smoke, rain, and snow, one can collect a radioactive deposit without the aid of a strong electrical field. These nuclei apparently act only as carriers and are drawn to the wire by diffusion, or wind. It is probably true that in case of smoke and dust particles the nuclei are sometimes charged. Negative nuclei would

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[1] Wilson, Phil. Mag., 1909.

[2] Harvey, Phys. Rev., 1909.

TABLE I.

Date.	Exposure in Hours.	Activity.	Remarks.
Nov. 3, 1914.....	74	448	W. wind, clear. Heavy deposits.
" 4, "	24	472	
" 5, "	24	512	
" 6, "	23	336	E. wind, clear.
" 7, "	25	384	W. " "
" 9, "	54	64	Rain.
" 10, "	22	168	W. wind, clear.
" 11, "	19	384	
" 12, "	24	256	
" 13, "	23	168	
" 14, "	25	268	E. wind, clear.
" 16, "	52	48	Rain.
" 18, "	52	344	W. wind, clear. Heavy deposits of soot.
" 20, "	48	512	
" 21, "	26	580	
" 23, "	53	308	
" 24, "	15	332	
" 25, "	29	308	Light rain.
" 30, "	124	108	Heavy rain.
	Average....	316	
April 1, 1915.....	24	121	W. wind, clear.
" 3, "	48	90	N. E. wind, stormy.
" 5, "	48	78	Light rain.
" 7, "	54	107	" "
" 8, "	24	117	Clear.
" 9, "	17	175	W. wind, clear.
" 13, "	96	168	Part rain.
" 14, "	24	68	Clear.
" 17, "	72	117	Part rain.
" 19, "	52	117	Clear.
" 26, "	168	207	Clear for 96 hours.
" 27, "	24	168	W. wind, clear. Very light deposits during month, except on the 26th.
" 28, "	24	117	
" 29, "	24	100	
	Average....	125	

therefore be the most efficient as collectors of the radioactive matter, which is positively charged. Any wire insulated in the air would be subject to the effect of the natural potential gradient of the atmosphere which is normally negative with respect to earth. Except in rare cases this potential gradient would not be comparable with the high ones usually used by experimenters in different parts of the world.

The author could get an active deposit whether the wire was insulated from, or connected to, the earth, and even a small trace when it was positively charged.

TABLE II.

Month.	**Activity.**			**Number of Observations.**	**Periods of Rain.**
	Maximum.	**Minimum.**	**Average.**		
March, 1914......	375	47	149	9	3
April, "	300	48	163	20	4
May, "	520	50	183	23	4
June, "	391	120	232	9	2
July, "	308	36	132	5	2
Sept., "	268	125	191	5	1
Oct., "	500	68	196	23	10
Nov., "	580	48	316	19	3
Dec., "	414	50	193	13	9
Jan., 1915......	443	54	223	18	8
Feb., "	410	28	181	13	5
March, "	268	0	170	18	5
April, "	207	68	125	14	4
May, "	396	100	242	18	10
Grand avera	....	....	193		

The present paper gives an account of further observations made during the last fourteen months under different atmospheric conditions. They are of considerable local interest, since for several years the Smoke Abatement League has labored to decrease the amount of smoke, and in the last year or two has been quite successful, so much so, that today the average clearness of the atmosphere at Cincinnati is greatly improved over what it was a few years ago. It was therefore of interest to see if this was having any effect upon the active deposit.

The experiments were carried out in exactly the same place, and with the same apparatus as those in 1908. The wire was 360 feet long, running as an endless belt over pulleys. The active deposit was rubbed off on to a piece of cotton, or linen, and tested in the null reading "balanced" electrometer, devised by the author. The electrometer was made very sensitive, and an uranium oxide standard used. Before each reading the "natural leak" was exactly balanced in the testing chamber and the standard, so that the only ionization measured was that of the active deposit.

By several preliminary experiments the conclusion was reached that nearly all the active deposit was removed from the wire by rubbing. At any rate only the outer layers of the deposit would be very active, since in a twenty-four-hour exposure the first active layers would have decayed away by the time of removal.

Observations were made on most days that the weather would permit. Those during the summer months are rather incomplete, as the author was absent from the university during that time. Exposures varied from about 15 hours to several days. An exposure longer than one day did not seem to make much difference in the active deposit obtainable. On many days decay curves were taken over as long a range as possible.

The results obtained for the last fourteen months are shown in Tables I. and II. Table I. gives the complete observations for the highest and lowest months; Table II. the summary for each month. The activity is given in terms of uranium oxide. A reading of 100 is equal to that of 1 sq. cm. of uranium oxide of about .9 the thickness of the McCoy Standard ($100 = 5.2 \times 10^{-13}$ amperes). A characteristic set of decay curves are also shown.

The following summary will serve as a discussion of the results obtained.

1. It has been clearly shown that an active deposit can be obtained on an uncharged wire in a smoky atmosphere such as exists at Cincinnati.

2. From the decay curves here shown it can be readily seen by careful examination that the active deposit consists of a mixture of the various disintegration products of radium. All the curves can be divided into three classes: those which are at first convex upward followed, by concave upward; those which are concave upward, then convex upward, and finally concave; and those which are very approximately exponential over their complete range. The periods (half decay values) vary from 8 to 50 minutes. A common exponential has a period about 28 minutes (Radium C). Of course the deposit is a chance mixture which accounts for the great variation in the decay curves. Deposits have been taken which differed by only a few hours, and which showed entirely different decay curves.

3. On no occasion could a decay curve be obtained which showed a period long enough to be taken for the thorium products. If it was present at any time it was never greater than the experimental error, certainly not greater than 10 per cent. This is in sharp contrast with the results shown in 1908 for a charged wire, when the thorium products were sometimes present in proportions as great as 30–40 per cent. It must therefore be that the thorium products are present in the air in small quantity compared with the radium products, or that they do not collect readily on smoke particles.

4. The amount of soot deposited varied through wide limits, the maximum amount collected in 24 hours being about 0.12 grams. It was found that in general a large amount of soot indicated a large amount of activity, although there were several cases where a large activity was obtained from a small deposit of soot. I think it can be safely said as a

rough approximation that, if the radioactive matter in the atmosphere remains the same, the amount of activity on the wire is proportional to the amount of soot deposited. This of course would be only true for a steady state up to equilibrium. An interesting case was observed once. On one day the activity was 90 with clear weather. During the night there occurred one of the heavy smoke fogs which used to be common to Cincinnati, but now more rare. In the morning the activity was 448. The next day was clear again, and the activity dropped to 100. The normal activity was thus low, but the heavy precipitation of smoke deposited a large activity momentarily on the wire.

5. The activity does not seem to vary greatly in summer and winter, but the results so far obtained would seem to indicate maxima and minima periods. A maximum occurs around November and December, and a minimum around April. A possible reason for this would be the following. At the approach of winter the production of smoke particles in the air is greatly increased due to inefficient stoking, and to the bad condition of the furnaces lying idle all summer. By late winter more efficient operation has been obtained with the result of much less smoke. It is a fact that the amount of soot in the air here in early winter is much greater than in late winter. In November the deposit was very thick and black, while in April it was light and gray, and mixed with dust.

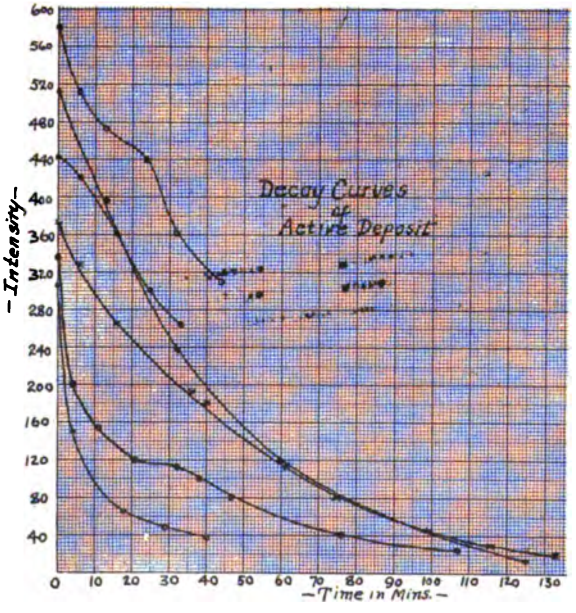

Fig. 1.

Both months were very fine. In May though the weather was very wet the activity was very much greater than in April. Whether actual maxima and minima occur in the amount of active matter in the air is not yet certain. Future observations over a long time may give results of interest.

6. Winds from the NW. to SW. in general give the largest activity, while those from the E. give the smallest. Winds from the former quarter in Cincinnati blow over the smoky part of the city and also bring fine weather. East winds come from over the residential parts.

7. The effect of rain is very certain. A heavy continued rain always clears the air of the activity. If the weather clears up with strong W. winds, the activity rises rapidly to its normal value. Before a rain the activity decreases gradually. Even during a rain storm one can often obtain a small amount of activity by rubbing the sooty water from the wire, and then drying the cloth. On one or two occasions even in fine weather not enough active deposit to be measured could be obtained.

In conclusion one can say that the results enumerated and discussed in this paper fall into line with the general results on the radioactivity of the atmosphere observed by different experimenters in many parts of the world, the deposit on an uncharged wire being due to the smoke nuclei in air acting as carriers. The actual deposit at any time is a chance diffusion mixture in all proportions, and states of decay, of the radium products present in the atmosphere.

UNIVERSITY OF CINCINNATI,
June, 1915.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE CHICAGO MEETING.

THE eightieth meeting of the American Physical Society was held in the Ryerson Physical Laboratory of the University of Chicago on Friday, November 26, and Saturday, November 27, 1915. On Friday an afternoon session only was held. On Saturday sessions were held both in the forenoon and afternoon. President Merritt presided.

A committee consisting of G. W. Stewart, of the University of Iowa, H. N. Randall, of the University of Michigan, and C. E. Mendenhall, of the University of Wisconsin, presented the following resolution and it was adopted by the society by a rising vote:

" Karl Eugen Guthe possessed an unusual combination of those qualities which came to the notice particularly of his associates in the American Physical Society. In him a strong desire and exceptional training for thoroughness and precision was broadened by a great diversity of scientific interests; an absorbing enthusiasm was tempered by a large measure of human interest which made his companionship peculiarly attractive. His energy and effort were genuinely centered in his science and his work, freeing him from false pride in his own attainments and making him always generous in his appreciation of the work of others. His original contributions to the development of Physics are conspicuously evident wherever he has labored, and bear witness to his keenness of intellect and skillful technique. Such a man will be very greatly missed as a friend, as a physicist and as a counselor.

(SIGNED) G. W. STEWART,
C. E. MENDENHALL,
H. M. RANDALL.

The managing editor of the PHYSICAL REVIEW informally presented some information concerning the present condition of the REVIEW and asked for suggestions, particularly with reference to the policy that should be adopted by the editorial board in attempting to provide an adequate channel for the publication of the increasing amount of material pressing for publication. He stated that the present income of the REVIEW had proved fully adequate for maintaining the REVIEW of the size that it has been maintained since the beginning of the present series and that with a continuance of this income a small increase in size might be made; but that, unless the income were augmented, no great increase in size could be made without drawing upon surplus. The society voted to express its approval of the conduct of the REVIEW thus far and its confidence in the editorial board and to record its feeling that the board should feel authorized to make such enlargement in the REVIEW as, in the opinion of the board, should seem desirable and expedient.

The following papers were presented:

Talbot's Bands and the Resolving Power of Spectroscopes. THOMAS E. DOUBT.

Sensitive Photoelectric Cells and a Photoelectric Relay. JAKOB KUNZ and J. STEBBINS.

Atomic Models in whose Behavior a Magnetic Field is an Important Factor. ERNEST MERRITT.

Experimental Evidence for the Essential Identity of the Normal and the Selective Photoelectric Effects. R. A. MILLIKAN and WILMER H. SOUDER.

The Theory of Photoelectric and Allied Effects. R. A. MILLIKAN.

The Cause of the Variation of the Emanation Content of Spring Water. R. R. RAMSEY.

Distribution of Transmitted and Reflected β-particles, Determined by the Statistical Method. ALOIS F. KOVARIK and L. W. MCMEEHAN.

The Condensation and Reflection of Molecular Impinging on the Solid Surface. IRVING LANGMUIR.

A Theory of the Initial Conditions of the Corona. JAKOB KUNZ.

Division of Current between a Gas Conductor and a Metallic Shunt. R. F. EARHART.

On the Optical Properties of an Isolated Crystal of Selenium. CHARLES H. SKINNER. (Introduced by L. P. Sieg.)

A Simple Method of Identifying Hexagonal Crystal Structure and an X-ray Analysis of a Hexagonal Selenium Crystal. ELMER DERSHEM. (Introduced by G. W. Stewart.)

Certain Cases of the Variation of Sound Intensity with Distance. G. W. STEWART.

The Distribution of Electrons in a Substance that Exists in the Conducting and the Non-conducting States. L. E. DODD.

A Study of Apparent Specific Volume in Solution. LEROY D. WELD. (By title.)

The Measurement of Optical Rotation in the Infra-Red. L. R. INGERSOLL.

The Maximum Frequency of X-rays at Constant Voltages between 30,000 and 100,000. ALBERT W. HULL.

The Magnetic Properties of Iron above the Curie Point. E. M. TERRY.

On a Supposed Allotropy of Copper. G. K. BURGESS and I. N. KELLBERG. (By title.)

The Reflection of Sound. F. R. WATSON.

On the Melting of Substances Under Pressure. M. A. ROSANOFF.

Preliminary Report on Conditions Affecting the Adsorption of Air by Cocoanut-shell Charcoal. HARVEY B. LEMON.

The Laws of Elastic and Viscoüs Flow. A. A. MICHELSON.

A. D. COLE, *Secretary.*

NEGATIVE RESISTANCE.[1]

BY A. W. HULL.

IT is well known that when electrons fall on a metal plate in vacuum they cause the emission of secondary electrons (sometimes called δ rays) and this secondary emission increases continuously with the velocity of the bombarding electrons, until at a certain velocity it becomes greater than unity; that is, the metal on which the electrons fall loses electrons instead of gaining them, and this loss continues to increase with increasing velocity up to three or four secondary electrons per primary electron.

If the primary electrons come from a hot filament, and the velocity with which they strike the plate is due to a voltage difference between the filament and plate, then it is clear that an increase of this voltage produces a decrease in the net current to the plate; *i. e.*, the resistance of the system filament-plate is negative.

If the system consists only of a filament and plate the secondary electrons will not be able to escape, since their velocity is too small to allow them to go back to the filament against the opposing potential, and they are compelled to return, after a short sally, to the place whence they started. In order to carry them away and realize the conditions of negative resistance sketched above, there must be present a third conductor more positive than the plate. This can most conveniently be made in the form of a perforated conductor placed between the filament and plate, maintained at constant potential higher than the plate, the perforations being large enough to allow a large fraction of the primary electrons from the filament to go through and strike the plate.

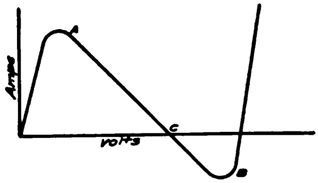

Fig. 1.

A large number of tubes of this type have been made, with considerable variations in form, and all show the general negative resistance characteristic given in Fig. 1. Here the current is plotted positive when positive electricity flows from high to low potential across the evacuated space. The part *AB* of the curve represents negative resistance over the range of voltage included, and obeys the equation

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[1] Abstract of a paper presented at the meeting of the American Physical Society at New York, October 30, 1915.

$$i = i_0 - \frac{V}{r}, \tag{1}$$

or, if the voltage v is measured from the point C, where the curve cuts the axis,

$$i = -\frac{v}{r}. \tag{2}$$

This negative resistance differs from devices having negative resistance characteristics, such as the arc, in three important respects, viz.:

1. It is completely reversible over the entire range.
2. It has no lag except that due to the capacity of the plate, which is exceedingly small.
3. It is aperiodic.

The scientific application of negative resistance in connection with systems possessing positive resistance is very interesting. A single example will serve to illustrate this.

Negative Resistance in Series with Positive Resistance.

Let the tube with negative resistance r be connected in series with an ohmic resistance R, and an E.M.F. V maintained across the terminals, as shown in Fig. 2. If v_r and v_R are the potentials across the negative and positive resistance respectively, and i the current, then

$$\begin{aligned} v_r &= r(i_0 - i) \quad \text{from} \quad \text{Eq. 1}, \\ v_R &= Ri, \\ \therefore \quad V &= v_r + v_R = ri_0 + i(R - r) \\ &= ri_0 + v_R(R - r/R), \\ \frac{dv_R}{dV} &= \frac{R}{R - r}. \end{aligned}$$

That is, small changes in V are magnified $R/(R - r)$ fold in v_R. The negative resistance used in this way is therefore a voltage amplifier, and the amplification $R/(R - r)$ can be made as large as desired by making R and r nearly equal. We obtained in our preliminary tests an amplification of 100 fold, at which point the variations in the constant voltage V_0 furnished by our generator

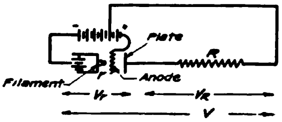

Fig. 2.

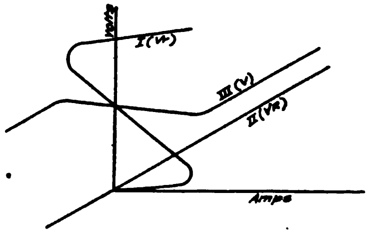

Fig. 3.

became so prominent as to mask the variations which we impressed upon it. But there should be no difficulty in obtaining 1,000 or even 10,000 fold amplification.

The operation is perhaps more evident from the graphical representation given in Fig. 3, where Curve I. represents the potential drop across the negative resistance as a function of the current, Curve II. that across the ohmic resistance, and Curve III, the sum of the two, which is the total potential V across the combination. It is evident that small changes in V correspond to very large changes in v and v_r.

Many other combinations of negative resistance with systems possessing positive resistance characteristics have been tried, with very interesting results, which will be published in the near future.

Research Laboratory,
General Electric Company.

Planck's Radiation Formula Deduced from Hypotheses Suggested by X-ray Phenomena.[1]

By William Duane.

According to the prevailing conception of the mechanism of X-radiation the cathode particles in an X-ray tube generate X-rays when they hit the atoms in the target.

Mr. Hunt, Dr. Webster and the writer have shown recently by experiments (*a*) that the maximum frequency of the X-rays multiplied by Planck's radiation constant h equals the energy of the cathode particles as given by the product of the voltage into the elementary charge, but (*b*) that a large fraction of the radiation has frequencies lying considerably below the maximum, even when all the cathode particles strike the target with the same velocity.

Several investigators claim to have produced X-rays by applying differences of potential of a few volts only to a tube; and, if the above laws hold in such experiments, the waves produced must lie in the ultra-violet region of the spectrum.

According to Moseley's law connecting the characteristic X-rays with the atomic numbers of the elements the α-lines in the K series of helium (if it has a K series) must have wave-lengths at about 1212 ångströms and the β and γ lines probably have wave-lengths several per cent. shorter than this. Using a discharge tube containing helium and hydrogen, Professor Lyman recently discovered some strong lines in this very region of the spectrum.

Further, granting that the above laws connecting the frequencies of X-rays with the energy of the cathode particles hold for small velocities, electrons having the kinetic energy attributed to atoms and molecules at incandescent temperatures on hitting atoms must produce radiation, the wave-lengths of which lie in the visible and infra-red spectra.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, October 30, 1915.

We are thus led to the general conception that radiation of all frequencies is produced by the bombardment of atoms by electrons.

The object of this paper is to discuss the distribution of energy in the spectrum of general, or black, body radiation at constant temperature on the following

Fundamental Assumption.

Some of the electrons in a radiating body fly about more or less freely, hitting the atoms and thereby producing the radiation; and the number dN of these electrons per c.c. that have speeds lying between v and $v + dv$ is given by Maxwell's distribution law, namely:

$$dN = Ae^{-\frac{v^2}{\alpha^2}}v^2dv, \tag{1}$$

where A is a constant at a given temperature, and α, the most probable velocity, is connected with the mass m of an electron and the temperature T by the equation

$$\tfrac{1}{2}m\alpha^2 = kT, \tag{2}$$

k being the well-known constant.

Before proceeding further with our deduction of the energy distribution we must get a relation between the frequency of a train of waves and the velocity of the electron that produces it, and a second relation between the amount of energy radiated and this velocity. The first relation I take from the above mentioned experiments of Mr. Hunt, Dr. Webster and myself, and state it as follows:

Frequency Hypothesis.

When an electron produces radiation by hitting an atom, the frequency ν of the radiation is given by the equation

$$\text{Energy} = \tfrac{1}{2}mv^2 = h\nu. \tag{3}$$

From this point of view the radiation of lower frequency that, as we discovered, came from the tube must be ascribed to secondary, tertiary, etc., radiations, and perhaps to electrons that have been slowed down in the surface layers of the target before they hit atoms in exactly the right way to produce radiation.

The frequency hypothesis says nothing about the *amount* of energy radiated. Equation (3) is not necessarily an energy equation in the sense that the whole kinetic energy $\frac{1}{2}mv^2$ is supposed to be radiated as a quantum $h\nu$ of radiant energy, although it may be so. The hypothesis simply states the relation between the frequency ν and the velocity v of the electron that produced the radiation.

The second hypothesis deals with the *amount* of energy radiated, and this I take from the so-called fourth power law. Sir J. J. Thomson first suggested this law, and some experimental evidence in favor of it has been obtained by estimating the total ionization due to X-rays and assuming proportionality between the total ionization and the total X-ray energy.

Energy Hypothesis.

When electrons of speed v produce X-rays by hitting atoms, the energy radiated per hit is on the average proportional to the fourth power of the speed: thus

$$\frac{\text{Energy radiated}}{\text{per hit}} = lv^4, \tag{4}$$

where l is a constant.

To deduce the distribution law we start with equation (1). The number of hits dH that the electrons of speed v make per second against the atoms must be proportional to dN and to v. Hence

$$dH = Be^{-\frac{v^2}{a^2}}v^3dv, \tag{5}$$

where B is a constant.

From the energy hypothesis, equation (4), the total energy radiated per second by these electrons is

$$dE = lv^4dH = lBe^{-\frac{v^2}{a^2}}v^7dv, \qquad (6)^7$$

and, substituting for a^2 from equation (2) and for v from equation (3), we have, since $mvdv = hd\nu$,

$$dE = 8lB\left(\frac{h}{m}\right)^4 e^{-\frac{h\nu}{kT}}\nu^3d\nu$$

$$= Ce^{-\frac{h\nu}{kT}}\nu^3d\nu, \tag{7}$$

where C is a constant.

This is Wien's equation, and we have deduced it from the assumption that some of the electrons in the radiating body possess the energy that atoms and molecules at the same temperature possess, and from two laws (equations (3) and (4)) suggested by X-ray phenomena.

In the following I propose to show that slight extensions of the frequency and the energy hypotheses, which are consistent with the known facts and laws of X-rays give us the distribution of energy in the spectrum in the form of Planck's equation.

The fact mentioned above that an X-ray tube produces rays of lower frequency than the maximum suggested to me that perhaps the mechanism which generates radiation might be such that electrons produce not only rays of the maximum frequency but also rays of 1/2, 1/3, $\cdots$, 1/nth $\cdots$, etc., of this frequency—a kind of resonance phenomenon. With this extension the first hypothesis becomes:

Frequency Hypothesis.

When electrons of speed v produce X-rays by hitting atoms the frequencies ν of the X-rays are given by the equation,

$$\text{Energy} = \tfrac{1}{2}mv^2 = nh\nu, \tag{8}$$

where n may be any positive integer. Here as above, according to the theory under discussion, radiation does not take place necessarily in quanta $h\nu$, although, the electron being a discrete particle producing individual shocks, the energy must be emitted in a more or less discontinuous manner.

The second hypothesis may be extended very simply as follows:

Energy Hypothesis.

When electrons of speed v produce X-rays by hitting atoms, on the average the energy radiated per hit at frequency ν is proportional to the fourth power of the quotient of v by n, thus

$$\frac{\text{Energy radiated}}{\text{per hit}} = l\left(\frac{v}{n}\right)^4, \tag{9}$$

where l is a constant and n is the same as in equation (8).

Let us calculate the energy radiated per second at frequencies lying between ν and $\nu + d\nu$. Some of this radiation comes from electrons having speeds ying between v given by equation (8) and $v + dv$. The number of hits these electrons make per second is given by equation (5) and from the energy hypothesis, equation (9), the energy radiated per second by them is

$$dE_n = \frac{lB}{n^4} e^{-\frac{v^2}{a^2}} v^7 dv. \tag{10}$$

Substituting from equations (2) and (8), and remembering that $mvdv = nhd\nu$, we have

$$dE_n = 8lB\left(\frac{h}{m}\right)^4 e^{-\frac{nh\nu}{kT}} \nu^3 d\nu$$

$$= Ce^{-\frac{nh\nu}{kT}} \nu^3 d\nu, \tag{11}$$

where C is a constant.

The total energy radiated per second between the frequencies ν and $\nu + d\nu$ is the sum of terms such as (11) where n has the values of the position integers. Hence

$$dE = C\nu^3 d\nu \left(e^{-\frac{h\nu}{kT}} + e^{-\frac{2h\nu}{kT}} + \cdots e^{-\frac{nh\nu}{kT}} + \cdots\right)$$

$$= C \frac{\nu^3 d\nu}{e^{\frac{h\nu}{kT}} - 1}, \tag{12}$$

which is in the form of Planck's equation.

Thus we have succeeded in deducing that equation for the distribution of energy in an emission spectrum which seems to fit the facts of black-body radiation better than any other equation hitherto suggested; and we have done this by making certain simple hypotheses which are consistent with X-ray phenomena as we know them, and which, as a matter of fact, were suggested by a study of X-rays.

According to the point of view adopted here the discontinuity that gives us the series (12) reducing to Planck's form of the equation comes from the frequency relations, and not from any law according to which radiant energy can be produced or exist only in quanta $h\nu$.

As stated above the discussion in this paper applies to the general radiation and not to the characteristic radiation. Apparently the characteristic X-rays do not obey accurately the law represented by equation (3), or (8). As I pointed out at a meeting of this Society last year, in general the energy of a cathode particle must be somewhat larger than $h\nu$, before it can excite a characteristic radiation of frequency ν.

H. A. Lorentz has deduced an expression for the absorption of radiation of sufficiently long wave-length by electrons distributed among the velocities according to Maxwell's law, and finds that the absorption is independent of the frequency. In such a case the ratio of the emission to the absorption expressed as a function of the frequency will be of the form represented by equation (12).

A further minute study of electrons in X-rays tubes may lead to an equation that represents the facts more accurately even than Planck's does.

Harvard University.

The Value of "h" Determined Photoelectrically from the Ordinary Metals.[1]

By W. H. Kadesch and A. E. Hennings.

In photoelectric observations on surfaces of Mg, Al, Sn, Cu, Fe and Zn freshly made in vacuo, it was found that the applied potential necessary to prevent the escape of electrons to the same Faraday Cylinder was, almost within the limits of error of the experiment, the same, for a given frequency, for all the metals tested. The slope of the curve relating the frequency of the light and the maximum emission velocity of electrons was the same for all, within the same limits, hence "h" determined in this way is also practically the same. Because of the greater range of frequencies to which they are sensitive, Mg and Al were chosen for more careful study. The value of "h" obtained from Mg was 6.45×10^{-27}, that from Al 6.41×10^{-27}.

Ryerson Laboratory,
University of Chicago.

The Relations of the Photo Potentials Assumed by Different Metals when Stimulated by Light of a Given Frequency.[1]

By A. E. Hennings and W. H. Kadesch.

Eight of the ordinary metals, Mg, Al, Sn, Cu, Fe, Zn, Ag and Brass, only the first six of which were used, were arranged on a wheel in a photoelectric cell in such a way that they could be turned at will toward a magnetically operated scraper, by means of which fresh surfaces could be readily made, and then toward a Faraday cylinder, placed approximately 180° from the scraper. The light was furnished by a mercury in quartz arc, and was analyzed by a quartz spectrometer. Settings on any spectral line were facilitated by means of a crystal of uranium sulphate. Absorbing screens were used to cut out diffuse light of shorter wave-length than that of the spectral line employed. The Faraday cylinder was connected with one pair of quadrants of an electrometer, and the illuminated electrode given any desired potential by means of a potentiometer arrangement. The electrometer had a sensitiveness of about 1,400 mm. divisions per volt.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, October 30, 1915.

The curves plotted with applied potentials as abscissæ and photo currents as ordinates plunge sharply into the potential axis, indicating the existence of a maximum velocity of emission. There is no indication of an asymptotic approach to the voltage axis.

For a given frequency the curves for the different metals, freshly scraped, all meet the voltage axis at very nearly the same point. When observations were made as soon as possible after scraping the curves for Mg and Al showed a tendency toward slightly higher electronic velocities. The differences however were not much greater than the errors of observation.

When the surfaces were allowed to age there was a gradual fall in the maximum velocities of the electrons. This is attributed to the formation of surface films.

RYERSON LABORATORY,
UNIVERSITY OF CHICAGO.

DEMONSTRATION OF A MODEL VISIBLY REPRESENTING GAS MOLECULES.[1]

BY E. F. NORTHRUP.

THIS is a piece of physical apparatus for illustrating many of the principles of the kinetic theory of gases. It is adapted also for making precise quantitative determinations. Approximately 16,000 one sixteenth inch steel balls in a cylindrical glass inclosure are maintained in motion in the precise manner in which the molecules of a gas are supposed to move. The apparatus enables one to show:

1. The change of pressure of a gas at constant volume with change of temperature.
2. The change of volume of a gas at constant pressure with change of temperature.
3. The viscosity of a gas as exhibited in damping out oscillations of an oscillating system suspended in it.
4. The property possessed by a gas and a liquid of causing a higgeldy-piggedly motion of small suspended particles and known as the Brownian movements.
5. The relation between gas pressure and number of impacts, at a given temperature, per unit time per unit area.

The glass cylinder is about 22 cm. in diameter and its height is 25 cm. Several experiments not here mentioned can be made with this apparatus. The apparatus was shown in operation and its principle features described.

PALMER PHYSICAL LABORATORY,
PRINCETON, N. J.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, October 30, 1915.

THE THEORY OF THE FLICKER PHOTOMETER.[1] II. UNSYMMETRICAL CONDITIONS.

By Herbert E. Ives and E. F. Kingsbury.

WHILE the primary purpose of this paper and the preceding one has been to develop a theory to explain previously ascertained experimental facts, the new experimental results, called forth by the predictions of the theory and the wish to verify all its consequences, are sufficiently valuable to demand that the summary be divided into two parts—" experimental " and " theoretical."

EXPERIMENTAL.

(a) *Phenomena of Frequency of Disappearance of Flicker.*

1. Different ratios of light to dark exposure under constant illumination call for varying critical speeds. At high illuminations a maximum occurs for approximately equal light and dark intervals, but the curves connecting relative period of light exposure and critical speed are not symmetrical.

2. At low illuminations, using blue light, the maximum at equal light and dark exposures is absent. Instead the critical speed increases continuously as the ratio of light to dark interval is decreased.

3. The relationship between illumination and critical speed is represented by the equation

$$\text{critical speed} = a \log \text{illumination} + b,$$

where "a" and "b" are constants, "a" being closely the same whatever the relative light and dark exposures, and "b" varying with the relative exposure.

4. When a flickering illumination is superposed on a steady one, the critical frequency is connected with the mean illumination by the same logarithmic relation as that just given, but the constant "a" varies with the ratio of flickering to steady illumination.

(b) *Behavior of the Flicker Photometer.*

5. The high sensibility of the flicker photometer is shown to be due to the very rapid increase in the critical frequency of disappearance of flicker on each side of the equality setting.

6. In a flicker photometer which exposes the compared colors for unequal periods, the less exposed color will be underrated.

7. Mechanical imperfections in the flicker photometer field are shown to seriously shift the equality point even with no color difference, emphasizing the necessity for strictly substitution methods in flicker photometry.

THEORETICAL.

The behavior of the visual apparatus toward intermittent light is closely parallel to the action of a layer of matter, obeying the Fourier conduction law, in which the diffusivity varies as the logarithm of the intensity of the illumination, this layer being exposed to the intermittent light in one side while its condition on the other side is measured by an instrument whose sensibility is governed by the Weber-Fechner law.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, October 30, 1915.

Or, given the experimental data on the behavior of the eye in perceiving flicker with equal dark and light sectors for different colors at various intensities, it is possible from the most general law of conduction, on the basis of the simple assumptions here made, to predict with considerable accuracy the phenomena occurring with discs of varying openings, at both high and low illuminations, the phenomena with superposed flickering and steady illuminations, the sensibility characteristics of the flicker photometer, the behavior of the flicker photometer toward different colors at various illuminations, the occurrence of color flicker and brightness flicker, the effect of unequal exposures of the colored lights under comparison, and the disturbing effects of mechanical defects in the photometric field.

The phenomena explained or predicted by the theory constitute in fact all known experimental facts in connection with the flicker photometer. Where the correspondence between theory and experimental fact is more qualitative than quantitative we have advanced reasons for believing these differences to be due to the approximate nature of the solutions of the mathematical work, or the insufficiency of our experimental knowledge of certain factors.

UNITED GAS IMPROVEMENT CO. PHYSICAL LABORATORY,
PHILADELPHIA, PA.

THE VELOCITY OF POLYMORPHIC CHANGES UNDER PRESSURE.[1]

BY P. W. BRIDGMAN.

IF two solid phases of any substance are in contact at the equilibrium pressure and temperature, and if the volume is changed by some external agency, an internal readjustment takes place such as to make the pressure return toward the original value. This readjustment is a growth of the appropriate phase at the expense of the other; the rate of growth is measured by the rate of return of pressure. If the displacement of pressure is so small as not to enter the region in which nuclei of one phase are spontaneously produced, the rate of return of pressure measures the rate of growth of the *surface* separating the two phases. Results have been obtained by this method for the time rate of a number of polymorphic transitions. Many substances show a band of finite width on either side of the equilibrium point within which the transition does not run, except for very minute effects of a different order. Furthermore the transition velocity curves are not symmetrical on the two sides of the equilibrium point, but that transition which is accompanied by decreasing volume in almost every case runs more rapidly than the reverse transition. This shows that the mechanism of a polymorphic transition must be entirely different from that of vaporization, for example, in which there is an equilibrium between streams of molecules in each direction. Except for these two points of similarity, different substances show great divergencies in the transition velocity. The velocity may become greater or less at higher pressures and temperatures, and the width of the band of indifference may also vary in either direction, or pass through a maximum or minimum. There is no definite connection between transition velocity and width of the band, but cases have been found where the transition increases in velocity at the same time that the band increases in width.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, October 30, 1915.

THE JEFFERSON PHYSICAL LABORATORY,
CAMBRIDGE, MASS.

HEAT CONDUCTIVITY OF TUNGSTEN AT HIGH TEMPERATURES AND THE WIEDMANN-FRANZ-LORENZ RELATION.[1]

BY IRVING LANGMUIR.

FROM the preceding paper it is seen that the resistance of a tungsten filament over a wide range of temperatures (600 to 2800°) can be expressed quite accurately by the equation $R = cT^\rho$, where c is a constant and $\rho = 1{,}242$ (average value of n between 600 and 2800°). Similarly the radiated power can be expressed with fair accuracy by the relation $W = cT^\omega$ where ω is a constant. By averaging the values of n_W from 2400° down to 400°, weighting each in proportion to the corresponding value of W, the value of ω is found to be $\omega = 4.96$.

If the heat conductivity (λ) of tungsten is assumed to be independent of the temperature, then the differential equation for the distribution of temperature along a filament near a cooling junction is

$$a^2\frac{d^2\theta}{dx^2} + \theta^\rho = \theta^\omega, \tag{1}$$

where θ is the ratio of the temperature (°K.) at the point x to that at a point so far removed from the junction that its temperature is unaffected by the junction. The value of a is

$$a = \frac{\sqrt{R\lambda T}}{v/l}. \tag{2}$$

Here T is the temperature of the filament far from the cooling junction; R is the resistivity of tungsten at the temperature T; and V/l is the potential gradient along the central part of the filament. It should be noted that the diameter of the filament does not occur explicitly in this equation.

Equation (1) apparently cannot be completely integrated, but if the numerical values of ρ and ω are introduced, the values of θ in terms of x/a can be calculated with any necessary degree of approximation. Tables of this function have been prepared and will be published.

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[1] Abstract of a paper read before the Physical Society at New York, October 30, 1915.

By means of this function Worthing's data on the heat conductivity of tungsten at high temperatures[1] have been recalculated. It is found that the experimental data are entirely consistent with the assumption that the heat conductivity is practically independent of the temperature. The value of λ found from Worthing's data for a filament at 2410° K. is $\lambda = 1.14$ watts per cm. per degree.

According to the Wiedemann-Franz-Lorenz relation the quantity $\lambda R/T$ should be a constant for all metals at all temperatures. Using the above value of λ we obtain, for tungsten at a mean temp. of 2200° K.:

$$\frac{\lambda R}{T} = 3.49 \times 10^{-8}.$$

Jaeger and Dieselhorst, by measurement on eight pure metals at 18° C. and 100° C., obtained the mean value (at 18° C.)

$$\frac{\lambda R}{T} = 2.43 \times 10^{-8}.$$

They found, however, that Lorenz's Law is not quite fulfilled, and that $\lambda R/T$ has a positive temperature coefficient, ranging, for different metals, from .0001 to .0010 (average .0004). If we assume that $\lambda R/T = 2.43 \times 10^{-8}$ for tungsten at room temperature then the value 3.49×10^{-8} at 2200° corresponds to a temperature coefficient of .00023, well within the range of values found by Jaeger and Diesselhorst.

We may therefore conclude that tungsten at high temperatures fits in as well with the Wiedemann-Franz-Lorenz relation as do most other pure metals.

Research Laboratory, General Electric Company,
Schenectady, N. Y.

Radiation from Tungsten Filaments and the Mechanical Equivalent of Light.[2]

By Irving Langmuir.[1]

Careful measurements of volts, amperes and candle-power as functions of filament temperature have been made on about fifteen specially constructed lamps. The length and diameter of the filaments were also measured. Corrections were made for the cooling effect of the leads. From these data the following quantities were calculated as functions of temperature:

1. Specific resistance, R ohms-cm.
2. Resistance exponent calculated from the relation $n_R = d \log R / d \log T$.
3. Watts radiated per sq. cm. of surface (bulb at 298° K.).
4. Watts exponent $n_W = d \log W / d \log T$ where W = watts radiated in space at 0° K.
5. Total emissivity (watts radiated expressed as a fraction of the total radiation from a black body at the same temperature).

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[1] Phys. Rev., *4*, 538 (1915).

[2] Abstract of paper presented at the New York Meeting of the Physical Society, October 30, 1915.

6. Intrinsic brilliancy in international candles per sq. cm. of *projected* area.
7. Color. Expressed as the temperature of a black body which emits light of the same color.

The temperature measurements were made with a Holborn-Kurlbaum pyrometer as described previously[1] using a monochromatic screen ($\lambda = .664\mu$) and taking the emissivity of tungsten for this wave length to be 0.46.

In calculating the total emissivity, the constant of the Stephan-Boltzmann equation was taken to be $\sigma = 5.633 \times 10^{-12}$ watts/cm².

To obtain the data on the color of the light a large image of a helically wound filament was thrown on a white screen. With a portable Weber photometer the light from different portions of the image was color-matched against the photometer lamp. It was found that the light from the inside of the helix (nearly black body) was distinctly redder than that from the outside. Quantitative measurements, corrected for the departure of the inside of the helix from true black body conditions, gave the results shown in the table. These results are to be looked upon as preliminary results only. More accurate experiments are in progress.

The intrinsic brilliancy (column 6) can be theoretically calculated from the Planck equation, provided the emissivity of tungsten, the visibility function and the mechanical equivalent of light are known. Conversely, the mechanical equivalent may be determined from the experimental data on the intrinsic brilliancy. Using Nutting's visibility data, and taking the constants of the Planck equation

$$C_1 = 3.72 \times 10^{-21} \text{ watts}; \quad C_2 = 1.4392$$

the mechanical equivalent of light is thus found to be .00121 watt per lumen.[2]

This result is in good agreement with Nutting's[3] value .00120, but differs considerably from that determined by Ives, Coblentz and Kingsbury,[4] namely, .00162.

The values of candles per sq. cm. given in the table have been calculated from the Planck equation and from Nutting's visibility data. The experimental results agree very closely with these values, showing no systematic error except at very low filament temperatures (below 1800°), where the Purkinje effect makes the observed candle-power too low.

To assume that the value .00162 is correct would lead to the conclusion either that the emissivity of tungsten is 0.67 (instead of 0.50) for $\lambda = 0.55$, or that the melting-point of tungsten is at least 3750° K. Both conclusions are altogether inconsistent with other observations.

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[1] Phys. Rev., 6, 138, 1915.

[2] The emissivity of tungsten is taken to be 0.46 for $\lambda = .664\mu$ and 0.50 for $\lambda = .55\mu$ and is assumed to be a linear function of λ.

[3] Trans. Ill. Eng. Soc., *9*, 633, 1914.

[4] Phys. Rev., *5*, 269, 1915.

Properties of Tungsten Filaments.

	1	2	3	4	5	6	7
T	Resistivity Ohm.-Cm.	n_R	Watts per Sq. Cm.	n_W	Total Emissivity.	Candles per Sq. Cm.	Temp. of Black Body Giving Color Match.
273	5.00×10^{-6}	1.40	——	——	——	——	——
300	5.69	1.32	——	——	.0237	——	——
400	8.19	1.305	.0036	5.116	.0330	——	——
600	13.53	1.233	.9360	5.171	.0514	——	——
800	19.28	1.228	.1725	5.433	.0755	——	——
1000	25.32	1.216	.6000	5.584	.1059	.00014	——
1200	31.60	1.215	1.674	5.571	.1435	.0074	——
1400	38.11	1.215	3.921	5.438	.1813	.1325	——
1600	44.87	1.226	8.012	5.265	.2170	1.179	——
1800	51.99	1.262	14.75	5.110	.2494	6.552	1841
2000	59.43	1.272	25.10	5.990	.2785	25.90	2050
2200	67.08	1.273	40.20	4.913	.3046	80.6	2256
2400	74.91	1.268	61.60	4.852	.3290	209.8	2464
2600	82.91	1.257	90.56	4.817	.3517	471.0	2672
2800	90.07	1.237	129.3	4.794	.3730	944.0	2877
3000	99.01	1.219	179.9	4.775	.3941	1729.	3082
3200	107.1	1.171	244.7	4.758	.4142	2941.	3285
3400	114.7	1.138	326.3	4.745	.4334	4725.	——
3540	120.0	——	295.0	——	——	6332.	——

RESEARCH LABORATORY OF THE GENERAL ELECTRIC CO.,
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EQUIPARTITION OF ENERGY AND RADIATION THEORY.[1]

BY W. F. G. SWANN.

THE state of equipartition is one which is infinitely probable on a dynamical scheme when we adopt a certain mathematical concept of probability. It is argued, however, that the infinite mathematical probability of this state is by no means a criterion for the likelihood of its existence. The reasonableness of this view is borne out when we observe that, in counting the number of ways in which a given state may be realized, the mathematics does not limit itself to those relations between the generalized coördinates which we should be willing to admit in a piece of matter while we continued to look upon it as one substance, but it includes in its survey, for example, all those configurations which would be possible when the matter, as we know it, had broken down by radio-active disintegration or otherwise, into its ultimate constituents.

Apart from objections to the theorem of equipartition itself, it is maintained that some of the results which are popularly supposed to follow from it, are by no means the conclusions to which a proper application of the theorem would lead. It is maintained, for example, that the theorem does not lead to equipartition of energy between the electrons inside and outside atoms, in the sense in which such equipartition is ordinarily understood.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, October 30, 1915.

In applying the theorem of equipartition of energy to radiation, apart from the criticisms which may be raised as to the legitimacy of considering the theorem applicable to an infinite number of degrees of freedom of the kind involved, it would appear that there is no justifiable reason for equating the average energy of the degree of freedom to $RT/2$ where R is the measured gas constant, and is a finite quantity. For the nature of affairs in the radiation field is not such that the energy can be expressed with the fine-grainedness necessary for the problem in hand and yet involve terms purporting to represent the energies of the molecules simply in terms of the velocities of centers of gravity.

Any attempt to divide the energy into two parts, a part corresponding to the radiation, and a part representing the energies of the molecules would, in view of the comparative crudeness of the latter, appear to be quite inconsistent with the refined analysis (down to infinitesimal wave lengths) to which the former is afterwards considered as subjected, and it is useless to say that we *define* a perfect gas molecule as a molecule all of whose actions are, for the purposes of our problem, capable of being discussed in terms of the motion of its center of gravity, for to do this would be to deny the molecules the very properties by which two gases, separated from each other, for example, by a non-conducting space, can come into temperature equilibrium through the medium of their radiation alone.

In spite of the foregoing remarks, and although Rayleigh's equipartitional formula $E_\lambda = 8\pi RT/\lambda^4$ cannot be correct (since it is wrong for short wavelengths) there remains the curious experimental fact that the constant R occurring in this formula, as applicable to long wave-lengths, is experimentally found to be the same number as that deduced from other considerations involving measurements on gases direct, without any appeal to radiation phenomena. In the ordinary deductions of Planck's formula, there appears a somewhat analogous difficulty in connecting the measured gas constant with the constant occurring in the radiation formula, a difficulty which is by no means overcome by the usual arguments connecting these constants. In fact, the molecule would appear to be too crude a thing to figure as such in any fundamental analysis from which a radiation formula is deduced; yet, this being so, how does it come about that the constant R associated with the molecule really does appear in the radiation formula?

A rather interesting light becomes thrown on the foregoing matter if we look upon the radiation formula as fundamental, for example, in the form

$$E_\lambda = \frac{8\pi}{\lambda^4} \frac{a\nu}{e^{\frac{b\nu}{T}} - 1} \tag{1}$$

where ν is the frequency and a and b are here considered as having no significance other than that they are constants.

Without concerning ourselves as to how this formula is supposed to be obtained, we may say that if a molecule or electron is placed in the radiation field and if we know only the approximate equations of motion of the molecule or electron under the influence of the field, we shall be able to deduce, to the same degree of approximation, the average energy (as ordinarily understood) which a degree of freedom of the molecule or electron will assume. It can be shown that, for the case of an electron, twice this energy is equal to

$$\mathrm{Lim}_{\nu=0}\left(\frac{\lambda^4 E_\lambda}{8\pi}\right) = \frac{a}{b}T$$

and if we tentatively generalize this result so as to apply to molecules, we see on writing $a/b = R$, that the fundamental origin of the constant R is to be sought in the radiation formula rather than in the gas molecule, and the view to be taken is not that the constant R occurs in the radiation formula because it is the gas constant, but rather that the gas molecule as a whole is constrained by the radiation to move with the average energy $RT/2$, per degree of freedom, because this R is the constant which is contained in the appropriate position in the radiation formula. This view, of course, still allows us to look upon the radiation as originating ultimately in the internal motions of the molecules.

By a further extension of this idea we see that if in formula (1) we replace b/T by some unknown function of the temperature, say $f(T)$, we arrive at the relation.

Average energy per degree of freedom of gas molecule $= a/2f(T)$ and $f(T)$ then becomes determined as proportional to $1/T$ because we *measure* temperature as a quantity proportional to the ordinary kinetic energy of the gas molecule. The point seems of some interest because in those deductions of Planck's formula which involve the fewest assumptions, such deductions for example as that due to Sir Joseph Larmor,[1] the statistical analysis does not carry us beyond a stage in which the temperature is involved as an arbitrary function.

DEPARTMENT OF TERRESTRIAL MAGNETISM,
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THE MAXIMUM FREQUENCY OF X-RAYS AT CONSTANT VOLTAGES BETWEEN 30,000 AND 100,000.[2]

BY ALBERT W. HULL.

IN the *Philosophical Magazine* for September, 1915, Professor Rutherford has reported experiments in which the maximum frequency of X-Rays at different voltages was calculated from the absorption coefficients of the total radiation after a sufficient part of it had been absorbed to make the absorption coefficient nearly constant. He found that the maximum frequencies determined in this way did not increase linearly with the voltage, but less rapidly, and reached a maximum at 140,000 volts.

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[1] Roy. Soc. Proc., Ser. A, 83, pp. 82–95, Dec. 10, 1909.

[2] Abstract of a paper presented at the Chicago meeting of the Physical Society, November 26, 1915.

The measurements given below are taken from the curves of energy distribution in the X-Ray spectra at constant potentials,[1] which the author has been investigating. They were taken with the spectrometer, with an accuracy of about three or four per cent. Within this limit of error the maximum frequencies are proportional to voltage and are given by the quantum relation[2] $eV = h\nu_{\text{max}}$, where ν_{max} is the maximum frequency of X-rays produced at constant potential V, e is the charge of an electron, and h Planck's constant, which is taken as 6.59×10^{-17}. It is evident from the graph that there is no tendency to fall below this value at higher voltages, as Rutherford found. For convenience of comparison, Rutherford's values are reproduced on the dotted curve.

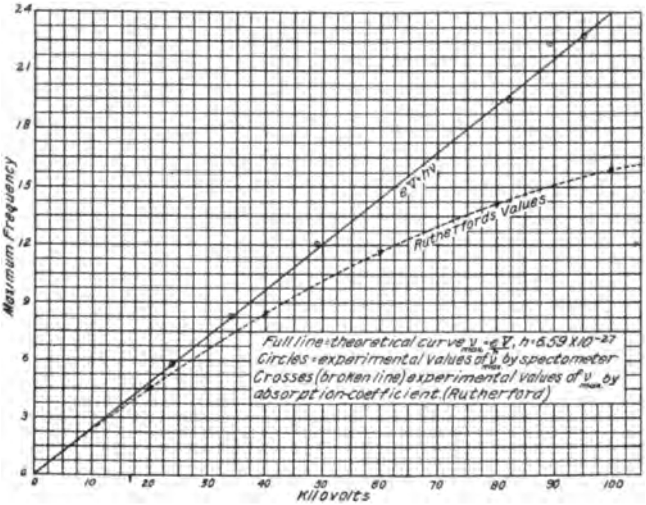

Fig. 1.

The cause of the difference between Rutherford's values and the author's is due to the fact, which Duane (*l. c.*) has already pointed out, that the absorption method does not give the maximum frequency. It can be shown from

Volts $\times 10^3$.	Maximum Frequency $\times 10^{18}$.	
	Theoretical.	Experimental.
24.0	5.8	5.78
34.1	8.21	8.27
48.8	11.78	12.14
82.0	19.8	19.5
89.0	21.4	22.4
95.0	22.9	22.8

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[1] To be published soon.

[2] Duane (Phys. Rev., 6, 166, Aug., 1915) has already shown this to be true for voltages up to 40,000.

the energy distribution curves that the absorption coefficient method leads to values of $v_{\max}$ which fall below the true values more and more as the voltage is raised. In fact, Rutherford's values can be calculated directly from the energy distribution curves and the law of absorption. This question will be more fully discussed in a future publication. The fact which it is desired to point out here is that the maximum frequency for voltages up to 100,000 is given accurately by the quantum relation, and that there is no reason to believe that it should not continue to increase with voltage indefinitely.

RESEARCH LABORATORY, GENERAL ELECTRIC CO.,
SCHENECTADY, N. Y.

NEW BOOKS.

Analytical Mechanics. By MILLER AND LILLY. Boston: D. C. Heath and Co., 1915. Pp. xv+297.

This text-book is the result of collaboration by a mathematician and an engineer and appears to be fairly successful in meeting the needs both of engineering students and of those whose chief interest is in applied mathematics. In the main it follows conventional lines of development, but the problems, which are numerous and apparently well chosen, have a distinctly "practical" character, especially those in statics. Gravitational units of force are used throughout.

A few criticisms of a minor character may be mentioned. On p. 4 and again on p. 127, acceleration is defined by the equations

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2};$$

in the first case there is no intimation that this definition is restricted to motion in a straight line, and in the second case the restriction is not made very conspicuous. The normal component of the acceleration is first introduced on p. 163 by a process which seems unnecessarily complicated. An earlier introduction of work and energy might be desirable; they do not appear until Chapter XI. is reached. In the treatment of the motion of a rigid body, p. 236 et seq., a rather formidable array of mathematics is used, considering the very simple applications which follow. This and some other similar cases appear to be deliberate since, in the preface, the authors say: "We have chosen the older methods of treatment, such as the resolution of forces, etc., because we believe that in this way the student keeps in a little closer touch, at least in the beginning, with the physics of the problem." There is room for difference of opinion as to the validity of this conclusion in all cases; certainly there are several places in the book where adherence to the method of resolution along the coördinate axes does not conduce to perspicuity or physical insight.

H. A. B.

Elements of Optics. By GEORGE W. PARKER. New York: Longmans, Green and Co., 1915. Pp. 1+122. Price, $.75 net.

After a student has mastered Euclid, he is at the threshold of physical theory, and how to introduce him to this realm is the problem of the teacher. The author's solution is simple geometrical optics. Nothing could be more appropriate. The fact that light travels in straight lines immediately enables us to apply geometry. The laws of reflection and refraction are extremely simple, and follow directly from experiment. With Euclid in one hand and these two laws in the other, he can lead the admiring student past a wealth of phenomena, understanding all as he goes. In no other field of physics is such immediate success possible. One cannot but admire the conception behind this little book.

The author carefully preserves the essential sameness of physical theory and mathematical deduction, and yet strongly emphasizes the essential difference, approximation, which he keeps carefully before the reader. The first chapter describes rectilinear propagation, and states the laws of reflection as experimental facts, and then applies them to the phenomena of reflection. The second chapter does the same for refraction. The eye and optical instruments are explained in the third chapter, and the book ends with a description of Newton's experiment which discovered the dispersion of light, and with this basis briefly treats of achromatic lenses.

P. V. W.

Second Series. *February, 1916* *Vol. VII., No. 2.*

THE

PHYSICAL REVIEW.

A SPECTRAL LUMINOSITY CURVE EQUATION AND ITS USE.

By E. F. Kingsbury.

SEVERAL attempts have been made to obtain an equation that would express the luminosity curve of the average eye as a function of an equal energy spectrum. Goldhammer,[1] Hertzsprung,[2] and Nutting[3] have each proposed equations that have the general form required. They have all been simple, transcendental equations and have fitted the experimental curves only approximately. The experimental curves have likewise been only approximate and there has been little value in obtaining an equation to fit any one closely.

Recently, however, considerable work[4] has been done which seems to define the luminosity curve rather better than heretofore. While much more work with a still larger number of observers remains to determine it even more accurately, it seemed worth while at this time to get an equation that would fit the latest determination within the limits of these values. Such an equation would be of value in a definition of the luminosity curve and in various computations of radiant efficiency, the mechanical equivalent of light, etc.

In an attempt to obtain an equation the application of a resonance, or absorption equation, at once suggests itself if we desire to solve it from a theoretical viewpoint. At the present time such an equation would necessarily be largely empirical in its details, and an attempt to make it fit the observed points closely would make it too complicated to be useful. The present effort was therefore to obtain by some simple means an arbitrary equation. The equations that have been suggested have probably included too few constants and where agreement has been obtained between the calculated and the experimental curves, it seems to have been accidental.

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[1] Ann. Ph., XVI., p. 621 (1905).

[2] Z. Wiss. Phot., IV., p. 43 (1906).

[3] "The Luminous Equivalent of Radiation," Nutting, Bull. of Bur. of Stds., V., 1908–9, p. 273.

[4] "Physical Photometry with a Thermopile Artificial Eye," Ives & Kingsbury, Phys. Review, VI, 5 (1915), p. 319.

For example, the form proposed by Goldhammer,

$$L_\lambda = R^n e^{n(1-R)}, \tag{1}$$

where $R = \lambda_{\max}/\lambda$, has two parameters, one of which, $\lambda_{\max}$, is the value of the argument at which the function is a maximum. This is fixed by experiment. Consequently, the only change that can be made is to vary the remaining parameter, n, which merely narrows or broadens the curve, fitting only part of the observed points.

Nutting[1] found that with $n = 181$, function (1) would fit the luminosity curve he had determined experimentally very well from about $\lambda = .48\mu$ to $\lambda = .65\mu$, using $\lambda_{\max} = .555$. This formula below $.48\mu$ gives values too small, and beyond $.65\mu$ too large, with a total area $(\mu \times L_\lambda)$ of about .103, which is practically the same as the experimental one. The calculated curve evidently adds to the red end what it lacks at the blue end. The accuracy of the luminosity curve at the red end is especially

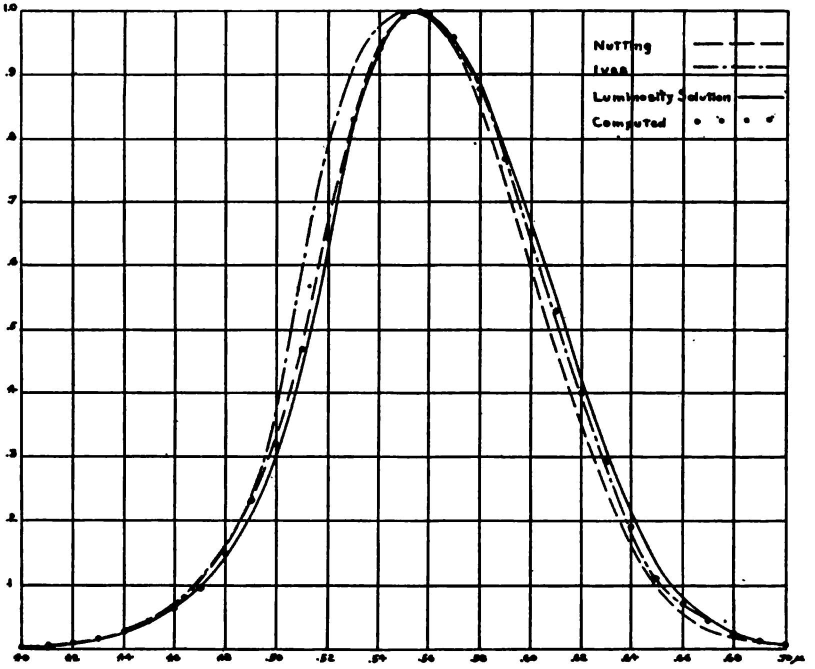

Fig. 1.

Various luminosity curves.

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[1] "The Visibility of Radiation," Nutting, Phil. Mag., 29, p. 301, 1915.

important, because most of our illuminants, at the temperatures we can obtain, have most of the energy on the red side and in multiplying the luminosity curve by the energy curves any errors in the former are magnified considerably.

The method of obtaining the equation given below was as follows:

The exponent, n, of the Goldhammer function (1) was given such a value that it filled the experimental curve throughout a good portion of the blue side and through the maximum, but was low at all other points, *i. e.*, the extreme blue end and all the red side. It was then assumed that the differences between the calculated and the true curves were in each case of the same general form as the one already used. If this were true, then by changing the exponent, n, and the maximum, $\lambda_{\max}$, and adding a constant coefficient to the whole expression, these new expressions could be added to the first one and their sum should be very close to the true value at all points. Of course chance wholly determined how many terms would be needed. As it happened three in all were sufficient. The equation obtained by the above method was as follows:

$$L_\lambda = .999(R_1 e^{1-R_1})^{200} + .04(R_2 e^{1-R_2})^{400} + .095(R_3 e^{1-R_3})^{1000}, \qquad (2)$$

where

$$R_1 = \frac{.556}{\lambda}, \quad R_2 = \frac{.465}{\lambda}, \quad R_3 = \frac{.610}{\lambda}.$$

The experimental curve taken as the criterion was the one described elsewhere and shown in the solid line in Fig. 1. The heavy dotted line is the curve computed according to equation (2). Fig. 2 shows each of the terms plotted separately and their sum. Where the calculated differs from the other it has been left so purposely as being within the best experimental points. In the future, as the luminosity curve becomes better determined, the calculated curve can be changed easily and more terms added if necessary.

Attention should be called to the fact that function (2) has no relation to the trichromatic theory of color vision and the terms have no connection with the three elementary sensation curves as given by Konig. It is true that if we knew these elemental curves sufficiently well we could obtain three spectral functions corresponding to them whose sum would be equal to the luminosity curve of the eye. This would be very desirable as we could carry the study of the luminosity curve to greater detail.

One of the most important applications of the above function is to compute the luminous power emitted by a black body at various temperatures. If J_λ be the quantity represented by Wien's or Planck's equation, and L_λ the quantity represented by the equation (2), then the luminous power, expressed in watts per steradian per square centimeter (projected area) of radiating surface can be expressed as follows:

$$\text{Light watts} = \frac{1}{\pi}\int_0^\infty L_\lambda J_\lambda d\lambda = \text{L}. \tag{3}$$

The factor, $1/\pi$, enters because it is the normal flux density, σ_0, of radiation per square centimeter per degree[4] we are considering and not the total hemispherical flux, σ, their relation being, $\sigma/\pi = \sigma_0$.

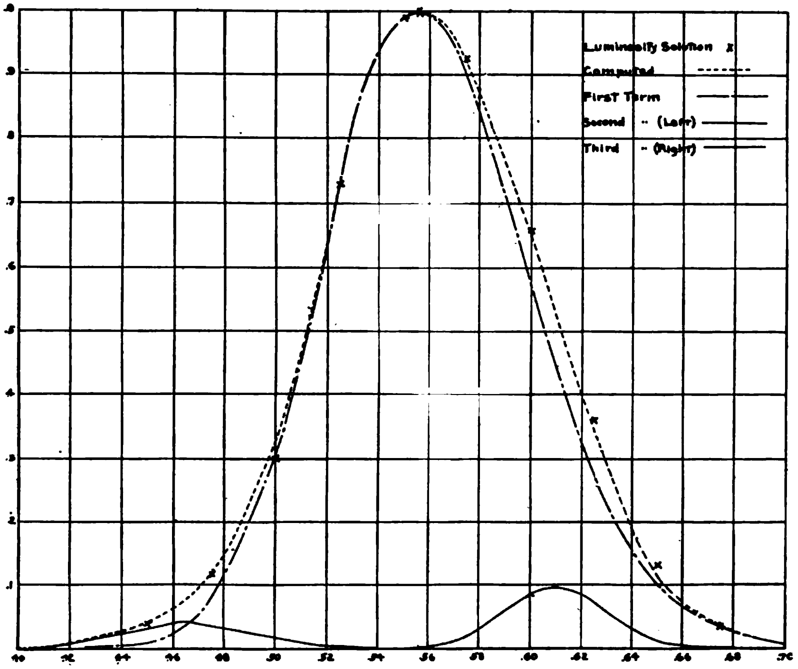

Fig. 2.

Computed luminosity curve showing effect of different terms.

Function (3) has heretofore been evaluated graphically. By means of the new luminosity curve equation it will now be integrated exactly. A formula for the integral will be obtained first by integrating the product of (1), which has the form of each term of (2), by J_λ

$$J_\lambda = c_1\lambda^{-a}(e^{c_2/\lambda k} - 1)^{-1}. \tag{4}$$

Remembering that

$$c_1 = \frac{\sigma c_2^{a-1}}{1.0823(a-2)!}$$

we get

$$L = \frac{\sigma}{\pi} A \sum_{\infty}^{m=1} \left(\frac{1}{\frac{mB}{k} + 1}\right)^{n+a-1}, \tag{5}$$

where

$$A = \frac{e^n}{1.0823(a-2)!}\left(\frac{c_2}{\lambda_m}\right)^{a-1}\frac{\Gamma(n+a-1)}{n^{n+a-1}}$$

and

$$B = \frac{c_2}{n\lambda_m}.$$

Since $\Gamma(n+a-1) = (n+a-2)!$ and from Stirling's formula[1] for large values of n

$$(n+a-2)! = \frac{\sqrt{2\pi}}{e^{n+a-2}}(n+a-2)^{n+a-3/2}$$

we have

$$A = \frac{\sqrt{2\pi}e^{2-a}}{1.0823(a-2)!}\left(\frac{c_2}{\lambda_m}\right)^{a-1}\frac{(n+a-2)^{n+a-3/2}}{n^{n+a-1}}$$

For Wien's law $a = b$ and $m = 1$ (5) reduces to

$$L = \frac{\sigma}{\pi}\frac{A}{\left(\frac{B}{k}+1\right)^{n+4}}, \tag{6}$$

where

$$B = \frac{c_2}{n\lambda_m}$$

and

$$A = .01922\left(\frac{c_2}{\lambda_m}\right)^4\frac{1}{\sqrt{n}}\left(\frac{n+3}{n}\right)^{n+7/2}.$$

If $c_2 = 14370$ and $\sigma = 5.65 \times 10^{-12}$ watts per sq. cm. per degree[4] and using (2) for L_λ instead of (1), (6) becomes

$$L = \frac{5.65 \times 10^4}{\pi}\left[\frac{1.254}{\left(\frac{129.23}{k}+1\right)^{204}} + \frac{.0715}{\left(\frac{77.26}{k}+1\right)^{404}} + \frac{.0359}{\left(\frac{23.56}{k}+1\right)^{1004}}\right]. \tag{7}$$

In Fig. 3 the logarithm of this function is plotted against the logarithm of the absolute temperature, k, and marked A. B is the total energy radiated normally, calculated from the Stefan-Boltzmann law. C is the log of the radiant luminous efficiency, the difference between B and A. The maximum efficiency is about 13.5 per cent. at about 6500° K.

The values used in plotting curves A and D are tabulated in Table 1.

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[1] Planck's Heat Radiation, Eng. ed., p. 218.

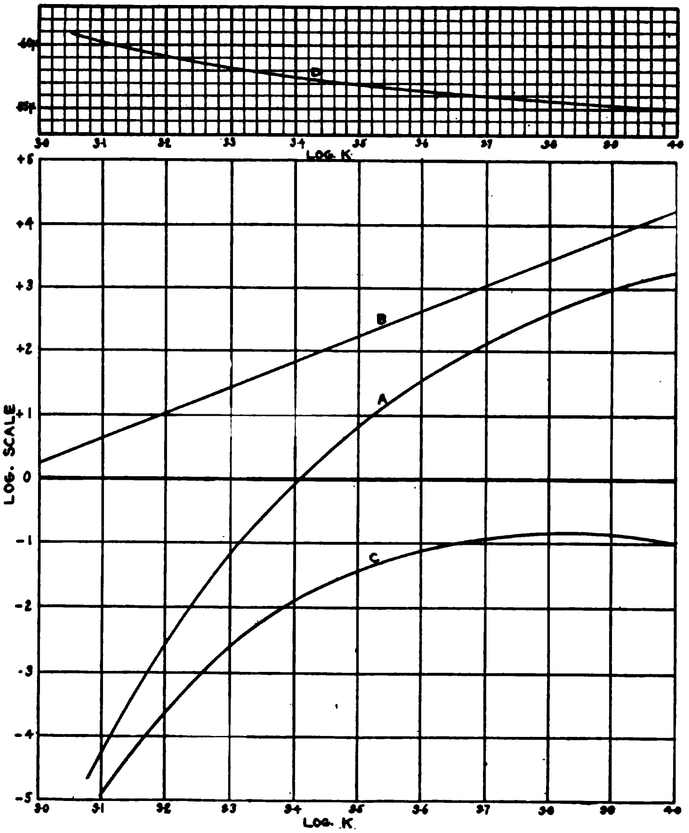

Fig. 3.

A is the log. of the light watts, *L*, computed from equation (7). *B* is the log. of the total energy calculated from the Stefan-Boltzmann law. *C* is the log. of the radiant luminous efficiency = *A* − *B*. *D* is the Crova wave-length calculated from equation (13). K. = degrees Kelvin.

It is possible to transform these values, which are in terms of light watts per steradian, to candle-power, which is lumens per steradian. The relation is

$$b_0 = \frac{L}{m}, \tag{8}$$

where m is the constant known as the "mechanical equivalent of light," *i. e.*, the number of watts of luminous flux in the lumen. Conversely, from experimental values for the brightness of the black body the mechanical equivalent of light may be derived.

Experimental values of the candle power per mm.^{2} of a B. B. have been obtained by Lummer and Pringsheim[1] and by Nernst.[2] The values of

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[1] Phys. Z., 3, 97 (1901–02).

[2] "The Brightness of a B. B.," Nernst, Physikal. Zeit., Vol. 7, p. 380 (1906).

TABLE I.

K.	L.	Log K.	Log L.	Crova Wave-length.	Luminous Efficiency.
1,200	2.173×10^{-5}	3.079	-4.663	.605μ	.00000582
1,400	3.75×10^{-4}	3.146	-3.426	.597	.0000555
1,600	3.25×10^{-3}	3.204	-2.488	.591	.000275
1,800	1.76×10^{-2}	3.255	-1.754	.586	.000933
2,000	6.95×10^{-2}	3.301	-1.158	.582	.00247
2,500	8.30×10^{-1}	3.397	$-.081$	.576	.0118
3,000	4.45	3.477	$+.648$	.570	.0305
4,000	3.66×10	3.602	$+1.563$	.564	.0794
5,000	1.32×10^{2}	3.700	$+2.121$	.560	.118
6,000	3.12×10^{2}	3.778	$+2.494$	.557	.134
7,000	5.78×10^{2}	3.845	$+2.762$	.555	.134
8,000	9.18×10^{2}	3.903	$+2.963$	.553	.125
10,000	1.76×10^{3}	4.000	$+3.246$	.551	.098

the mechanical equivalent obtained by using their figures in (8) scatter widely on both sides of the recent experimental values. They appear also to correspond to a different luminosity scale than that used by us as shown from a consideration of the equivalent or Crova wave-lengths to which these values were supposed to correspond. Fig. 3, curve *D*, shows the variation of the Crova wave-length with the log. of the temperature for equation (7). This was derived from the following definition.

The Crova wave-length applying to a black body over a small temperature interval is that wave-length whose radiant power varies with any change of temperature by the same fractional part that the total luminous power varies for that temperature change. Thus

$$\frac{\Delta_k J_\lambda}{J_\lambda} = \frac{\Delta_k L}{L}.$$

For the limit, as the increments approach zero, we may write[1]

$$\frac{\partial_k J_\lambda}{J_\lambda} = \frac{\partial_k L}{L}. \qquad (9)^2$$

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[1] This definition and equation can be generalized to apply to radiators whose light is varying from other causes than temperature changes. For example, for different Welsbach mantles we can substitute for J_λ, a function of the wave-length and the percentage Ceria. Then we could get a Crova wave-length for mantles of varying composition, which has been done experimentally. (Washington Convention I. E. S., Sept., 1915.)

[2] By the following steps the result obtained by Foote (Journal Wash. Acad. of Sciences, V, 15 (1915), p. 526) can be derived.
Since

$$\partial_k L = \frac{c_2}{\pi k^2} \int_0^\infty \frac{L_\lambda J_\lambda}{\lambda} d\lambda dk$$

(10) becomes

$$\lambda_c = \frac{L}{\frac{1}{\pi} \int_0^\infty \frac{L_\lambda J_\lambda}{\lambda} d\lambda},$$

which is the same as his equation (5).

Since

$$\partial_k J_\lambda = \frac{c_2 J_\lambda}{\lambda k^2} dk$$

we get

$$\frac{c_2}{\lambda k^2} dk = \frac{\partial_k L}{L}. \tag{10}$$

From (10) we get the relation to the Crova wave-length of the slope of Curve *A* in Fig. 3, thus

$$\frac{\partial_k \log L}{d \log k} = \frac{c_2}{\lambda_c k}. \tag{11}$$

If we differentiate (6) to get $\partial_k L$, we get for the Crova value the simple form

$$\lambda_c = \frac{n \lambda m}{n + 4}\left(\frac{B}{k} + 1\right). \tag{12}$$

If we take (7) instead of (6) we get

$$\lambda_c = \frac{L}{\frac{\sigma}{\pi}\left[\frac{2.3005}{\left(\frac{129.23}{k} + 1\right)^{205}} + \frac{.1553}{\left(\frac{77.26}{k} + 1\right)^{405}} + \frac{.0591}{\left(\frac{23.56}{k} + 1\right)^{1005}}\right]}. \tag{13}$$

From this equation was calculated the Crova wave-lengths given in the table. Now, Nernst's extrapolation formula

$$k = \frac{11,230}{5.367 - \log b_1}$$

calls for a value of λ_c about .56μ and Lummer and Pringsheim's for λ_c about .54, while .58 is called for by the present luminosity curve for the neighborhood of 2000° K. A Crova value of .577μ has been found experimentally for the 1.25 w.p.c. tungsten lamp, which is in excellent agreement with the above luminosity curve.[1]

Detailed discussion of the application of this work to the determination of the mechanical equivalent of light is reserved for a subsequent paper in which will be given new experimental work on the brightness of the black body.

The thanks of the author are due Dr. H. E. Ives for his interest and numerous suggestions in preparing this paper.

THE UNITED GAS IMPROVEMENT COMPANY PHYSICAL LABORATORY,
PHILADELPHIA, PA.,
June, 1915.

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[1] The Application of Crova's Method of Colored Light Photometry to Modern Incandescent Illuminants, Ives & Kingsbury, Washington Convention I. E. S., Sept., 1915.

THE HALL EFFECT AND ALLIED PHENOMENA IN TELLURIUM.

By Peter I. Wold.

THE following investigation of the galvano- and thermo-magnetic properties of tellurium was prompted originally by a desire to find if any trace of the Hall effect could be detected in a liquid metal.

In view of the fact that the Hall effect has been found in gases[1] and that the theories on this subject give no reason to believe that it should not be found in liquids, experiments have been made at various times attempting to prove or disprove its existence. See for example the work of Everdingen,[2] Roiti,[3] Chiavassa,[4] and Bagard.[5] Tellurium was selected for the study because of its exceedingly large Hall effect at ordinary temperatures.

In the preliminary tests made on tellurium with the above object in view the specimen used showed such unusual and interesting properties that a more thorough investigation of the various galvano- and thermomagnetic constants seemed justified. This was particularly true in view of the fact that the apparatus necessary for the investigation at the melting point lent itself well to investigation at other temperatures.

Apparatus.

A Weiss electro-magnet, with pole pieces 10 centimeters in diameter, was used. With these pole pieces a very uniform magnetic field, about 7 centimeters in diameter, could be obtained. This gave a maximum field strength of 10 to 15 thousand gausses per cm.^{2} for the distances between the pole faces which were ordinarily used.

The various effects were all measured by the potentiometer method and the most sensitive galvanometer used had a sensitivity of 130 mm. per microvolt at a scale distance of 4 meters.

Preliminary experiments with the magnet showed that there was considerable residual magnetism on breaking the magnetizing current and that the amount of residual magnetism depends on the current strength and distance between the poles. It was apparent that for accurate work one could not rely upon a previously plotted curve giving the magnetic field for different values of current. The importance of this will appear later in considering certain parts of the work. Accordingly, arrangements were made to measure the change in the magnetic field, both on making and on breaking the magnetic circuit and at the time of taking observations on the Hall or other effect.

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[1] Wilson, Phys. Rev., 3, 375, 1914.

[2] Leiden Comm., No. 41, 1898.

[3] Journ. de Phys., 1883.

[4] Ellettricista, 6, 1897.

[5] Journ. de Phys., Ser. 3, T. 5, p. 499, 1896.

The specimen of tellurium to be studied was fused in a hydrogen atmosphere to prevent oxidation. Generally it was found best to place the tellurium in a test tube of combustion tubing with a small hole blown in the side, about four centimeters from the bottom. Hydrogen was then allowed to flow over the metal and escape through the hole, where it was ignited. The tube was then placed in an electric furnace and the molten metal was quickly poured into a mold, with the hydrogen flowing over it continually.

In its final form the mold consisted of a plate of asbestos slate 4 mm. thick in which a hole 4 cm. by 2 cm. was made. This was closed by cementing to the bottom a thin piece of the same material. The various electrodes, shown in Fig. 1, were embedded in grooves in the plate before the casting was made, being held in place by plaster of paris.

Heating was obtained by means of a double electric furnace. The inner furnace consisted of two brass plates 15 cm. square and spaced 5 mm. apart. Nichrome ribbon was wound around this for a heating coil, being insulated from the brass plates by mica. The second furnace was made of sheets of asbestos board properly spaced to just admit the first furnace and was also wound with nichrome ribbon, the windings being at right angles to those of the first furnace. By this arrangement the temperature gradient through the ends of the furnace was very much reduced and a very uniform temperature was obtained throughout the central portion of the furnace. The furnace, complete and mounted, was about 2 cm. thick. The currents through the two furnace windings were independently controlled.

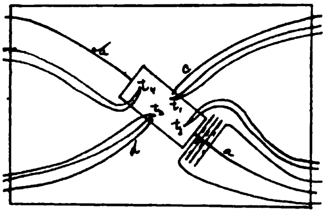

Fig. 1.

In order to obtain the various readings desired four electrodes and four thermo-junctions were connected to the specimen, as shown in Fig. 1. The primary electrodes, *a* and *b*, consisted of platinum ribbon extending across the ends of the specimen, which became fused to the specimen in casting. Copper wires, fused to these platinum strips, served as leads.

The Hall and Nernst effects were measured by means of the electrodes *c* and *d*, consisting of copper wires fused to short pieces of platinum. Copper cannot be used directly in contact with tellurium for it dissolves readily in molten tellurium.

As near to the last two electrodes as possible were placed two copper constantan junctions, t_1 and t_2. For reasons which will appear later, these two junctions were insulated from but embedded in the tellurium. The insulation consisted of a glass sheath about 1/20 mm. thick. Arranged longitudinally in the specimen were two platinum-platinum rhodium junctions, t_3 and t_4. These junctions were fused into the tellurium, being in metallic contact therewith.

The temperature gradient required in the specimen for the thermomagnetic effects was obtained by a small heating coil of 8 turns threaded through holes in the base plate near one end of the specimen, as shown in Fig. 1. By suitable control of the current in this coil any reasonable temperature gradient could be obtained.

HALL EFFECT.

The preliminary observations were made on a specimen of tellurium, obtained from Baker and Co., of fair degree of purity. The specimen was 2 cm. square. It was repeatedly heated to a molten condition and search made for the Hall effect. With the galvanometer then in use no definite results were obtained. Certain deflections were noted but they were irregular and may well have been due to mechanical motion of the liquid metal. Readings were taken at various temperatures during the heating and the results consistently showed a change from a positive to a negative Hall effect at a temperature in the neighborhood of 125° C. followed by a reversal to a positive value in the neighborhood of 225°. These results were so surprising that it seemed well to subject them to more careful examination. It was thought that the effect might be due to impurities and consequently work was started to secure especially pure tellurium. This part of the work was done by Mr. D. T. Wilber, of Cornell University. A new supply of tellurium was obtained from Eimer and Amend and subjected to the following treatment:

$$Te + 4HNO_3 = TeO_2 + 4NO_2.2H_2O,$$

$$3TeO_2 + 2CrO_3 + 6HNO_3 = 3TeO_3 + 2Cr(NO_3)_3 + 3H_2O.$$

This was evaporated with an excess of HNO_3 until the telluric acid, $H_2TeO_4.2H_2O$ or $TeO_3.3H_2O$, separated out. This was recrystallized six times by dissolving in hot water, then adding nitric acid and cooling. It was washed once with alcohol to remove traces of $Cr(NO_3)_3$.

The pure acid was reduced to the metal by boiling with recrystallized hydrazine hydrochloride:

$$2TeO_3 + 3N_2H_4.2HCl = 2Te + 6HCl + 3N_2 = 6H_2O.$$

The finely divided metal was washed with water to free it from hydrochloric acid, hydrazine or telluric acid.

The metal was then reduced to the massive form by melting in a section of combustion tubing, there being a continuous stream of hydrogen passing over and through the mass. This product was treated in the manner described on page 3 in order to get it in the desired form.

Measurements were made to find the relation between the Hall effect and the magnetic field at constant temperature. The results obtained, by taking a number of runs at different temperatures, are given in Table I., in which

H = strength of field in gausses per cm.^{2}
E_a = Hall effect in microvolts with the magnetic field in one direction.
E_b = Hall effect with field in other direction.
E = mean of these two.
R = Hall constant.
$D = E_a - E_b$.

R is obtained from the usual expression for the relation between the various quantities involved; viz.,

$$R = \frac{E}{Hib},$$

where i is the current density and b is the distance between the Hall electrodes.

Fig. 2 shows graphically the relation between the Hall effect, repre-

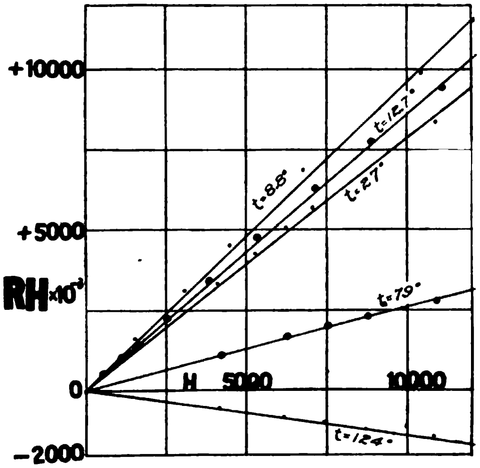

Fig. 2.
Hall Effect and Magnetic Field.

TABLE I.

Hall Effect.

t = 12.7° C. Current = .234 amp.

H	E_a	E_b	E	$RH \times 10^{-3}$	R	D	$D/H^2 \times 10^{+7}$
586	+259	+263	+261	+516	+881	− 4	
1,151	524	506	515	1,015	881	+18	
1,684	726	724	725	1,413	839	2	7
2,595	1,154	1,139	1,147	2,222	857	15	22
3,884	1,747	1,722	1,735	3,375	868	25	17
5,365	2,460	2,415	2,438	4,740	884	45	16
7,193	3,301	3,204	3,253	6,250	869	97	19
8,930	4,043	3,870	3,957	7,710	864	173	22
11,134	4,936	4,692	4,814	9,390	844	244	20
t = 27.1° C. Current = .236 amp.							
577	+229	+228.5	+229	+446	+773	+ 0.5	15
1,660	674	655	665	1,289	776	19	69
3,060	1,235	1,220	1,228	2,360	771	15	16
4,160	1,724	1,697	1,711	3,300	793	27	16
5,320	2,217	2,166	2,192	4,240	798	51	18
6,280	2,648	2,564	2,606	5,050	804	84	21
7,080	2,975	2,864	2,920	5,650	798	111	22
8,820	3,636	3,474	3,555	6,890	781	162	21
10,950	4,432	4,174	4,303	8,350	763	258	22
t = 79.0° C. Current = .234 amp.							
2,160	+290	+276	+283	+535	+248	+14	30
4,265	580	539	560	1,080	254	41	23
6,310	865	789	827	1,650	262	76	19
7,580	1,069	952	1,011	1,970	260	117	20
8,790	1,207	1,039	1,123	2,260	257	168	22
10,900	1,511	1,299	1,405	2,755	253	212	18

sented by RH, and the magnetic field. The curves are straight lines, the slope of each being the sum of the ordinates divided by the sum of the abscissæ for the points shown for that curve. It will be noted that in each case the points for intermediate values of field lie above the line, this departure from the straight line relation can also be noted by observing the values for R in Table I. In each case, even where the effect is reversed, the intermediate values for R have a higher value, considered positively. The discrepancy appearing in the first two lines of the first set is probably due to the greater relative error in taking observations in weak fields.

A certain amount of dissymetry was observed in the Hall effect as obtained for fields of different directions. This is shown under the column headed D and will be discussed later in connection with the change of resistance.

ETTINGHAUSEN EFFECT.

The Ettinghausen effect, or the transverse galvanomagnetic temperature difference, was measured by means of the copper constantan thermojunctions, t_1 and t_2 of Fig. 1. By connecting these two junctions in opposition it was possible to increase the accuracy of the work greatly. To do this, however, it was necessary that the junctions should be insulated from each other. This was done in the manner described elsewhere. The arrangement was such that a scale deflection of 1 mm. was equivalent to .0002°. As a rule measurements on the Ettinghausen effect were taken along with those on the Hall effect. Upon application of the field the reading for the Hall effect was taken quickly—a few seconds being sufficient—and then the attention was turned to the Ettinghausen effect. This effect was given at least a minute in which

TABLE II.

Ettinghausen Effect.

H	Δt	$P \times 10^6$
1,668	.006	+ 8.06
3,316	.009	6.05
6,330	.014	4.93
9,632	.029	6.71
11,750	.033	6.27
13,450	.035	5.81
15,190	.041	6.03
15,640	.043	6.14
	$t = 8.8°$.	
1,577	.0058	+ 7.12
3,110	.0116	7.20
4,520	.018	7.55
6,807	.025	7.05
10,482	.038	6.88
12,625	.046	7.03
15,390	.060	7.47
	$t = 27.1°$.	
577	.0035	+11.8
1,660	.011	12.7
3,060	.020	12.7
4,160	.024	11.2
5,320	.029	10.6
6,280	.035	10.9
7,080	.039	10.7
8,820	.042	9.3
10,950	.059	10.5

to build up. Considerable difficulty was found in deciding when the deflection had reached its maximum. This was due to a drift caused by slow temperature changes in the specimen, which might be in one direction or the other. In finding the Ettinghausen effect under any conditions temperature readings were taken in the order and with the magnetic quality of 0, +, 0, −, 0. This gave four temperature changes. With the primary current reversed a similar set of four temperature changes was obtained. The mean of these eight readings is given in the table as a value for Δt under the given conditions.

The values obtained in a number of different runs, showing the relation between this effect and the magnetic field, is given in Table II.

In this table Δt is the transverse temperature difference set up or caused by the magnetic field. P is the Ettinghausen constant as defined in the usual manner by the equation

$$P = \frac{\Delta t}{Hib},$$

in which i is the current density and b the distance between the junctions. The results are shown graphically in Fig. 3, in which Δt is used for ordinates and field strength for abscissæ. Considering the relatively large errors which enter into the measurement of such small temperature changes under such conditions the results seem to justify the conclusion that the Ettinghausen effect in tellurium is proportional to the magnetic field.

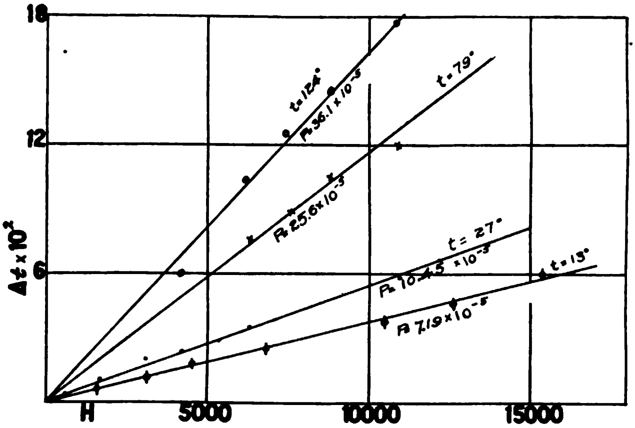

Fig. 3.

Ettinghausen Effect and Magnetic Field.

The value for P found for tellurium by Ettinghausen is $+ 20 \times 10^{-5}$. Lloyd[1] gives $P = 14 \times 10^{-5}$ to 29×10^{-5} at 65° and for $H = 5500$.

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[1] Amer. Journal of Science, 12, p. 57, 1907.

LONGITUDINAL EFFECT.

A third electromagnetic effect is that commonly known as the longitudinal effect, and which is commonly expressed as change in resistance. By means of the platinum wires of the junctions t_3 and t_4 the potential between two points in the specimen, arranged longitudinally, could be found. In series with the specimen was a standard resistance. Connections were made from this to a potentiometer connected to a very sensitive galvanometer and currents could then be measured accurately to one part in 20,000. Two potentiometers were adjusted simultaneously and, in spite of variations in the battery current, instantaneous values of the current and of the potential difference between the two points in the specimen were obtained. Upon application of the magnetic field there was a change in the potential difference in the specimen indicating a change in resistance. The results obtained in two runs are given in Table III., in which H is the field strength, W_0 is the resistance in zero magnetic field during the observations at the value of field indicated, w' is the change in resistance with the magnetic field in one direction, w'' is the change with the field in the opposite direction, and w is the mean of these. D is the difference between w' and w''. The positive value of w/W_0 indicates an increase of resistance.

TABLE III.

Resistance.

H	W_0	$w'/W_0 \times 10^5$	$w''/W_0 \times 10^5$	$w/W_0 \times 10^5$	$w/W_0H^2 \times 10^{18}$	$D \times 10^5$	$D/RH \times 10^{12}$
1,588	.56335	+ 74.6	− 40.0	+ 17.3	+687	115	751
3,085	313	+ 194.8	− 26.6	+ 84.1	+890	221	748
4,430	666	+ 317	− 2.6	+ 157.2	+801	320	750
6,730	429	+ 636	+ 129.4	+ 383	+847	507	782
10,352	231	+1208	+ 439	+ 824	+768	769	770
12,515	222	+1548	+ 646	+1097	+700	902	749
15,300	222	+2092	+1028	+1560	+666	1064	722
1,690	.58918	+ 56.8	− 28.0	+ 14.4	+505	85	642
3,120	916	+ 155	− 27.0	+ 64.0	+657	182	744
4,340	913	+ 239	− 4.9	+ 117	+624	244	717
5,660	892	+ 372	+ 48.3	+ 210	+655	324	732
6,340	895	+ 455	+ 90.7	+ 273	+679	364	732
7,130	735	+ 560	+ 162	+ 361	+710	398	713
8,840	752	+ 755	+ 251	+ 503	+645	504	728
10,960	758	+1049	+ 438	+ 744	+620	611	725

The change in the value of W_0 during the course of a run is due to slight temperature drift. It will be shown later that the resistance of tellurium is very senstitive to temperature changes.

In view of the fact that this longitudinal effect does not change direction with the change in direction of the magnetic field it has been assumed that the effect should be proportional to the square of the magnetic field. This law is fairly well obeyed in some substances. In case of bismuth, which has been studied the most carefully in this respect, it appears that the law holds for weak fields, but for stronger fields the effect is nearly proportional to the first power of the magnetic field. Column six in the table shows how this law holds in this case. The variations between the different values of w/W_0H^2 seem to be entirely irregular and are probably due to experimental errors.

Referring to Table I. it is to be noticed that there is a dissymmetry in the value of the Hall effect, depending on the direction of the magnetic field. In his work on bismuth Van Everdingen[1] has shown that this dissymmetry can be explained by means of the change in resistance. If the Hall terminals are not on an equipotential line, but are slightly displaced, a component of the total potential drop through the specimen, due to the primary current, will show itself at the Hall terminals and, of course, will not reverse with the magnetic field. Upon the application of the magnetic field the Hall effect will be superimposed upon this potential difference, in the one case adding to it and in the other case subtracting from it. The Hall effect is obtained by taking the difference between the potentiometer reading for zero field and for field H. If the component of the potential at the Hall terminals remains constant there will be no error from this source. If, however, the current remains constant and the resistance changes on the application of the field it is obvious that the component potential across the Hall terminals will change in the same proportion. Consequently the value of the Hall effect obtained by taking the difference between the potentiometer readings with and without the field will be in error. If the change in resistance reversed with the magnetic field it would be impossible to separate it from the Hall effect and so find the true value of the Hall effect. Since it does not reverse it is apparent that in the one case it will increase the value of the Hall effect and in the other case decrease it so that a correct value for the Hall effect is obtained by taking the mean of the two values so obtained. This method has been followed out as shown in Table I., in which E_a gives the Hall effect obtained with the field in one direction and E_b that obtained with the field in the opposite direction. The correct value of the Hall effect is the mean of these two, given under E.

If this dissymmetry is due to the change in resistance it should follow the same law. Assuming, from the results in Table III., that the change in resistance is proportional to H^2 then there should be a linear relation between D and H^2, or D/H^2 should be a constant. Column 8 of Table I. shows the values obtained for D/H^2. Excluding the values for very weak fields, in which the dissymmetry is very small and subject to relatively large errors, it will be noticed that the agreement among the values of D/H^2 is very satisfactory, thus giving a verification of Van Everdingen's theory.

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[1] Leiden Communication, No. 26, 1896.

A dissymmetry in the change of resistance was also found, the value being always larger with the magnetic field in one direction than in the other. It even occurred that in weak fields there was an apparent decrease in resistance with the field in one direction. Columns 3 and 4 of Table III. show this. The mean of the two values was, however, positive in all cases. It occurred to the author that a similar line of reasoning may be used to explain this dissymmetry as was used in the case of the Hall effect. If the two leads, by means of which the potential drop along the specimen is measured, are not exactly on a line of current flow it will occur that, upon the application of the magnetic field, a component of the Hall effect will be found across these potential leads. Since the resistance change does not reverse with the magnetic field and since the Hall effect does reverse, in the one case the resistance will be increased by the component of the Hall effect and in the other case will be decreased. Consequently the correct value of the resistance change will be obtained by taking the mean of the two values so obtained. Since the Hall effect is very large in tellurium it is not surprising that the small component of it which comes across the potential leads should more than overbalance the resistance increase, giving an apparent decrease in resistance. Since the resistance increases as the square of the magnetic field while the Hall effect increases with the first power only the increase in resistance finally outstrips the Hall effect. If this dissymmetry is due to the Hall effect it should be proportional to it and consequently there should be a linear relation between D and RH, or D/RH should be a constant. In column 8 of Table III. will be found the values of D/RH. The agreement is very satisfactory indeed.

One other galvanomagnetic effect, *i. e.*, the longitudinal change in temperature, was sought for but, if present, was masked by a component of the Hall effect. If the junctions t_3 and t_4 had been insulated from the specimen, as were t_1 and t_2, it might have been possible to find it.

THERMOMAGNETIC EFFECTS.

In order to study the thermomagnetic effects a small heating coil near one end of the specimen was arranged as described on page 3. Since there was no corresponding cooling agent at the other end the average temperature of the specimen was raised and thus the study at the lower temperatures was limited. The temperature gradient was measured by means of junctions t_3 and t_4, which are spaced 2.6 cm. apart and approximately equidistant from the center.

NERNST EFFECT.

The relation between the Nernst effect and the magnetic field is shown in Table IV. The average temperature is given by t and the temperature gradient by dt/dl. The second column gives the Nernst effect in absolute units for a plate of unit dimensions in which there is unit temperature gradient. The third column gives the constant for the Nernst effect, where Q has the usual significance expressed by the relation

TABLE IV.

Nernst Effect.

$t = 45.8°$ C., $dt/dl = 21.4/2.6 = 8.23$.

H	$QH \times 10^{-2}$	Q
2,230	6.3	.283
4,340	12.5	.287
6,520	18.8	.288
7,890	22.3	.283
9,080	25.3	.279
11,250	29.7	.264
$t = 204°$ C., $dt/dl = 17.8/2.6 = 6.85$.		
2,180	5.8	.264
4,330	11.6	.268
6,500	17.7	.273
7,870	20.9	.266
9,120	23.7	.260
11,210	28.4	.253
$t = 43.1°$ C., $dt/dl = 19.6/2.6 = 7.54$.		
2,160	11.1	.512
4,270	22.1	.517
6,400	33.5	.523
7,830	40.8	.522
9,030	46.4	.514
11,200	55.2	.492

$$Q = \frac{E}{H\frac{dt}{dl}b}.$$

Lloyd[1] gives $Q = .36$ at 33°.

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[1] Amer. Journ. of Sci., 12, p. 57, 1907.

The relation between QH and H is also shown graphically in Fig. 4. The slopes of the straight lines shown are equal to the sum of all the ordinates of the points shown divided by the sum of all the abscissæ. It will be noted that there is a slight departure from the straight line relationship, there being a slight curvature upwards. This same is apparent from a perusal of the values of Q in Table IV., the values of Q increasing slightly with increase of field and then decreasing. The same effect was noted in the case of the Hall effect.

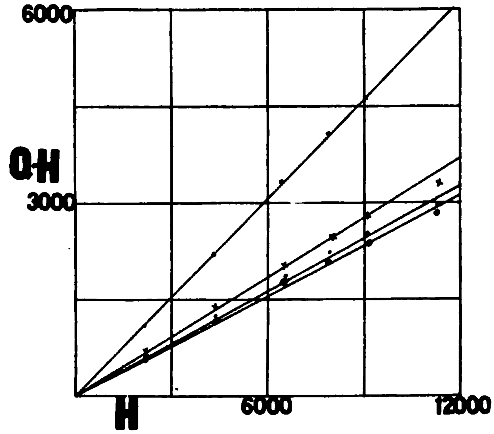

Fig. 4.
Nernst Effect and Magnetic Field.

LEDUC EFFECT.

The relation between the Leduc effect and the magnetic field is shown in Table V. The manner of taking observations was the same as that given for the Ettinghausen effect, the same order of magnetic changes being followed and being repeated. Each value of Δt is therefore the mean of eight readings.

TABLE V.

Leduc Effect.

$t = 114°.$, $dt/dl = 30°/2.6 = 11.5°$, $Q = 1.07$.

H	Δt	$SH \times 10^{+3}$	$S \times 10^{+6}$
3,120	.412'	20.4	6.5
6,390	.729	36.1	5.7
7,750	.973	48.1	6.2
8,920	1.162	57.5	6.5
10,700	1.243	61.6	5.8

The Leduc constant is obtained from the usual equation

$$S = \frac{\Delta t}{H\dfrac{dt}{dl}b}.$$

The values for S show a good agreement considering the great difficulties in the measurement of such small temperature differences. The conclusion is justified that the Leduc effect is proportional to the magnetic field.

Lloyd[1] found $S = +4 \times 10^{-6}$ for tellurium at 34°.

An unusually large value for the Nernst effect was obtained at this time, being given by $Q = 1.07$.

Relationship Between the Various Effects and Temperature.

The dependence of these various effects upon temperature has been studied up to the melting point of tellurium and will be discussed in the same order as for the dependence upon the magnetic field.

Hall Effect and Temperature.

In their original investigations on tellurium Ettinghausen and Nernst[2] stated that the Hall effect decreases with an increase of temperature. Smith[3] has also recently investigated the effect of temperature up to 370° C. Smith finds a rapid drop as the temperature rises until 180° C. is reached. Between 180° and 275° the effect decreases slowly. At 275° Smith notes a sudden rise of about 100 per cent. after which there is again a gradual drop. Smith connects this discontinuity with the fact that there are probably two crystalline forms present in tellurium, this fact being pointed out by Haken[4] as a result of work done on the resistance and the thermo-electric power of tellurium.

In the preliminary work, mentioned previously, the author was surprised to find that the Hall effect started with a positive value and diminished rapidly with rise of temperature, changing into a negative value and later, with further rise in temperature, reversing to a positive value again. The same results were obtained repeatedly. The fact that a reversal in the Hall effect in tellurium had never been noted led to the suspicion that it was due, in this case, to some impurity in the specimen used. At the same time further investigation of this unusual phenomenon seemed justified. Accordingly tellurium, purified in the manner previously indicated, was subjected to careful study. The results of the preliminary trial were fully verified, as shown in Fig. 5, in which the ordinates represent Hall effect for a constant magnetic field of 7,580 C.G.S. units, and a constant current of .2425 ampere. The initial value of the Hall effect differs radically in different successive runs but in every case there is a definite double reversal in the sign of the Hall effect. The initial value depends, apparently, upon the heat treatment. The runs were taken in the order in which the curves are numbered. The runs 1, 2 and 3 followed each other on successive days and it appeared as though the extent to which the specimen became negative was decreasing and that finally the change would not appear. After this, various other properties were studied and the specimen was subjected to a varied heat treatment. Another run was then made on the Hall effect resulting in Curve 4 which shows an enormous negative value. Not expecting such results large temperature steps were taken and consequently the turning point of the curve was entirely missed. One can therefore only estimate the magnitude of the negative Hall effect attained in this run. It is of importance to note that, while there is a great difference in the curves at lower temperatures, the curves gradually approach each other and cross to a

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[1] Amer. Journ. of Sci., 12, p. 57, 1907.

[2] Acad. Wiss., Wien, Anz., p. 173, 1886.

[3] Phys. Rev., 5, p. 351, 1913.

[4] Ann. d. Phys., 32, p. 291, 1910.

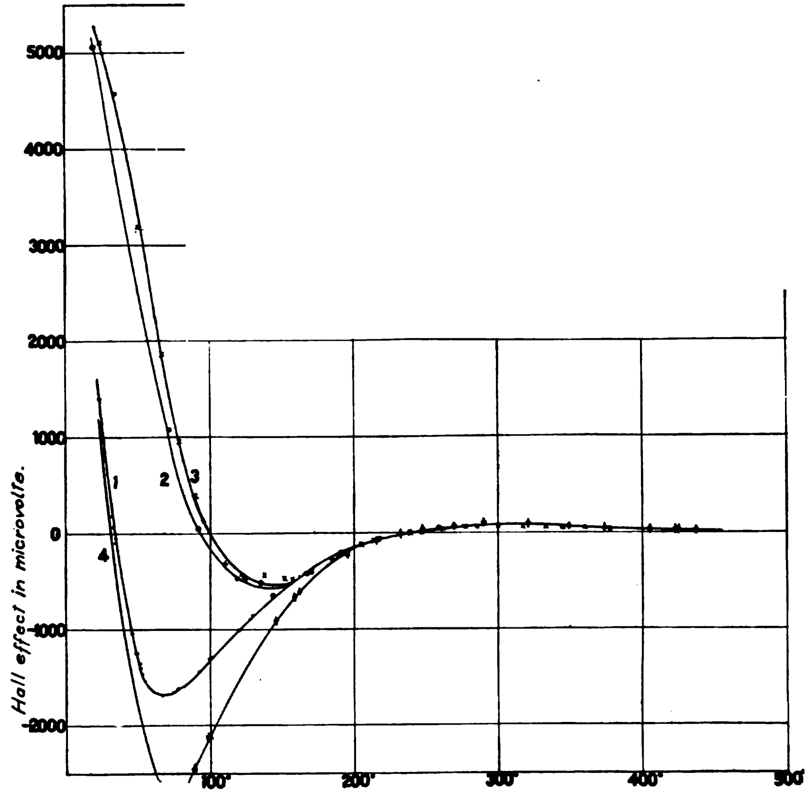

Fig. 5.

Relation between Hall Effect and Temperature.

positive value at the same temperature, viz., 245° C. From there on the curves agree within the errors of observation.[1]

According to Haken and others tellurium occurs in two crystalline modifications. The α modification is stable below 354° C. and the β modification is stable above this temperature. As tellurium is cooled from this temperature the β form goes over into the α form, but the process is rather slow and, in general, some of the β form will not have time to pass over before it is cooled to such an extent that it cannot make the transition. At ordinary temperatures, then, both forms are present and the relative amounts will depend on the previous heat treatment.

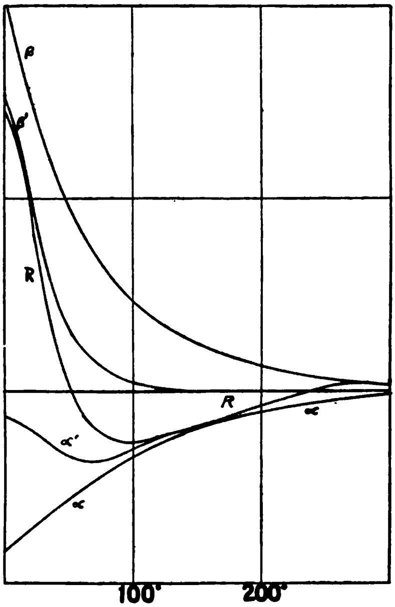

Fig. 6.

Relation between Hall Effect and Temperature.

The peculiar effects here obtained may be tentatively explained on the basis that the β modification has a positive and the α modification a negative Hall effect. Referring to Fig. 6 we may assume, for the present, that the Hall effect for pure β tellurium would be given by the curve β and that for pure α tellurium by the curve α. At room temperature there may be a large proportion of the β tellurium, but with rise in temperature the mobility of the molecules increases and they go over in the α form. Assuming the mass law to hold in this case and bearing in mind the continually decreasing amount of this form, the actual Hall effect contributed by the β tellurium would be given by curve β'. Similarly, in view of the relatively small initial but continually increasing amount of α tellurium, the actual Hall effect due to it would be given by curve α′. The sum of these two curves, *i. e.*, curve *R*, would give the resultant Hall effect, which shows a reversal to the negative sign at a temperature depending on the heat treatment.

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[1] Since writing the above a chemical analysis of the tellurium used in this work has been made by Prof. R. P. Anderson, of Cornell University. In spite of the superiority of the method used and the great care exercised in the preparation of the tellurium it seemed possible that the unusual results obtained were due to the presence of some impurities, in particular tellurium dioxide. The analysis showed that, aside from tellurium dioxide, the impurities were too small to be detected. The powdered tellurium contained about 17 per cent. of the dioxide. The average of two tests on the fused metal gave a tellurium content of 100.5 per cent. The analyst states that this high result is, without doubt, due to a slight oxidation of the tellurium during the drying operation at the close of the analysis. The error is in the direction to indicate high purity and in any event the amount of oxide present in the fused metal is exceedingly small and is certainly no larger than in that used by previous experimenters.

At a higher temperature the β tellurium becomes stable and consequently the α reverts into the β form, with its positive Hall effect. As a result we find the curve *R* crossing the axis to a positive value. Having reached a maximum when all the α has been transformed in β tellurium, it follows the regular law of decrease for the β tellurium.

In Fig. 6 no attempt has been made to give the curves quantitative values. The figure is qualitative only.

The above hypothesis can probably be tried out by giving the specimen a systematic and varied heat treatment, and it is hoped that a thorough investigation along this line can soon be made.

It is to be noted that the actual temperatures of reversal are materially below the temperature of 354° C., given by Haken as the transition temperature.

ETTINGHAUSEN EFFECT AND TEMPERATURE.

Simultaneously with the readings on the Hall effect, resulting in the curves just discussed, readings were taken on the Ettinghausen effect. The values obtained for the constant *P* are plotted in Fig. 7. Curve 1 corresponds to Curve 1 of Fig. 5. The points obtained during the runs corresponding to Curves 2 and 3 are shown, respectively, by circles and by crosses. While the values do not agree with those of Curve 1 they show in all cases a decided increase in the value of the effect as the temperature increases. Both of the last two runs show a big drop in value in the neighborhood of 200° C. and then an increase. In view of the difficulties in taking these readings there is some doubt as to whether this drop is a real effect or is due to experimental errors.

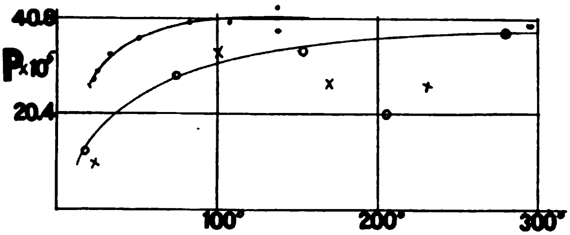

Fig. 7.

Relation between Ettinghausen Effect and Temperature.

Resistance and Longitudinal Effect with Temperature.

While readings were being taken on the Hall effect and the Ettinghausen effect readings were also taken on the resistance and the change of resistance in the magnetic field. The method used has been described.

Fig. 8 shows the relation between specific resistance and temperature.

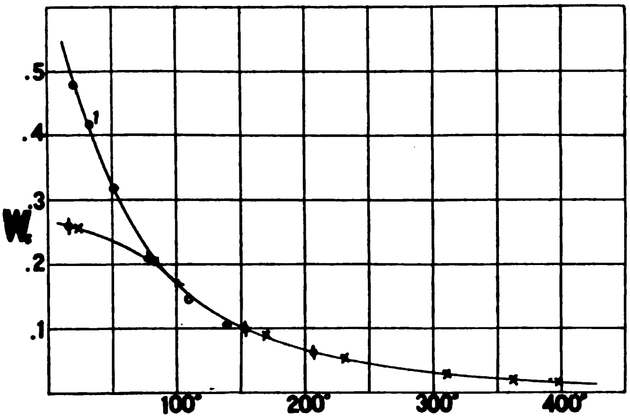

Fig. 8.

Relation between Resistance and Temperature.

The ordinates are expressed in ohms per cm.^{3} Curve 1 corresponds to Curve 1 of Fig. 5. It will be seen that the resistance at room temperature depends on the heat treatment and that in the neighborhood of 150° C. the curves come together after which they repeat consistently. The results obtained from the runs 2 and 3 agree throughout. The specific resistance at room temperatures is enormous, being about 500,000 times that of copper.

Tellurium has been classed by Koenigsberger[1] with certain other materials as a conductor of the second class. Its decrease of resistance with increase of temperature has been explained as due to the fact that the number of free electrons, which are effective in producing metallic conduction, is not constant, as is assumed in ordinary metals, but increases with increase of temperature. In ordinary metallic conduction the increase of resistance with temperature is due to the increased collision activity of the electrons. This factor is also effective in conductors of the second class but is masked by the great increase in the number of free electrons. According to Koenigsberger a temperature is reached at which the further increase of electrons is much decreased or ceases entirely. From this point on these conductors should show an increase of resistance with rise in temperature. No such effect can be detected in the resistance curve of tellurium up to a temperature of 400° C., and there seems to be no indication of its appearance before the liquefaction temperature of about 450° C. is reached.

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[1] Ann. d. Phys., 32, p. 179, 1910.

The dependence of the magnetic change of resistance on the temperature is shown in Fig. 9, in which the ordinates give $\Delta w/W$, *i. e.*, the ratio of the increase of resistance in the magnetic field to the resistance in zero field. The constant field strength used was 7,580 gausses and the current was .2425 ampere. As in the case of the resistance curves, the results repeat above about 150° C. Curve 1 corresponds to Curves 1 of Figs. 5 and 8. The largest relative increase occurs when the resistance is largest.

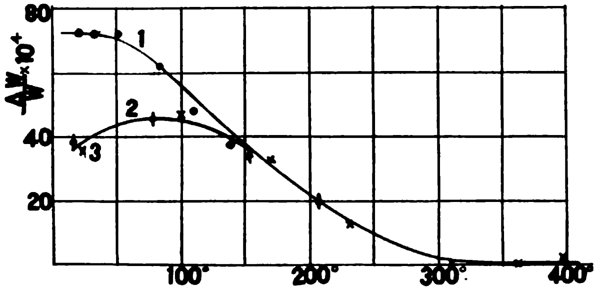

Fig. 9.

Relation between Longitudinal Effect and Temperature.

NERNST EFFECT AND TEMPERATURE.

With increase of temperature the Nernst effect is found to increase quite rapidly to a maximum and then decrease continually to the liquefaction temperature. This is shown in Fig. 11.

THERMO-ELECTRIC POWER AND TEMPERATURE.

Measurements were made on the thermo-electric power of tellurium, use being made of the platinum terminals of the thermo-junctions 3 and 4. In view of the very large thermo-electric power measurements could be made with small temperature difference between the two junctions, ranging from 2 to 10 degrees. The small temperature gradient still permitted measurements of the Nernst effect. In view of the comparative smallness of the Nernst effect, the Hall effect could also be obtained by first measuring the Nernst effect and then sending a primary current through the specimen. The potential difference last found would give the sum of the Nernst and Hall effect. Knowing the former the Hall effect could then be obtained. Thus it was possible to obtain, simultaneously, values of the Hall and the Nernst effects and of the thermoelectric power. The results are shown in Fig. 10 and Fig. 11. Curves

1 and 2 of Fig. 10 and Curve 3 of Fig. 11 show, respectively, the Hall effect, the thermo-electric power and the Nernst effect. Curves 1′, 2′ and 3′ are the corresponding curves for a similar run. There seems to be no particular relationship between the curve for the Nernst effect and the other two curves, but the relationship between the Hall effect and the thermo-electric power is most striking.

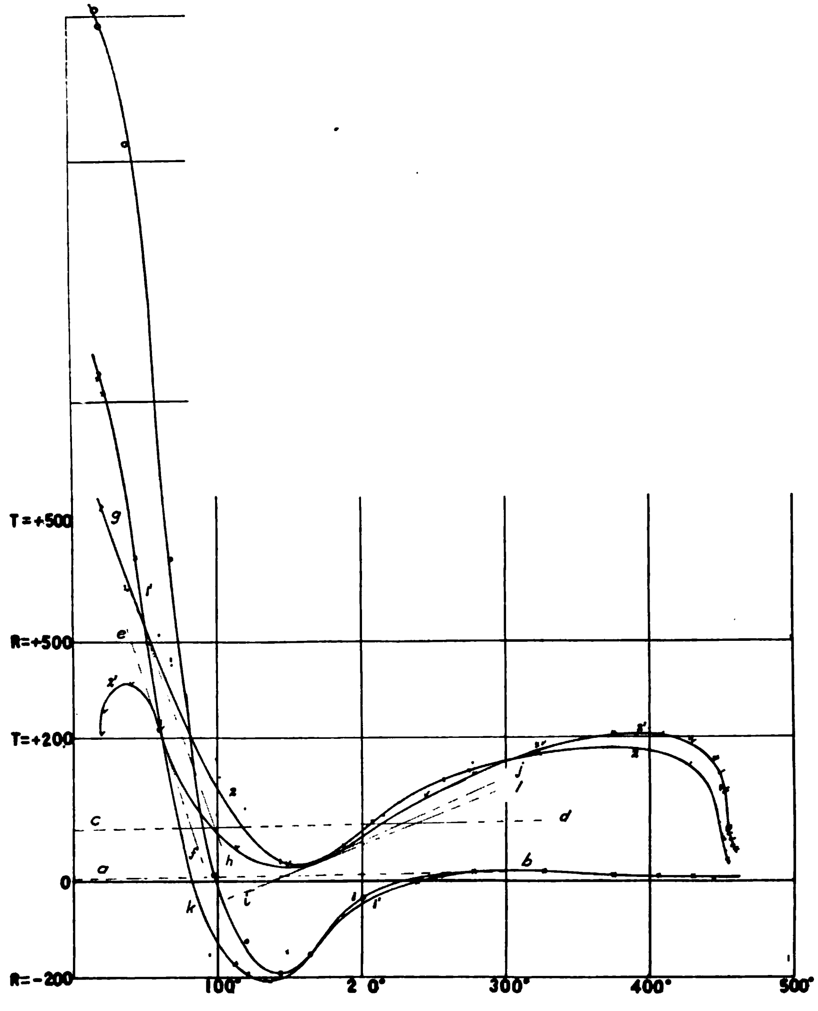

Fig. 10.

Comparison of Hall Effect and Thermo-electric Power at Different Temperatures.

It has been remarked by a number of investigators that, in general, a high thermo-electric power is accompanied by a high Hall effect. Wick[1] has arranged a large number of substances according to their thermo-electric powers and finds that, with few exceptions, the Hall effect follows the same order, both in magnitude and in sign.

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[1] Phys. Rev., 27, p. 76, 1908.

In this work, however, we have an unusual and remarkable curve for the Hall effect accompanied by an unusual and remarkable curve for thermo-electric power (which is ordinarily an approximately straight line) and following it so closely in form that the two curves might well be mistaken for each other. The author is not aware of any case in which such an intimate relationship between Hall effect and thermo-electric power has been shown. One is almost compelled to believe that the thermo-electric power should reverse with the Hall effect. On one or two occasions, earlier in the work, a reversal in the thermal E.M.F. was found, but at that time was laid down to error in observation. During the more careful test this reversal has not occurred, but the form of the two curves together suggest that this might well be expected and is a matter depending on the heat treatment.

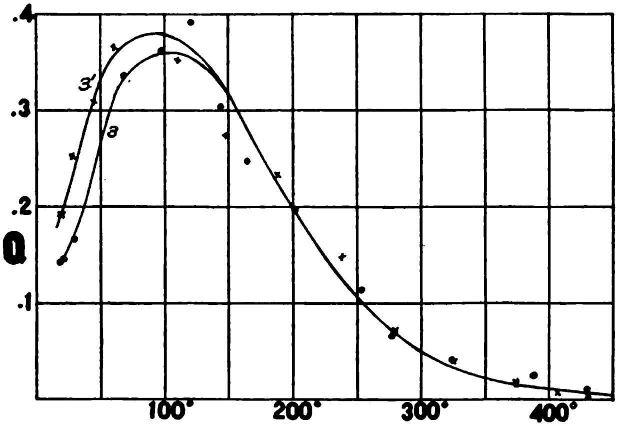

Fig. 11.

Relation between Nernst Effect and Temperature.

The thermo-electric power is here given with respect to platinum. The lead line, *a* — *b*, is shown slightly above it. The lead line has been taken as the standard in thermo-electric measurements because of the fact that its Thomson effect is very nearly zero. The slope of the thermo-electric power line, with respect to lead, gives the Thomson effect for any given material. So far as the Thomson effect is concerned we might equally well take any line parallel to the lead line as our standard and lead has been chosen because it is that metal which has the smallest Thomson effect, as a result of which its thermo-electric power line is practically horizontal. It is apparent then that lead is not a fundamental standard but one of convenience. A line, *c–d*, has been drawn parallel to the lead line and, for the present, will be considered as the base line for thermo-electric power. If tellurium occurs in two crystalline forms, with a relatively large proportion of β tellurium "caught" at room temperature, as previously suggested, the first part of the curves shown will apply closely to the properties of the β modification. A gradual transition commences as soon as the temperature rises. The tangent to the Curve 2 in its early stages, shown by the dotted ling *g–h*, represents the thermo-electric power for the relatively pure β modification. The point where this line intersects our new base line indicates the temperature at which the thermo-electric power reverses with respect to this base line. It will be noted that this is the same temperature—about 99° C. —as that at which the corresponding Hall effect reverses, as seen by Curve 1.

The line *e–f* similarly represents the thermoelectric power in the early stages of Curve 2′. This line intersects the new base line at the temperature 83° C., which is the same temperature as that at which the corresponding Hall effect reverses in sign, as shown by Curve 1′.

Going to the other side of the minimum points of these curves, we must conclude that the transformation into the α modification is complete when a temperature of about 185° C. has been reached. The lines *i–j* and *k–l* seem to fairly represent the slopes of the thermo-electric power curves for this temperature region, and therefore are the thermo-electric power curves for the pure α modification. The temperatures at which these lines *i–j* and *k–l* intersect the base line *c–d* are the same as those at which the corresponding Hall effects reverse from the negative to the positive signs.

It is apparent that there is a reasonable amount of freedom in choosing the location of the line *c–d* as well as the tangents *e–f* and *g–h*, at the same time there was no difficulty in bringing about the remarkable agreement between the points of intersection noted above and it is an interesting possibility that we may here have the basis for the location of a more fundamental line than the lead line from which to measure thermo-electric forces. The author has, however, no theoretical considerations to support the experimental evidence.

The change in slope of the thermo-electric power lines beyond 250° C. is to be noted. Here we are having the change from the α into the β tellurium. There is probably a large time lag in this transformation, and therefore the slope of the pure β tellurium is not reached till we are near the melting point.

LEDUC AND LONGITUDINAL THERMOMAGNETIC EFFECTS.

It was expected that the relation between temperature and the Leduc effect would be found. Unfortunately one of the transverse thermojunctions was broken during one of the heat runs and consequently no readings have been taken.

Attempts were also made to find the effect of the magnetic field upon the thermo-electric power as well as upon the longitudinal temperature gradient. There was clear evidence of the latter at least, but the results have been so inconsistent and irregular that they are not given.

HALL EFFECT IN LIQUID TELLURIUM.

Mention was made earlier of preliminary attempts to find the Hall effect in liquid tellurium. These attempts were repeated with the more elaborate apparatus but with substantially the same results. In spite of the greater precautions to eliminate disturbances it was found impossible to obtain consistent results. Transverse potential differences were obtained which were large compared to those which were obtained just previous to the melting of the metal; but these deflections were irregular in size and, at times, in direction. The author is of the opinion that the deflections noted were due chiefly, if not entirely, to mechanical motions of the liquid under the action of the magnetic field upon the primary current. Furthermore, that in the form of apparatus used, these disturbances would always be so large as to mask any Hall effect which might be present.

As far as the author is aware the Hall effect has never been found in an amorphous substance, with the exception of gases, mentioned previously. It would, therefore, be of as much interest to find it in a solid amorphous substance as in a liquid. It also seemed that, if tellurium could be obtained in an amorphous condition, the finely divided metallic precipitate, as obtained after the chemical purification, was certainly amorphous. Accordingly a specimen was prepared from some of this precipitate by pressure, sufficient to make a fairly compact mass but far from sufficient to cause the metal to flow. A Hall constant of $R = 60$ was found. A micro-photograph was then taken of some of the material with the result shown in Fig. 12. It is clear that the material used, in spite of its finely divided state, is crystalline and not amorphous.

In this connection it may be mentioned that the conviction has been growing in the mind of the author that the crystalline structure of a substance is not merely important but is necessary for the presence of the Hall effect, as we know it in general. It is true that it has been found in gases and it is possible that it exists to a slight extent in liquids; but such

Hall effects are simple and conform to simple theories, whereas in solids we are dealing with phenomena of great complexity the explanation of which must be on an entirely different basis so that it is a question whether they should even be looked upon as the same phenomena.

HEAT CONDUCTIVITY.

Measurements were made on the heat conductivity of the specimen last under investigation, both with and without a magnetic field. The work was carried out with Dr. R. W. King[1] and by an ingenious method recently developed by him for finding the heat conductivity. This method consists, essentially, in measuring the velocity of propagation of a sinusoidal heat wave through the specimen. The method is particularly adapted for measurement in such a case as this. This work, which has been done only recently, must be considered as preliminary and it is hoped that the matter may be carried further at a later time. The results are as follows:

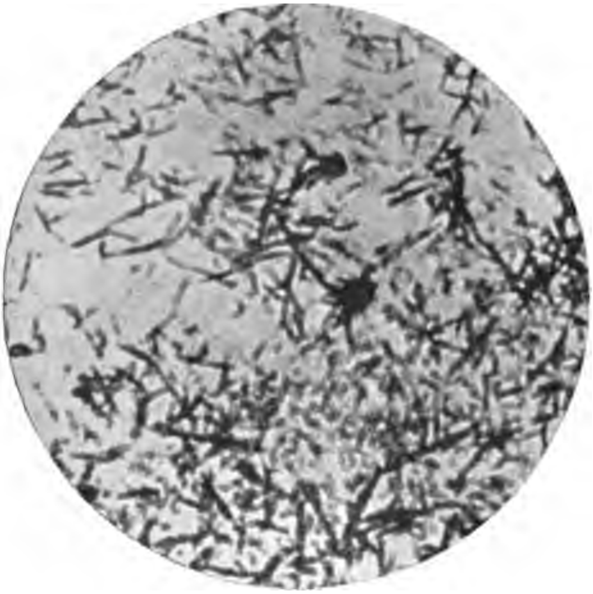

Fig. 12.

Microphotograph of Finely Divided Tellurium. (Magnification = 300 diameters.)

It has been assumed that the specific heat remains constant, that is, it is independent of the magnetic field. The last value for the conductivity was found after the magnetic field had been removed.

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[1] King, Phys. Rev.

$t = 45.3°$, Density = 6.25, Spec. heat = .05.

Magnetic Field.	Coef. of Heat Conductivity.	Per Cent. Change of Conductivity.
0	.0145	
6650	.0120	19
0	.0140	

The increase of electrical resistance, or decrease in electrical conductivity, under the same magnetic field was about .4 per cent.

The only datum on this matter which has come to the attention of the writer is that given by Lloyd[1] in which he states that he found a decrease of 10 per cent. in the heat conductivity in a magnetic field of 4,700 gausses. This being a longitudinal effect we may assume that it follows the H^2 law. Using the figures above we have $19/(6,650)^2 = 43 \times 10^{-8}$. Using Lloyd's figures we have $10/(4,700)^2 = 45 \times 10^{-8}$.

SUMMARY.

The results of this experimental work lead to the following conclusions:

1. Up to 12,000 gausses the Hall and the Nernst constants show themselves to be nearly independent of the magnetic field, with indications of a slight maximum value in the neighborhood of 6,000 gausses. Within the errors of observation the Ettinghausen effect and the Leduc effect are proportional to the magnetic field.

2. The magnetic change of resistance shows itself closely proportional to the square of the magnetic field.

3. A dissymetry of the Hall effect was found to be proportional to the square of the field strength, *i. e.*, to the longitudinal effect, thus verifying Van Everdingen's explanation of the dissymetry.

4. A marked dissymetry in the longitudinal effect was found. This dissymetry is proportional to the field strength and, therefore, to the Hall effect. This dissymetry can be explained in a manner similar to Van Everdingen's explanation of the Hall dissymetry.

5. The value of the various constants, at room temperature, depends greatly upon the previous heat treatment. This is particularly true of the Hall constant for which, under certain conditions, enormous values —as high as 2,000—were obtained.

6. The Hall effect, initially positive, decreases rapidly with rise of temperature, reverses to a negative value at a temperature below 100° C., depending on the heat treatment, and reverses to a positive value again in the neighborhood of 240° C. An explanation can be given on the assumption that tellurium occurs in two crystalline forms which may be

called the α tellurium and the β tellurium. The α modification is stable below about 200° C. and the β modification is stable above this temperature. There is a large time lag in the transformation from the one form into the other which accounts for the great influence of the heat treatment.

7. The Ettinghausen effect and the Nernst effect increase quite rapidly with rise in temperature. The Nernst effect reaches a maximum at about 100° C. after which it decreases rapidly. There are indications that the Ettinghausen effect also reaches a maximum at this temperature and then decreases.

8. The resistance and the longitudinal effect decrease rapidly with rise in temperature but show no reversals and no discontinuities.

9. The Hall effect and the thermo-electric power with respect to temperature give the same form of curves, showing a very intimate relationship between the Hall effect and the thermo-electric power. By the choice of a new standard from which to make measurements the thermo-electric power has a double reversal at the same temperatures as for the Hall effect.

10. Attempts were made to detect the presence of the Hall effect in liquid tellurium but without definite results. Rather large potential differences were found but they were irregular in magnitude and direction so that it was concluded that the effects noted were due to mechanical motions of the liquid.

11. Measurements were made on the heat conductivity of tellurium, both with and without a magnetic field. A large decrease in the heat conductivity was found when in the magnetic field. This decrease was much greater than the decrease in electrical conductivity under the same conditions.

In conclusion I wish to express my thanks to the Rumford Committee, which bore part of the expense of this investigation. My thanks are also due to Professor Nichols and Professor Merritt for the continual interest and assistance which they have given to this work.

Cornell University, 1915.

THE EFFECT OF TEMPERATURE UPON THE COEFFICIENT OF ABSORPTION OF CERTAIN GLASSES OF KNOWN COMPOSITION.

BY K. S. GIBSON.

INTRODUCTORY.

THE effect of temperature upon the absorption spectra of various kinds of colored glass has been investigated by Houstoun[1] and by Gibbs.[2] Their results show that in general the absorption increases with temperature, though in some instances, in certain parts of the spectrum, it decreases. In some specimens, upon heating, the absorption bands were found to remain stationary, and in others to shift toward the red. In only one case, found by Gibbs, has a shift of the absorption band toward the violet been observed, and in this specimen a permanent change in the spectrum was produced. Usually there was a complete recovery upon coming back to the original temperature.

The purpose of this investigation has been to study the changes that take place in the absorption spectra of certain glass specimens between the temperatures of −180° and 430° C. These glasses were kindly furnished by the Corning Glass Works. There are five of them, their color and material being listed as follows:

No.	Color.	Coloring Material.	Glass.	Thickness.
1	Red	Cadmium, selenium	Zinc	1 mm.
2	Orange	Cadmium, selenium	Non-lead	2 mm.
3	Light amber	Cadmium, selenium	Borosilicate	2 mm.
4	Lemon yellow	Cadmium sulphide	Zinc	2 mm.
5	Canary	Uranium	Lead	2 mm.

By mistake No. 5 was at first listed as Cd Se and hence was studied with the others. The first four comprise an interesting and instructive series since the edge of the absorption band is at a different position in the spectrum of each, though of the same general character. Conclusions may thus be drawn from a study of this series, which might not be possible with specimens of various composition whose absorption curves are entirely different in shape from one another.

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[1] Houstoun, Ann. der Phys., 21, p. 535, 1906.

[2] Gibbs, R. C., PHYS. REV., Vol. XXXI., No. 4, p. 463.

Apparatus.

These glasses were ground in the shape of circular discs about 22 mm. in diameter and from one to two mm. thick. The faces were highly polished and made parallel to within .01 mm. These precautions were found necessary in order to obtain the same and therefore the correct per cent. transmission when the specimen was moved around in its position.

To study the effect of heating, the glass *N* (Fig. 1) was placed between two iron washers and heated in the furnace *H*. A thin cylindrical shell of iron K_1 kept the specimen and washers in position. The furnace was an iron core, hollowed out as indicated. Heating was produced by passing a current through wires wound on the outside. These wires were embedded in asbestos cement and the furnace then wrapped in many layers of asbestos paper, which prevented an excessive loss of heat by radiation. A piece of mica at each end prevented air currents and thus helped to keep the temperature on the inside steady. All temperatures were measured by means of a copper-advance thermo-junction which was inserted as indicated at J_1. This was used in the usual way with a potentiometer. It was calibrated by taking readings at the known temperatures of 0° and 100°, and at 232°, 327.5° and 419° C., the melting points of tin, lead and zinc respectively. At low temperatures it was calibrated by comparing it with a thermojunction whose calibration was known, readings being taken at a number of temperatures between 0° C. and the temperature of liquid air.

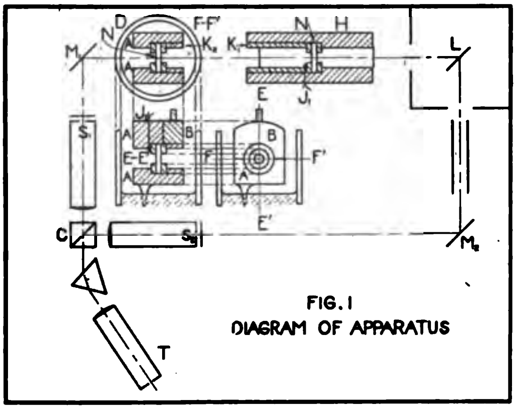

Fig. 1.

From the above readings a curve was drawn, and from this the potentiometer readings for any desired temperature could be obtained. The accuracy of this calibration is such that the maximum variation from any of the temperatures as given is probably not more than two or three degrees.

To study the effect of cooling, apparatus was used which had been devised and constructed by Professor Gibbs for work at low temperatures. This consisted of a solid iron support A which could be lowered and clamped into position inside of a Dewar bulb D. This support was hollowed out and part of the upper half B was cut out and made removable. The glass N could then be lowered into its position between two washers and held in position by a cylindrical iron shell K_2. Then the cover B could be lowered into place. The liquid air was pumped into the bottom of the Dewar through a glass tube and the temperature varied by varying the amount of liquid air. An iron pointer fastened to the bottom of A helped to lower the temperature quickly and keep it steady. Windows of mica were not needed here as there were no convection currents in the air, and no trouble was experienced in keeping the temperature steady. The thermo-junction was inserted at J_2 as indicated. Precautions were taken to prevent dust from settling on the inside of the Dewar in the path of the light beam or upon the specimen during the observations. Also it was found necessary, in order to prevent condensation of moisture from the air on the outside of the Dewar, to fasten heating coils on each side about the places where the light entered and left.

The source of light used was an acetylene flame L whose pressure was kept constant by means of a water manometer. The light from this source, after passing through the specimen in the furnace or in the Dewar, was reflected by a mirror M_1 into the slit S_1 of one of the collimators of a Lummer-Brodhun spectrophotometer. The intensity of this light was compared by means of the Lummer-Brodhun cube C with the intensity of light from the same source L reflected by means of the mirror M_2 into the slit S_2 of the other collimator. A uniform contrast field was obtained by the use of ground glass screens placed in front of the slits. This field was viewed by the telescope T, used without the eye piece.

To exclude stray light, the acetylene flame and the spectrophotometer were boxed in and other screens used where needed, so that errors from this source were eliminated so far as the eye could detect.

During all observations the slits in the collimator S_1 and in the telescope T were kept constant at .80 mm., and the width of S_2 varied until a match was obtained. The amount of spectrum included by a slit of .80 mm. at wave-lengths .46μ, .56μ and .66μ is indicated at the bottom of each figure. A calibration curve was first obtained by taking observations throughout the spectrum with the specimen removed but everything else in position. By repeated trials and adjustments of apparatus, a curve was obtained which varied but little throughout the spectrum, which kept constant from week to week, and which was practically independent of the acetylene pressure over a considerable range, though as stated before the pressure was kept as nearly constant as possible. Thus the errors due to calibration were small. At least three settings of the slit S_2 were made for any particular wave-length, care being taken to avoid any lost motion in the screw. If these three did not agree closely, more were taken. This was almost always necessary at the red and violet ends of the spectrum.

If the average of the settings with the specimen in position is D, and the setting with the specimen removed is I (taken from the calibration curve), then D/I is the per cent. of the incident light transmitted. But since about 4 per cent. of the light incident on each surface of the glass is reflected at that surface, the true per cent. transmission P is approximately equal to $D/.96^2I = D/I'$.

The value of the coefficient of absorption K is derived as follows. If dI' is the change in intensity of light transmitted through a thickness dx, then dI' is proportional to $-dx$ and to I', the original intensity; *i. e.*, $-dI' = KI'dx$ where K is called the coefficient of absorption of the substance for any particular wave-length. From this

$$\int_{I'}^{D} dI'/I' = -\int_{0}^{x} Kdx,$$

$$\log D/I' = -Kx,$$

$$\log 1/P = Kx,$$

$$K = \frac{\log 1/P}{x},$$

where x is the thickness of the glass specimen. In computing K, the values of $\log 1/P$ were taken from a curve plotted between P and $\log 1/P$.

Results.

In Figs. 2 to 9, the results of the investigation are shown graphically. In Figs. 2 to 6, the values of P, the per cent. transmission and of K, the coefficient of absorption are plotted against wave-lengths for various temperatures. In each figure, both for P and for K, there are seven curves corresponding to the following temperatures, with the two exceptions noted below.

Curve No. 1 was taken at $-180°$, No. 2 at $-80°$, No. 3 at $+20°$, No. 4 at 100°, No. 5 at 210°, No. 6 at 320°, and No. 7 at 430° C.

With specimen No. 3, Fig. 4, however, it happened that curve 2 was taken at $-95°$ instead of $-80°$ and in specimen No. 4, Fig. 5, curve 2 was taken at $-60°$ instead of $-80°$ C. The values for 20° are the average of the values taken just before heating, and just before cooling. The cooling effect was studied about a year after the heating effect. In Fig. 2 all the curves are numbered. In the other figures curves 1 and 7 only are numbered.

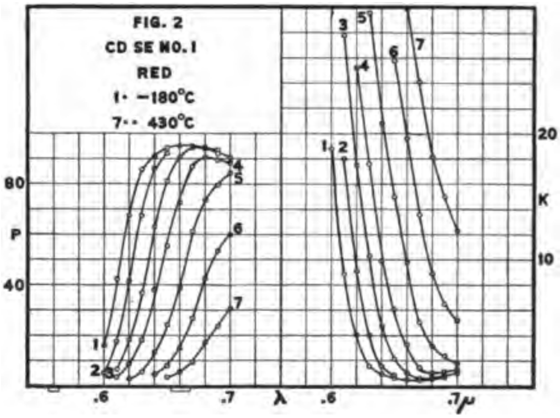

Fig. 2.

Glass No. 1. Cd, Se. Variation of the per cent. transmission P and of the coefficient of absorption K with wave-length λ at the temperatures $-180°$, $-80°$, $+20°$, 100°, 210°, 320° and 430° C.

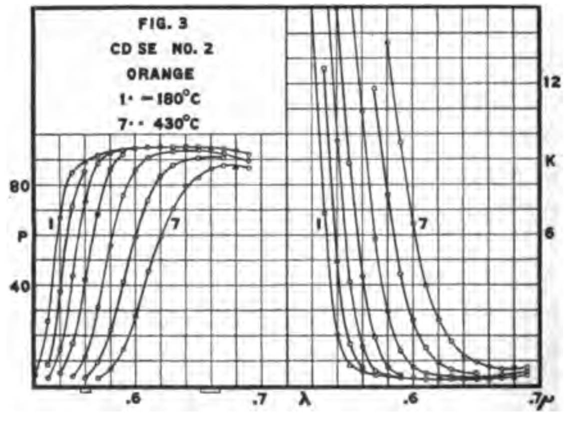

Fig. 3.

Glass No. 2. Cd, Se. Variation of the per cent. transmission P and of the coefficient of absorption K with wave-length λ at the temperatures $-180°$, $-80°$, $+20°$, 100°, 210°, 320° and 430° C.

In Figs. 7 and 8, values of K are plotted against temperature T for those wave-lengths which show the greatest changes. These curves are for specimens Nos. 1 to 4 as indicated. In Fig. 9 are plotted values of K at the extreme temperatures, $-180°$ and $430°$, for the four *Cd* specimens.

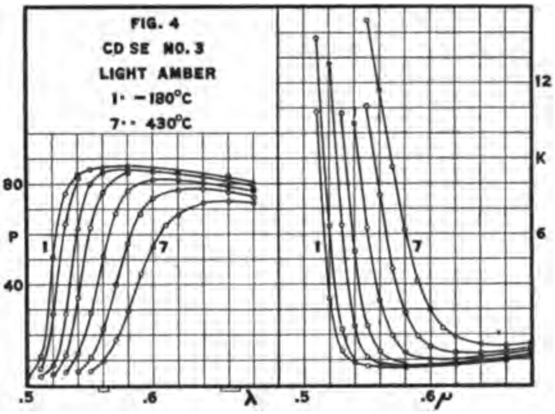

Fig. 4.

Glass No. 3. Cd, Se. Variation of the per cent. transmission P and of the coefficient of absorption K with wave-length λ at the temperatures $-180°$, $-95°$, $+20°$, $100°$, $210°$, $320°$ and $430°$ C.

Perhaps the most interesting and important result found in this in-

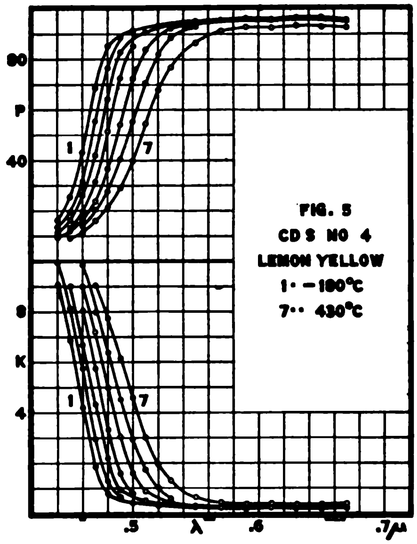

Fig. 5.

Glass No. 4. Cd S. Variation of the per cent. transmission P and of the coefficient of absorption K with wave-length λ at the temperatures $-180°$, $-60°$, $+20°$, $100°$, $210°$, $320°$ and $430°$ C.

vestigation is the enormous increase in the coefficient of absorption in certain parts of the spectrum. In the red glass for instance at certain

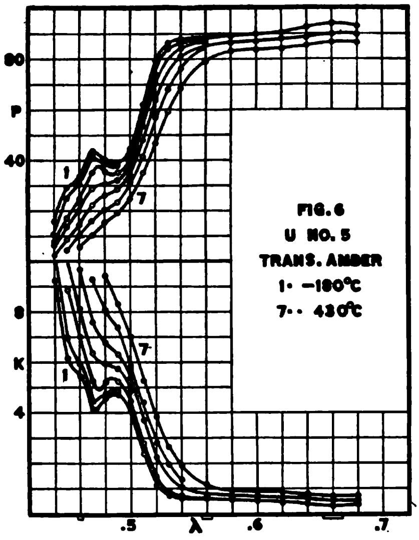

Fig. 6.

Glass No. 5. Uranium. Variation of the per cent. transmission P and of the coefficient of absorption K with wave-length λ at the temperatures −180°, −80°, +20°, 100°, 210°, 320° and 430° C.

wave-lengths, K is increased at 430° by more than fifty times its value at −180°; and what was nearly a transparent glass in the region of .66μ

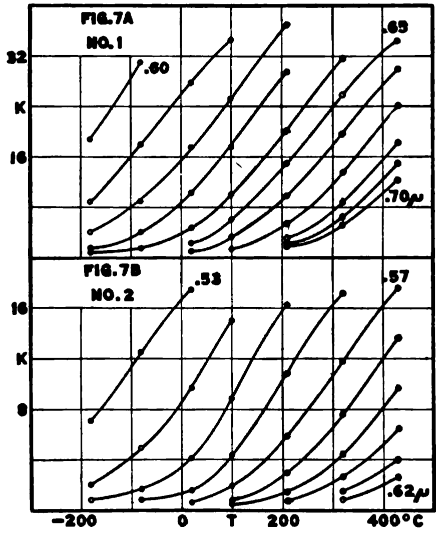

Fig. 7.

Glasses Nos. 1 and 2. Variation of the coefficient of absorption K with temperature T at certain wave-lengths.

became nearly opaque. In spite of these big changes, there was in all the specimens nearly a complete recovery after the heating; and after cooling no change at all was found in the room temperature curves.

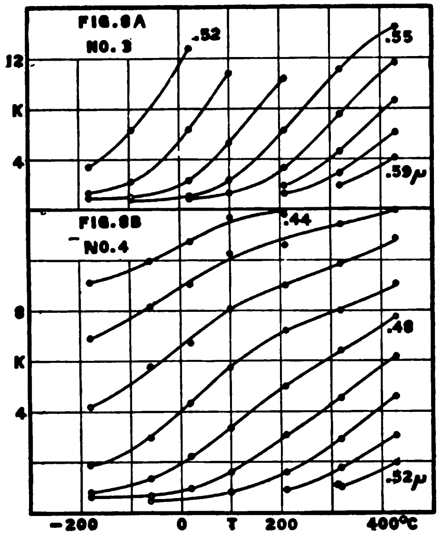

Fig. 8.

Glasses Nos. 3 and 4. Variation of the coefficient of absorption K with temperature T at certain wave-lengths.

The biggest changes occur on the edge of the general absorption region,

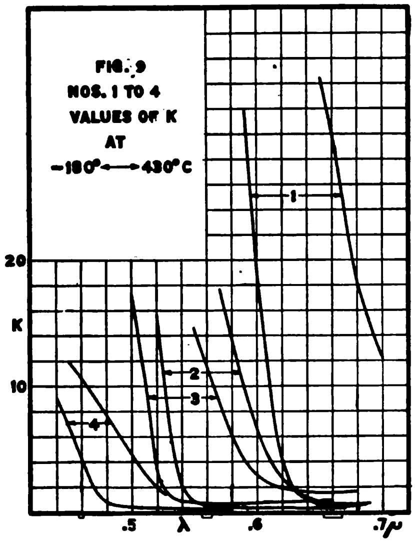

Fig. 9.

Glasses Nos. 1 to 4. Variation of the coefficient of absorption K with wave-length λ at the extreme temperatures, −180° and 430°, only.

and this results in some cases in a shift of the point of maximum per cent. transmission as can be seen from the curves. Also as might be expected, in some of the specimens there was a change in color, the orange glass becoming yellow at −180° and turning to a brilliant red at 430°. The light amber glass became pink at 430°. The other specimens did not show marked color changes, though the red glass changed from a bright to a very deep red at 430°.

It is also interesting to notice (Fig. 9) that the edge of the absorption band is much sharper in the red glass than in the lemon yellow one, where the edge is in the violet, and that the curves for the other two specimens are intermediate in slope. But as the specimens are heated and the edge of the absorption band shifts toward the red, it becomes less sharp, though the difference is not great.

Also it will be noticed that the red specimen, which has the steepest curve, shows the greatest change in the value of K at different temperatures, the change becoming less and less in the other specimens as we go toward the violet.

A change in the per cent. transmission of "red pyrometer glass, presumably made with copper oxide" has recently been observed by Hyde, Cady and Forsythe.[1] Values of the coefficient of absorption at 20° and at 81° C., computed from their curves, show a change of the same order of magnitude as those noted above for the red specimen.

The specimen of canary glass containing uranium, No. 5, Fig. 6, though of different material and with a very irregular curve, yet shows the same effects upon change of temperature as the others. This glass is also interesting as showing how the absorption bands near the region of general absorption disappear upon heating. The specimen shows a green fluorescence, probably due to the absorption band at .49μ. It would be interesting to study the fluorescence and see if it disappeared at the same temperature as this absorption band.

CORNELL UNIVERSITY,
Sept., 1915.

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[1] E. P. Hyde, F. E. Cady and W. E. Forsythe, PHYS. REV., p. 74, July, 1915, and Astrophysical Journal, Nov., 1915.

THE X-RAY SPECTRUM OF TUNGSTEN.

By W. S. Gorton.

In the work on the X-ray spectra of the elements, tungsten has certainly been used as frequently, with the possible exception of platinum, as has any other element but the results obtained by various workers do not agree very well. De Broglie[1] has photographed the spectrum of tungsten using a rotating crystal of rock salt as an analyzer. He gives two tables of lines, but the values given in the two tables differ considerably and, moreover, one table gives four lines and two bands while the other gives five lines and two bands. Herweg[2] finds three lines in the tungsten spectrum, while Moseley,[3] who has done much work in this direction reports only one line. He could, moreover, find no trace of the bands reported by de Broglie. Bragg,[4] who used an ionization method of detection (as have also Moseley and Darwin[5]) reports only one line, the wave-length of which differs considerably from the wave-length as given by Moseley. One would expect tungsten to have a spectrum at least as extensive as that found by de Broglie for, according to Moseley, the heavy metals in general have in their spectra five lines: α, β, γ, δ, ϵ in order of decreasing intensity. There is always a faint companion α' on the long wave-length side of α, a rather faint line ϕ between γ and β of the rare earth elements and a number of very faint lines of wave-length greater than α. All the workers in this field agree that the characteristic radiation of tungsten is rather weak. Now all previous work seems to have been done with X-ray tubes excited by induction coils; consequently, it seemed very probable that with the use of more powerful apparatus, such as the author had at his disposal, more lines could be detected and the ones already found confirmed.

In the present work a Coolidge tube was used as the source of X-rays and the tube was excited by a 10 K.W. interrupterless generator of the Snook Roentgen Co. This machine consists of a 10 K.W. step-up transformer and a synchronously revolving switch which rectifies the high tension E.M.F. This combination of tube and generator is very convenient for X-ray work as it will run for long periods of time without the slightest attention, meanwhile maintaining all conditions perfectly constant.

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[1] De Broglie, Comptes Rendus, 157, 1413, 1913; 158, 177, 1914.

[2] Herweg, Verh. d. D. Phys. Ges., 16, 73, 1914.

[3] Moseley, Phil. Mag., (6), 26, 1024, 1913; 27, 703, 1914.

[4] W. H. Bragg, Proc. Roy. Soc., 89A, 246, 1913.

[5] Moseley and Darwin, Phil. Mag., (6), 26, 210, 1913.

The spectrometer used was of the type in which slit and detecting device are on the circumference of a circle through the center of which passes the face of the analyzing crystal; the face of the crystal is perpendicular to the plane of the circle. This arrangement has the advantage, as shown by Bragg,[1] that the position of any line is independent of the position of the crystal. In the present work the detecting device was a photographic film bent to the radius of the circle mentioned. On the table which carried the crystal was a small brass cone, the axis of which coincided with the axis of the spectrometer. The shadow of this cone was cast on the film and defined the point on the film diametrically opposite the slit. This device enabled all measurements to be carried out on the film itself and obviated the necessity of an accurately graduated circle which is necessary to determine the position of the plate-holder if plates be used. A circle graduated to degrees was provided for setting the crystal. Fig. 1 shows the general plan of the spectrometer and Fig. 2 the design of the crystal holder. The width of the slit was $\frac{1}{4}$ mm., the material of the jaws being $\frac{1}{8}$ in. (3 mm.) lead. The photographic film was inclosed in a black paper shield and the whole spectrometer was protected by a covering of $\frac{1}{8}$ in. sheet lead.

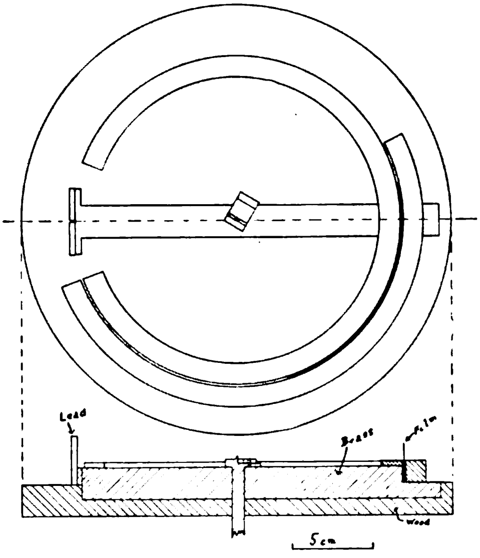

Fig. 1.

In the present investigation the crystal was kept stationary during the exposure. In some previous photographic work, notably that of de

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[1] W. H. and W. L. Bragg, Proc. Roy. Soc., 88A, 428, 1913.

Broglie, the crystal was kept in rotation during the exposure. This latter procedure has the advantage of recording all the lines on the film simultaneously but requires as a rule long exposures, sometimes as much as six hours. In the present work long exposures were inexpedient for various reasons, and so all photographs were made with the crystal at rest. This method records only a small portion of the spectrum at any one exposure but experience has shown that, in the first order, exposures made with the particular crystal used in the present work, in successive positions one degree apart, will record all of the spectral lines. The reason for this may readily be seen by reference to Fig. 3. The X-rays come not from a point on the anti-cathode of the bulb but from an extended area. Thus, different rays strike the crystal at different angles, and, as a consequence, all rays having wave-lengths lying within a certain range affect the film. The present method has the further advantage, as may readily be seen, that very faint lines can be detected much more easily than with a rotating crystal unless extremely long exposures are used. In the present work the usual length of exposure was five minutes. Fairly good results were obtained, however, with exposures as short as one minute, while no gain was experienced by increasing the exposure to fifteen minutes.

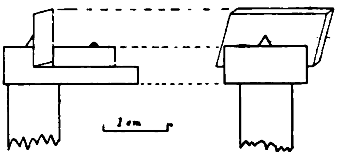

Fig. 2.

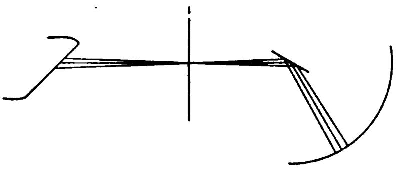

Fig. 3.

The usual current through the X-ray bulb was 0.020 ampere and the parallel spark gap was 4 in. (10 cm.), corresponding to a maximum potential difference on the tube of 76,000 volts. No essential change was produced by raising the gap to 8 in. (20 cm.) (122,000 volts). The analyzing crystals were calcite and rock salt, two of each kind being used. Calcite was used because it is comparatively easy to get a calcite crystal which gives clear, sharp lines when used as an analyzer of X-rays. It is not so easy to get a good rock salt crystal but rock salt was also used in the present work because most of the previous workers have used rock salt. It gives lines darker but not quite so sharp as those of calcite. The two calcite crystals gave equally good results. One of the rock salt crystals, however, gave much better definition than the other. The crystal giving the better definition was the one which had the optically better surfaces. The crystals were bounded entirely by cleavage planes. The dimensions of the crystals were 2.2 × 0.8 × 0.3 cm. in the case of the rock salt, and 1.5 × 0.9 × 0.3 cm. in the case of the calcite. These dimensions are much less than those of the crystals used in most of the previous investigations on the subject. This has the disadvantage of allowing a less extended range of wave lengths to be recorded for any one position of the crystal but makes it far easier to find a suitable crystal.

The results of the work are given in Table I. The values of the wavelength λ were calculated from the formula:

$$n\lambda = 2d \sin \theta$$

where n is the order of the spectrum (always unity in the present work), d the distance apart of the atomic planes and θ the angle between the face of the crystal and the beam of X-rays. The value of d for the cubic (100) face of rock salt has been given by W. L. Bragg[1] as 2.814×10^{-8} cm. The value of d for the rhombohedral (100) face of calcite was calculated from the results of the present work to be 3.028×10^{-8} cm. The measurement of θ for each line was accurate to one-third of one per cent. As each value of θ given in the tables was the average of measurements made on several films, the values of λ may be considered accurate to one-fifth of one per cent. The lines are classified, according to intensity as being strong, medium, faint, or very faint. Some of the lines are too faint to show when reproduced but many show up well. Several prints of the original films are shown in Plate I. An effort was made to obtain prints in which the very faint lines would be intensified, by the process of photographing the films using a slow plate and underexposing it, but the results were not noticeably better than those shown in Plate I. No retouching of any sort has been done. The hazy bands adjacent to some of the lines are due to the general radiation from the tube.

Table I.

Lines.		Calcite Analysis.		Rock Salt Analysis.		Average Value of $\lambda \times 10^8$
Name.	Strength.	θ	$\lambda \times 10^8$	θ	$\lambda \times 10^8$	
a_1	faint	14.10°	1.474	15.21°	1.477	1.476
a	strong	14.01	1.466	15.10	1.465	1.466
b_1	faint	12.34	1.294	13.28	1.290	1.292
b_2	very faint	12.24	1.283	13.17	1.283	1.283
b	strong	12.14	1.273	13.11	1.277	1.275
c	faint	11.97	1.257	12.88	1.255	1.256
d	strong	11.77	1.235	12.70	1.238	1.237
g	strong	10.39	1.092	11.21	1.095	1.094
h	medium	10.06	1.057	10.83	1.057	1.057
k	faint	9.75	1.026	10.48	1.024	1.025

The number of lines found in the present investigation is larger than

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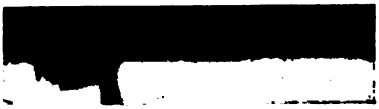
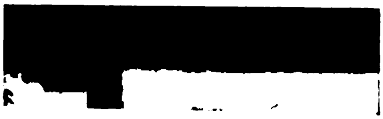
[1] Bragg, Proc. Roy. Soc., 89A, 248, 1913.

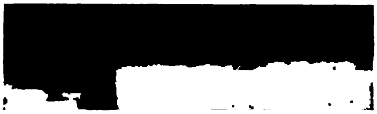


Calcite. Lines a_1 and a.

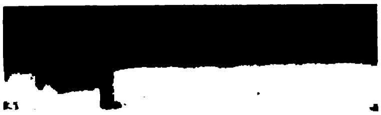
Rock salt. Lines a_1 and a.



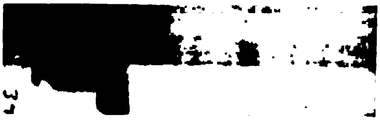
Calcite. Lines b_1, b_2, b, c, and d.



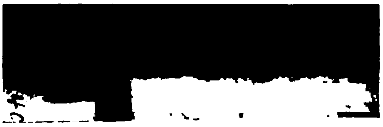
Rock salt. Lines b_1, b_2, b, c and d.



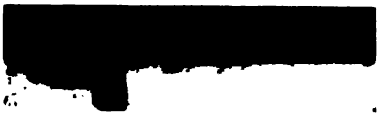
Calcite. Lines g and h.



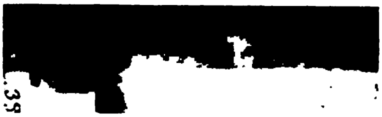
Rock salt. Line g.



Calcite. Lines h and k.



Rock salt. Lines g and h.



Rock salt. Line k.

Fig. 5.

X-ray Spectrum of Tungsten.

W. S. GORTON.

has been hitherto reported. Moseley[1] reported only one line for tungsten; this was the α line. He gave the wave-length as 1.486×10^{-8} cm. This is probably the *a* line of the present work. The β and γ lines, according to Moseley's formula, should have the wave-lengths 1.286×10^{-8} and 1.248×10^{-8} respectively. The ϕ line is between β and γ. The *b*, *d*, and *c* lines seem most nearly to fill the calculated places of the β, γ, and ϕ lines respectively. No lines having wave-lengths greater than that of $\alpha'(a_1)$ were found. De Broglie[2] in the latter of his two tables (which differ considerably) reports the lines; θ = 11° 14′ s., 12° 44′ s., 12° 55′ f., 13° 08′ s., and 15° 09′ s., rock salt being the crystal. These values agree very closely with those for the *g*, *d*, *c*, *b*, and *a* lines of the present article. Since the completion of the present work but before the writing of this paper, Barnes[3] has given the results of an investigation carried out by him using a rotating rock salt crystal as analyzer. He reports the following lines:

θ	$\lambda \times 10^8$	Intensity.
15° 13′	1.477	strong
13° 29′	1.312	weak
13° 19′	1.296	strong
13° 07′	1.277	weak
12° 55′	1.258	strong
11° 24′	1.113	strong
11° 05′	1.082	weak

These values seem to fit best the *a*, b_1 *b*, *c*, *d*, *g*, and *h* lines but the values of θ are uniformly higher than those found by the author. The a_1, b_2, and *k* lines were apparently not found.

It is important to inquire into the cause of the discrepancies between Barnes's results and those of the author. While the results of de Broglie do not differ from the author's by more than 0.3 per cent., Barnes's results are uniformly higher by from 1.0 to 2.3 per cent. The difference is not due to any differences in the velocities of the cathode rays which conceivably might cause a difference in the wave-lengths of the characteristic radiation; the author has tested this point. Moreover, no such difference as one per cent. would be expected. The tubes used in the investigations were both Coolidge tubes; consequently, the cause is not to be sought in any variation or impurity in the material of the cathode. This leaves instrumental errors as the only other likely cause. Barnes does not give a diagram of his apparatus but the arrangement, as well as can be made out from his description, is that represented in Fig. 4.

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[1] Moseley, *l. c.*

[2] De Broglie, *l. c.*

[3] Barnes, Phil. Mag. (6), 30, 368, Sept., 1915.

(This arrangement has also been used by de Broglie.) *S* represents the slit (0.5 mm. in width), *O* the axis of the spectrometer, and *PP′* the photographic plate. His procedure was to make an exposure with the crystal rotating slowly in the neighborhood of the position represented by *OC′*, then to change over to *OC* and make a second exposure, thus producing two lines on the plate. This method is bad because it is necessary to use a very narrow beam of X-rays in order to get sharp lines. All the reflected beams of a certain wave-length pass through, say, either *F* or *F′*. If we have a very narrow beam of rays passing through *O*, this will produce lines at A_1 and A_2'. A beam *SCC′* would produce lines at *A* and *A′*. Consequently, if a beam of rays having an angle of several degrees is used, broad diffuse lines will result making measurement difficult. It is easily seen, too, that the position of the maximum intensity in the lines is dependent on the distribution of intensity in the incident beam of X-rays. Any variation in the velocity of rotation of the crystal would likewise have an effect on the distribution of intensity in the lines. None of these objections apply to the present work, for the photographic film was everywhere on the circumference of the circle. The device of the small brass cone for marking on the film the intersection of the line *SO* with the circle does not introduce any error. Films exposed, as in Barnes's work, so as to produce symmetrically placed lines gave the same results as those of Table I. within the limit of error of the method.

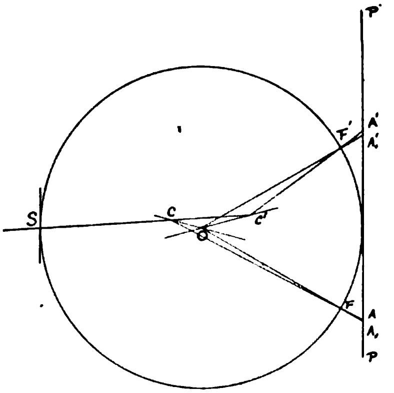

Fig. 4.

The conclusion of the present work is that tungsten shows the same general type of spectrum as do the rest of the heavy metals. The number of lines here reported is greater than has been found hitherto; it is possible that with much longer exposures still further additions to the number may be made.

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THE TEMPERATURE COEFFICIENT OF CONTACT POTENTIAL.

By K. T. Compton.

FROM a consideration of the conditions of statistical equilibrium of the electrons inside and outside different metals in contact, Professor Richardson[1] has derived the equation

$$V_m - V_s = \frac{1}{e}(\phi_m - \phi_s) + P \tag{1}$$

for the contact difference of potential between the metals m and s. In this equation ϕ is the difference between the average potential energy of an electron inside and outside the metal. The latent heat of evaporation of an electron w which has been directly measured in several cases differs from ϕ only by the addition of a very small term denoting the difference between the rates of transfer of energy per electron by currents inside and outside the metal, if such a difference exists. This quantity, introduced, by Professor Richardson, appears to define one of the most important characteristics of the metal, since it is the measure of the work required to extract an electron from the metal. w has been measured directly[2] for platinum, osmium and tungsten and calculated for a large number of metals from data of thermionic[3] and photoelectric[4] emission.

The last term P which represents the Peltier difference of potential between the metals is comparatively very small. Neglecting this term, equation (1) is supported by such evidence as we have on the subject. A direct experimental test of the complete equation appears impossible owing to the extreme smallness of P in comparison with the limits of accuracy attainable in measurements of w. Equation (1) may, however, be transformed[5] to involve only the first and last terms, giving the equation

$$\frac{\partial}{\partial T}(V_m - V_s) = \frac{P}{T}, \tag{2}$$

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[1] "Electron Theory of Matter," pp. 455, 456.

[2] O. W. Richardson and H. L. Cooke, Phil. Mag., 20, p. 173, 1910; 21, p. 404, 1911; 25, p. 624, 1913; 26, p. 472, 1913; H. A. Wilson, Phil. Trans., A, 202, p. 243, 1903.

[3] O. W. Richardson, Phil. Trans., A, 201, p. 497, 1903.

[4] O. W. Richardson and K. T. Compton, Phil. Mag., 24, p. 575, 1912.

[5] "Electron Theory of Matter," p. 459 (37).

which should be capable of experimental test.[1] The object of this paper is to discuss the results of attempts to test this equation.

According to equation (2) we should expect the temperature coefficient of contact difference of potential to be very small. For iron and bismuth and iron and nickel, for instance, equation (2) leads to the values, at 50° C.,

$$\frac{\partial}{\partial T}(V_{\mathrm{Fe}} - V_{\mathrm{Bi}}) = 0.0000887 \text{ volt per degree,}$$

$$\frac{\partial}{\partial T}(V_{\mathrm{Fe}} - V_{\mathrm{Ni}}) = 0.0000324 \text{ volt per degree.}$$

For none of the common metals are the thermoelectric powers larger than these. Thus on changing the temperature from 0° C. to 100° C. we should expect to observe variations in the contact differences of potential of approximately 0.00887 and 0.00324 volt respectively, which are large enough to measure with suitable apparatus.

We have both direct and indirect experimental evidence on which to base a test of the theory. From certain photoelectric[2] and thermionic[3] experiments we may conclude that the temperature coefficient of contact difference of potential is less than the influence of disturbing factors and experimental error. But as the magnitude of these uncertainties considerably exceeds that of the effect under consideration we can obtain no positive information from this source.

Such direct experiments as have heretofore been made lead to values of the temperature coefficient which are entirely too large to be consistent with equation (2). Probably the most reliable results have been obtained by Erskine-Murray[4] and Burbridge,[5] who found temperature coefficients of contact potential varying between −0.0022 and 0.0043 volt per degree for different metals. These results are of the order of a hundred times too large to fit equation (2) and the measured values bear no obvious relation to the thermoelectric powers. Accurate measurements are difficult to make owing to the smallness of the effect in comparison with superposed effects due to insulation charges, time changes and oxidization. Yet there seems to be no doubt but that the experimental results are not in accord with the theory, assuming the usual interpretation of these results to be correct.

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[1] It is perhaps worth while to call attention to the recent criticism of the reasoning employed by Professor Richardson by W. Schottky, Verh. d. D. Phys. Ges., 17, p. 109, 1915. This criticism is based largely on the results obtained by Schottky when he applies the energy principle to a thermodynamic cycle similar to that treated by Richardson and obtains an equation inconsistent with the corresponding one derived by Richardson. Schottky, however, seems to have misapplied the energy principle when he places the total heat absorbed equal to zero in the reversible cycle which is not isothermal (Equation (6), p. 113). When this is corrected his results are not inconsistent with those given by Richardson.

[2] R. A. Millikan and G. Winchester, Phil. Mag., 14, p. 188, 1907.

[3] W. Schottky, Ann. d. Physik, 44, p. 1011, 1914.

[4] Phil. Mag., 45, p. 424, 1898.

[5] PHYS. REV., 2, p. 183, 1913.

Apparatus.—The experimental measurements thus far made have been by some form or modification of the variable condenser method. It seemed worth while to make measurements by the ionization method, in which the air between two plates is ionized by γ rays and the potentials of the plates so adjusted by a potentiometer that no current passes between them, the applied difference of potential being then equal to the contact difference of potential. As compared with the older method, this method has the advantages of greater accuracy, elimination of static charges developed by moving parts and quickness and convenience in manipulation.

A cross section of the apparatus and a diagram of the connections are shown in Fig. 1. *NN* and *FF* are the opposing nickel and iron faces, respectively, of two hollow metal boxes, each about 6×4×½ inch. These boxes are surrounded by hard rubber insulation *I* and fit in the cavity of the lead block *LL*. A tube of radium was placed at *R* so that the γ rays passing up through the fan-shaped groove *G* ionized the air between the plates *NN* and *FF*. Two tubes passing into each box served as intake and overflow for the stream of water which was used to regulate the temperature of the plates. This device, while clumsy, was preferable to heating in an oven because it permitted very rapid variations in the temperature. One box was connected to the potentiometer and subdivided millivoltmeter and the other to one pair of quadrants of an electrometer. The sensitiveness of the apparatus was such that the contact difference of potential could be measured accurately to the ten thousandth of a volt.

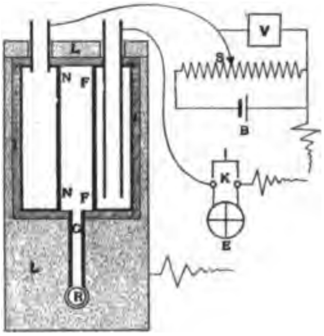

Fig. 1.

Time Changes.—It was first planned to polish the plates (with emery paper, then glass paper, then clean cotton), place them in position immediately and measure the contact difference of potential at different temperatures within a few minutes after polishing. When this was attempted, however, the measurements were very erratic and bore no resemblance to the true contact difference of potential. This effect did not disappear until four or five hours had elapsed after polishing and reappeared whenever the apparatus was slightly jarred or disturbed. Various tests made it evident that frictional charges, probably on the rubber insulation, were responsible for this spurious effect. Inability to avoid them when putting the boxes in place or polishing them in position made it necessary to wait several hours after polishing before making the desired measurements. During this interval as well as later the value of the contact difference of potential changed with time, the iron becoming more electronegative as is shown by Fig. 2. The solid curve is a true logarithmic curve whose equation is

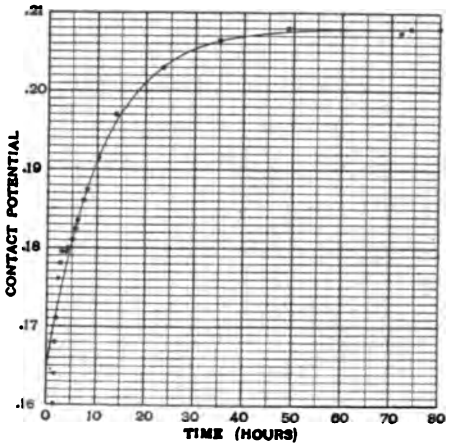

Fig. 2.

$$V_\infty - V = (V_\infty - V_0)e^{-kt} = (0.2080 - 0.1646)e^{-0.0925t} \qquad (3)$$

The experimental readings beyond five hours from the time of polishing fall almost exactly on the curve. The extreme regularity of the curve proves an accuracy of the experimental arrangement beyond anything hitherto attained in such measurements. Other similar tests yielded curves with the same values (approximately) of V_∞ and k but the values of V_0 varied somewhat.

The significant feature of these results is the fact that the iron became continually more electronegative with respect to nickel. This was probably due to oxidation of the iron—an explanation supported by equation (3) which is of the form typical of chemical reactions of this kind. This oxide film, however, must have been extremely thin since the surface of the iron appeared as brightly polished at the end as at the beginning of the test.

The Temperature Coefficient of Contact Difference of Potential.—In view of these results the effect of temperature was tried only after the initial erratic variations had subsided and usually after the lapse of an interval of about twenty-four hours from the time of polishing. In various tests the temperature was varied within the interval 20° C. to 60° C. with the following results:

0.00216 volt per degree.
0.00154
0.00166
0.00113
0.00177
Mean = 0.00165 volt per degree.

While the large variations in these results indicate the effect of some disturbing factor, their general indication is plain. They are in good accord with the results obtained by the other method and are about fifty times as large as the theoretical coefficient deduced from equation (2).

Discussion.—There are two alternatives in interpreting the bearing of these results. Either equation (2) and therefore equation (1) does not conform to the facts or else what is actually measured is not the real contact difference of potential between the metals. Various lines of evidence support this second alternative. The rapid formation of an oxide film, very thin yet sufficient to alter the normal contact difference of potential, has occurred during the progress of all experiments of this sort. Thus we actually measure the contact difference of potential between opposing oxide surfaces (possibly so thin that the effects of metal and oxide superpose), so that we should expect to find temperature coefficients equal to or largely affected by the thermoelectric powers of the oxides. It has been recently shown by Bidwell[1] that the metallic oxides in general possess extremely high thermoelectric powers, some of them being sufficiently large to account for the observed values of the temperature coefficient of contact difference of potential by the theory expressed by equations (1) and (2).

It is possible that surface layers of a gaseous nature contribute to increase the variation of surface potential with temperature. If this were a predominating factor, however, we might expect a "lag" in the potential change as the temperature is varied. No certain indication of such a lag was detected in these experiments.

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[1] Phys. Rev., 3, p. 204, 1914.

Recently Professor Richardson[1] has shown that the presence on metals of surface films with large thermoelectric powers would account for certain peculiarities in the thermionic emission from metals.

Thus the status of the problem appears to be that no certain test of equation (2) has yet been made. The suggested explanation of the experimental results is certainly true to some extent but whether or not the entire discrepancy between theory and experiment may be thus accounted for is a question which at present cannot be answered. Preparation of surfaces and measurements in extremely high vacua, as were carried on successfully by Hennings[2] and Kadesch[3] can alone lead to a reliable test of the validity of equation (2).

The measurements described above were made in the physical laboratory of Reed College with the assistance of Mr. Ellis Jones of the senior class. I take pleasure in thanking him for his aid and for valuable suggestions.

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PRINCETON, N. J.

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[1] Roy. Soc. Proc., A, 91, p. 524, 1915.

[2] PHYS. REV., 4, p. 228, 1914.

[3] PHYS. REV., 3, p. 367, 1914.

ON THE EFFECT OF GENERAL MECHANICAL STRESS ON THE TEMPERATURE OF TRANSITION OF TWO PHASES, WITH A DISCUSSION OF PLASTICITY.

By P. W. Bridgman.

THIS note is to present a thermodynamic formula of considerable generality, and to discuss some of its special cases. The need for such a formula arose in connection with experiments on the effect of high pressures on melting points and polymorphic transitions.[1] In these experiments the specimens were placed in open containing vessels of steel, which may be expected to offer some restriction to the free propagation of changes of hydrostatic pressure throughout the mass of the specimen. The question at issue was: what is the effect on a transition (or melting) point of unknown extra stresses not hydrostatic in nature? It was a surprise to me, after a careful search, to find that this problem has received very meager attention, although it is one capable of solution by ordinary thermodynamic methods. For instance, Voigt's encyclopædic work on " Krystall Physik " does not mention the problem, although it contains numerous examples of the use of the thermodynamic potential for solid bodies under strain. Formulas have, however, been given by James Thomson[2] and Gibbs,[3] among others, for the restricted case of a fluid in contact with an isotropic solid under elastic stress.

The conditions to which the general formula applies are as follows. Two phases of a pure substance, either gaseous, liquid, or crystalline solid, are initially in equilibrium under a hydrostatic pressure p and at absolute temperature θ_0. The two phases are in contact across a membrane of such a nature as to be permeable to the reacting phases, so that the transition from one phase to the other may run across the membrane, but such that it may support a difference of stress between the two sides, so that the normal component of stress on the two phases on opposite sides of the membrane need not be the same. A mechanical stress system, ${}_1X_x$, ${}_1Y_y$, ${}_1Z_z$, ${}_1Y_z$, ${}_1Z_y$, ${}_1X_y$, in addition to the hydrostatic pressure is applied to the phase (1), and a different stress system ${}_2X_x$, ${}_2Y_y$, etc., to the second phase. The *total* normal stresses (including the original hydrostatic pressure) across the surfaces of separation are now ${}_1N_\nu$, and ${}_2N_\nu$. In accordance with the usual practice of elasticity, the stress is considered positive if a tension. It is required to find the change of temperature τ necessary to bring the phases into equilibrium again across the surface of separation. The formula follows.

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[1] P. W. Bridgman, Proc. Amer. Acad., 47, 345 and 439, 1912; 51, 53, 1915. Phys. Rev., N. S., 3, 126 and 163, 1914; 6, 1 and 94, 1915.

[2] James Thomson, Phil. Mag. (4), 24, 395.

[3] J. Willard Gibbs, Scientific Papers, vol. 1, p. 184.

$$\tau = \frac{\begin{array}{l} W_1 + \frac{1}{3}\left(\frac{\partial v_1}{\partial p}\right)_\tau ({}_1X_x + {}_1Y_y + {}_1Z_z)(p + {}_1N_\nu) - v_1({}_1N_\nu + p) \\ \qquad - W_2 - \frac{1}{3}\left(\frac{dv_2}{\partial p}\right)_\tau ({}_2X_x + {}_2X_y + {}_2Z_z)(p + {}_2N_\nu) + v_2({}_2N_\nu + p) \end{array}}{\begin{array}{l} \frac{\lambda'}{\theta_0} + \left(\frac{\partial v_2}{\partial \tau}\right)_p \cdot \left\{\frac{1}{3}({}_2X_x + {}_2Y_y + {}_2Z_z) - p - {}_2N_\nu\right\} \\ \qquad - \left(\frac{\partial v_1}{\partial \tau}\right)_p \left\{\frac{1}{3}({}_1X_x + {}_1Y_y + {}_1Z_z) - p - {}_1N_\nu\right\} \end{array}}.$$

This formula is rigorously exact within the limits over which the coefficients remain constant, but the following approximation is good to small quantities of the first order:

$$\tau = \frac{\theta_0}{\lambda}\Big[W_1 + \frac{1}{3}\left(\frac{\partial v_1}{\partial p}\right)_\tau ({}_1X_x + {}_1Y_y + {}_1Z_z)(p + {}_1N_\nu) - v_1({}_1N_\nu + p)$$

$$- W_2 - \frac{1}{3}\left(\frac{\partial v_2}{\partial p}\right)_\tau ({}_2X_x + {}_2Y_y + {}_2Z_z)(p + {}_2N_\nu) + v_2({}_2N_\nu + p)\Big].$$

The following additional terms used in the formula need explanation. λ is the heat absorbed (in the appropriate mechanical units per gm. of substance) when (2) changes to (1) at p, θ_0, and λ' is the corresponding heat of transition under equilibrium conditions under the altered stress at $\theta_0 + \tau$. W_1 is the work done by the stress system ${}_1X_x$, ${}_1Y_y$, etc., when applied isothermally to ONE GRAM of the phase (1), and similarly for W_2. W is to be calculated by the ordinary rules; in the case of a solid it may be found accurately enough by supposing the extra stress system applied to the phase at $p = 0$. It is to be noticed that because of the conditions of mechanical stability, W is essentially positive. $(\partial v/\partial p)_\tau$ is the "compressibility" of 1 *gm.* of the substance, and $(\partial v/\partial \tau)_p$ the "dilatation" of 1 gm.

It is easy to find the change in the latent heat under the changed conditions of equilibrium.

$$\Delta\lambda = \lambda' - \lambda = \frac{\tau}{\theta_0}\lambda + \tau({}_1\gamma_X - {}_2\gamma_X) + \theta_0\left(\frac{\partial v_1}{\partial \tau}\right)_p\left[\frac{{}_1X_x + {}_1Y_y + {}_1Z_z}{3}\right]$$

$$- \theta_0\left(\frac{\partial v_2}{\partial \tau}\right)_p\left[\frac{{}_2X_x + {}_2Y_y + {}_2Z_z}{3}\right].$$

${}_1\gamma_X$ and ${}_2\gamma_X$ are the specific heats under constant stress.

The formulas were derived by ordinary thermodynamic methods; it is hardly worth while to reproduce the wearisome details. The method is to carry the substance around a complete cycle, writing down the two conditions that the total change of energy and of entropy of the cycle shall be zero. The changes of energy and of entropy of the two phases separately were obtained from Voigt's general thermodynamic potential for solids under stress. It is important to notice that we cannot in this general case write down the conditions of equilibrium by demanding that the thermodynamic potentials of the two solid phases be equal, as we can when dealing with fluid phases by using the potential $Z = E + pv - \theta s$. The thermodynamic potential as usually given for elastic solids is good only within the range of the small strains contemplated by the theory of elasticity, and is not good for the comparatively large changes of volume involved in a transition. It probably would not be difficult to extend the thermodynamic potential for solid bodies so as to attain this greater generality, but it was not necessary for the purpose in hand.

The formulas in the form given above apply only to homogeneous isotropic solids, or more generally to crystalline solids characterized by only one constant of thermal dilatation. The extension is immediate, however, to the most general crystalline solid with its 28 constants, comprising 21 elastic constants, 6 thermal expansion constants, and one specific heat. The 21 elastic constants are buried in the expression for W. The only formal change necessary is to replace, wherever it occurs,

$$\tfrac{1}{3}(\partial v/\partial \tau)_p[X_x + Y_y + Z_z] \text{ by } a_1X_x + a_2Y_y + a_3Z_z + a_4Y_z + a_5Z_x + a_6X_y,$$

where the "a's" are the six constants of thermal expansion. This produces no change in the approximate value for τ, but does enter the formula for $\Delta\lambda$.

It is evident that by a double application of the formula the still more general case may be treated of two phases initially in equilibrium under any stress system not hydrostatic. It is not worth while to further complicate matters by giving a single formula which shall apply to this general case.

SPECIAL CASES.

1. If the stress applied to the two phases is the same increment of hydrostatic pressure dp, the expression immediately reduces to Clapeyron's equation, as it should, by putting ${}_1N_\nu = {}_2N_\nu = -(p + dp)$; ${}_1X_x + {}_1Y_y + {}_1Z_z = {}_2X_x + {}_2Y_y + {}_2Z_z = -3dp$.

2. If the pressure on (1) is left unaltered and the stress on (2) is raised

to the hydrostatic pressure $p + dp$, we find immediately

$$\tau = -\frac{\theta_0}{\lambda} v_2 dp.$$

This is the well-known expression for the depression of the transition point by a pressure exerted on one phase only. It has been discussed by Planck, Poynting, Ostwald, Tammann, Lewis, and has recently been made the basis of a theory of plasticity by Johnston. We will return to this matter.

3. The formula for τ contains a theorem by its very form; the tangential stress on the surface of separation does not enter. The transition temperature of two phases under a given stress is therefore the same across all planes over which the normal stress is the same, and is not affected by the tangential stress at the surface of separation. A special case of this is that if shearing stresses are applied to two opposite faces of a cubical solid in equilibrium with a liquid or another solid the transition point is depressed by the same amount on all six faces of the cube, irrespective of whether these are the faces to which the shearing stress is applied or not.

4. Consider now the problem of the introductory paragraph for the case of a melting solid. A liquid (1) and a solid (2) in the form of a uniform cylinder are in equilibrium at p, θ_0. A mechanical tension is applied to the solid. We have to consider two faces of the solid, that freely exposed to the liquid, and that where the tension is applied. At the free surface we put

$$ {}_1N_\nu = {}_2N_\nu = -p,$$

and find

$$\tau = -\frac{\theta_0 W_2}{\lambda},$$

a result independent of the initial pressure. The freezing point is therefore lowered, and the solid melts at the unstressed surface. It is to be noticed that W varies as the square of the applied stress, but it is not necessary for this reason to restrict the *stresses* to which the formula is applicable to small quantities of the first order. It is only necessary that the strains produced by the stresses remain small. Thus for steel, a stress as high as the elastic limit produces a strain of only 0.001.

At the stressed surface, if we suppose the membrane such that the solid would melt to a liquid at normal pressure p, we find,

$$\tau = \frac{\theta_0}{\lambda} v_2 X_x$$

or the melting point is raised, and no transition takes place at this surface, unless there is liquid present to freeze to the solid.

5. Imagine a cylinder of solid at its melting point and in equilibrium with it at one end only the liquid. Additional hydrostatic pressure dp is applied to the liquid, producing in the solid a one sided stress ${}_2X_z = -dp$. The normal stresses in solid and liquid across the surface of separation are in this problem equal. We find that at the surface of separation the melting point is raised by

$$\tau = \frac{\theta_0}{\lambda}(v_1 - v_2)dp,$$

and at the free surface it is depressed by the same amount as in the preceding problem. At the stressed surface the change of melting point is exactly the same as if the entire solid were subjected to the increment of hydrostatic pressure dp.

This has application to problems like that of a weighted wire melting its way through a block of ice, or the melting produced under the runners of a sleigh by its own weight. The usual treatments are faulty in that they assume hydrostatic pressure in the solid, whereas the stresses in these cases are not hydrostatic pressures, but are one-sided stresses to which the usual Clapeyron's equation does not apply. The above discussion shows, however, that immediately at the point where the pressure is applied, the usual expression of Clapeyron does hold.

A combination of the results of these last two paragraphs has immediate application to the experimental question mentioned in the introduction which started this inquiry. Suppose we have a liquid and solid in equilibrium, and we change the pressure supposedly equally on both phases by an increment dp of hydrostatic pressure, but that, because of imperfections in the apparatus, we actually apply an additional stress not hydrostatic to limited portions of the surface of the system. It is easy to see that melting or freezing takes place at the free surfaces with such changes of volume as to tend to produce throughout the mass just that change of hydrostatic pressure which we thought we had applied.

6. If instead of a liquid and a solid, we consider two solid phases, we see that the transition point may be either raised or lowered. If the normal stress across the surface of separation is unaltered by the applied stress system, then the transition point is always lowered when a stress system is applied exclusively to (2), and raised when applied to (1). The differentiation between (1) and (2) is provided by the sign of λ. There is, however, this difference between a liquid and a solid, and two solids. If the two solids are in equilibrium under hydrostatic pressure

and we try to increase pressure uniformly on both phases, but because of restraints in the apparatus actually apply it to only one of the phases, the reaction will in some cases *not* so run as to redistribute the pressure uniformly throughout the mass. That is, one phase may act as a protecting covering for another, so that the reaction need not run on passing the transition line, but will run at some remoter pressure, when the strength of the covering is exceeded. Such cases are not common, but do occur sometimes; ice V and VI afford an example.

7. Let us now consider the behavior of *crystalline* phases, either two polymorphic solids in contact, or a solid and liquid. We restrict ourselves to the cases usually met in practise where the phases are in elastic equilibrium at the surface of separation without the interposition of a permeable membrane; that is, we assume ${}_1N_\nu = {}_2N_\nu$. For those cases in which ${}_1N_\nu = -p$, the depression of the transition point is of the second order in the applied stress, and for those for which ${}_1N_\nu \neq -p$, the depression is of the first order. In both cases, the effect of the orientation and the specific crystalline constants on the depression is of a higher order. That is, the fact that the substance is crystalline enters the result only in so far as the surface stresses are determined by the orientation of the crystal. Given a crystal under a specified state of internal stress, and with specified stresses across the surface of separation from another phase, and the shift of the transition point is entirely determined by these elements alone (to quantities of the order concerned), and does not involve at all the orientation of the surface with respect to any of the elements of symmetry of the crystal. This result is evidently of importance as affecting the uniqueness of transition or melting points determined experimentally.

APPLICATION TO PLASTICITY.

The fact that the melting point of a solid at a free surface is always depressed by any stress system whatever is suggestive and of considerable importance.[1] This fact offers the possibility of a theory of plasticity, in some respects similar to a previous theory, but free from some of its objections. This previous theory, suggested by Poynting,[2] and lately developed in much greater detail by Johnston,[3] depends on the fact that the melting point is always lowered by the application of hydrostatic pressure to the solid phase only, and lowered by an amount greatly exceeding the rise that would be produced by the same pressure applied hydrostatically to both phases. This formula was given above under special case (2). For the purposes of this theory we consider that when a solid is stressed the pressure is taken up unequally by the miscroscopic crystalline grains, and that on some of these grains the pressure may be very intense. If it is so intense as to depress the melting point to the temperature of the surroundings, melting takes place at these grains, and we have flow of the metal as a whole, with the phenomena of plasticity.

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[1] This fact was stated by Gibbs.

[2] J. H. Poynting, Phil. Mag. (5), 12, 32, 1887.

[3] J. Johnston, Jour. Amer. Chem. Soc., 34, 788, 1912.

The theory in this form seems to me to be open to several objections. It is most difficult to conceive how in any actual solid there is anything corresponding to the fictitious membrane which maintains the solid under hydrostatic pressure, while permitting the liquid to flow freely away. Certainly if in the solid there are any grains about which the surrounding material is not closely packed, the conditions of elastic equilibrium demand that at the surface at which the melting is to occur the normal stress be less than the average. In other words, no matter how intense the average pressure throughout any individual grain, at the free surface of this grain where the melting is to take place there can be no normal stress, and hence the conditions of the theorem do not apply. Another objection is that experimentally the plastic flow of a well-annealed metal begins at the same numerical value of the stress, whether in compression or tension. The fundamental formula gives a change of melting point directly proportional to the stress; under a tension the melting point is raised, and there should be no plastic flow at all. In other words, the theory accounts qualitatively for plasticity in compression but not in tension.

The theory modified as above is evidently not open to these objections. A solid subjected to any stress system whatever, either tension, compression, or shear, experiences a *depression* of the melting point at the *unstressed* surface (or at a surface stressed by less than the average amount) and melting will take place at that surface if the stress is high enough. This statement is free from all hypothesis as to the nature of the contact between the surfaces of the grains. The picture we form of a plastic flow is essentially the same as in the previous theory; that is, under some conditions flow is produced by an actual local melting, followed by regelation with equalization of pressure on all the grains. This local melting takes place when any grain is subjected to more than its share of the stress and so is subjected to a stress not hydrostatic.

It is now of interest to inquire of how wide application this contributory agent in plasticity is. Several reasons lead me to the opinion that it has not the universal importance that Johnston maintains in his theory, but that in most solids at points far below the melting point the more important part of the mechanism of plastic flow is of quite distinct origin.

One of these reasons is the effect of temperature on rate of flow. Tammann and his pupils[1] have found that the rate of flow under constant stress very nearly doubles for every 10° rise of temperature. This holds over considerable temperature ranges—to far below the melting point. If plasticity is really due to local melting, one would not expect so rapid a change, but would rather expect the rate to perhaps more nearly double when the distance to the melting point is approximately halved. Another reason is to be found in the values of the maximum stresses that solids will support without plastic yield. It is well known that the flow point may be raised by the previous application of stress. This is in all probability due to a readjustment of the crystalline grains so that the load is distributed more equally. On the above theory this readjustment is brought about by actual melting and regelation. But it is also well known that there is an upper limit beyond which the plastic limit cannot be raised. This upper limit is presumably reached when the readjustment between the grains has proceeded so far that each grain is uniformly stressed by its proper share. According to the theory under discussion, this upper limit should be reached at the theoretical stress, assumed uniformly distributed, requisite to depress the melting point to the prevalent temperature. As a matter of fact, the upper plastic limit is never so high. The following table shows the upper limit found experimentally by Tammann and Faust,[2] and the computed values taken from Johnston's paper. The table includes all the metals common to the two papers. The actual values are from 8 to 40 times too low, divergences so large as to belong to another order of magnitude. If Johnston's theory is correct, this would mean that after prolonged flow the internal readjustment of stress between the grains is still so imperfect that some grains always bear from 8 to 40 times their share of stress. If the maximum stress is computed by the newer formula suggested above, values are found which are of the order of 10 times greater again than Johnston's. The

Metal.	Flow Point.	
	Observed, Kgm./cm.^{2}	Calculated, Kgm./cm.^{2}
Pb	102	1,760
Sn	55	2,200
Cd	109	3,300
Al	600	5,100
Zn	770	6,900
Cu	2,780	24,000

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[1] N. Werigin, J. Lewkojeff, and G. Tammann, Ann. Phys., 10, 647, 1902–03.

[2] O. Faust and G. Tammann, Zs. phys. Chem., 75, 108, 1910.

conclusion seems forced upon us that under ordinary conditions the mechanism of plasticity must be different from a partial melting. Pertiment suggestions as to what the mechanism may be are common in the literature.[1]

In some cases, however, plasticity by melting must be a real effect. Of course it must be of importance in the immediate neighborhood of the melting point. This means that no crystal continues rigid up close to the melting point, but softens in the immediate neighborhood. Experiments have been made on the plasticity of crystals near the melting point, and conclusions drawn as to the molecular mechanism. It is probable that a large part of this plasticity is due to minute traces of impurity just as is the increase of specific heat below the melting point. But quite apart from the effect of impurities, these considerations show that every crystal will be plastic near the melting point, independent of any special molecular mechanism, the plasticity being called into play by the very forces which must be applied to detect its existence. This kind of plasticity may also possibly enter at the very beginning of flow of a badly annealed piece of metal, in which the load is very unequally distributed among the grains.

Plasticity is to be expected not only near a melting point, but also near any polymorphic transition point, since any inequality of stress on the different grains produces a change of the transition point and consequently a reaction from one solid phase to another, accompanied by a change of volume and a readjustment that will show itself as flow. This plasticity will exist both above and below the transition point. In general, the range of plasticity about a transition point will be larger than about a melting point, because the heat of transition is usually less than the heat of melting. Because of the wide existence of polymorphism, this agent must be of geological importance.

To sum up these remarks on plasticity; under certain conditions plasticity is produced by an actual melting with regelation of those crystalline grains which bear the brunt of the stress. This local melting is governed by the equations for an unequally stressed solid, and not by the equations for a solid under hydrostatic pressure in equilibrium with a liquid not under pressure. This contribuory agent in plasticity does not seem, however, to be largely effective in the majority of the cases of practice. For some substances an analogous effect, due to displacement of polymorphic transition points, is of more importance.

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[1] J. A. Ewing and W. Rosenhain, Phil. Trans. 193 (A), 353, 1900. J. C. W. Humfrey, Phil. Trans. 200 (A), 225, 1903, for example.

A NEW RELATION DERIVED FROM PLANCK'S LAW.

BY PAUL D. FOOTE.

OF the many possible relations which may be derived from the radiation law of Planck only a few have received any attention. The most important of these are the two displacement laws of Wien $\lambda_{\max}\theta = \text{constant}$ and $J_{\max}\theta^{-5} = \text{constant}$. Salpeter[1] has defined a function of λ and θ as follows:

$$\eta(\lambda, \theta) = \frac{J_\lambda}{\int_0^\infty J_\lambda d\lambda}$$

where J_λ is the intensity of radiation of absolute temperature θ and wave-length λ as determined by Planck's law. The functions

$$\frac{\partial\eta}{\partial\lambda} = 0 \quad \text{and} \quad \frac{\partial\eta}{\partial\theta} = 0$$

have roots, the former expression giving $\lambda_m\theta = \text{constant} = 2{,}910$ micron degrees for $c_2 = 14{,}450$ micron degrees, *i. e.*, identical with Wien's displacement law, and the latter expression $\lambda\theta_{\max} = 3{,}686$ micron degrees for the same value of c_2.

It is evident that any number of displacement laws may be derived from the Planck equation.[2] Thus $\lambda'\theta = \text{constant}$ where λ' may refer to any kind of corresponding points on the energy curves at various temperatures, for example, the point of inflection on each side of the maximum ordinate, etc. The present paper concerns the relation that the product of the absolute temperature and the λ-coordinate of the center of gravity of the spectral energy curve is a constant.

Let

$$J_\lambda = c_1\lambda^{-5}(e^{c_2/\lambda\theta} - 1)^{-1} = \text{Planck's Law} \tag{1}$$

λ_c = the λ-coordinate of the center of gravity of the curve J versus λ. Then by the definition of center of gravity:

$$\lambda_c = \frac{\int_0^\infty J_\lambda \lambda d\lambda}{\int_0^\infty J_\lambda d\lambda} \tag{2}$$

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[1] Salpeter, Phys. Zeit., 15, 764–5, 1914.

[2] See Buckingham and Dellinger, Bureau of Standards, Scientific Paper No. 162, p. 405.

Put $x = c_2/\lambda\theta$. Equation (2) becomes:

$$\lambda_c = \frac{c_2}{\theta}\frac{\int_0^\infty x^2(e^x - 1)^{-1}dx}{\int_0^\infty x^3(e^x - 1)^{-1}dx}. \tag{3}$$

Expanding $(e^x - 1)^{-1}$ by division each term in both numerator and denominator takes the gamma function form and by the relation;

$$\int_0^\infty e^{-kx}x^{n-1}dx = \frac{\Gamma(n)}{k^n}$$

equation (3) may be expressed as follows:

$$\lambda_c = \frac{c_2}{\theta}\frac{\left(1 + \frac{1}{2^3} + \frac{1}{3^3} + \cdots\right)\Gamma(3)}{\left(1 + \frac{1}{2^4} + \frac{1}{3^4} + \cdots\right)\Gamma(4)} \tag{4}$$

whence

$$\lambda_c = 0.37021\frac{c_2}{\theta}. \tag{5}$$

Equation (5) is the new relation derived from Planck's law. Thus for $c_2 = 14,450$, $\lambda_c\theta = 5,350$ micron deg.

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A STUDY OF RIPPLE WAVE MOTION.

BY F. R. WATSON AND W. A. SHEWHART.

WAVE motion has been made the object of a large number of theoretical and experimental investigations to determine the physical properties of waves and also to explain the various phenomena of reflection, refraction, diffraction and interference. One of the most satisfactory experimental methods of attack has been found in the study of ripple waves, since in this case, we can visualize almost every phenomena of wave motion.

The object of this paper is to describe the development of a method by which ripple waves may be generated not only in steady patterns but also in patterns in which the waves apparently move very slowly. This allows a leisurely examination of the various phenomena. The method also allows the convenient exhibition of the waves to a lecture audience.

Historically, the investigation of ripples began in 1871 when Lord Kelvin[1] observed that the propagation of ripples depended on surface tension. Matthieson[2] tested the validity of Kelvin's formula, but, because of the rough measurements of the waves set up by a pin point piercing a jet of water, failed to obtain a great degree of accuracy. Ahrendt,[3] Riess[4] and others made similar experiments, but it remained for Lord Rayleigh[5] to develop the first accurate method of investigation. To make visible the extremely small disturbances in the plane of the liquid surface, he used a modified form of Foucault's method of testing plane surfaces. Furthermore, he used the stroboscopic method for making the waves appear to stand still. Dorsey[6] and Watson[7] have extended and improved Rayleigh's method in making investigations on the surface tension of liquids.

Tyndall[8] first made use of ripple waves to illustrate wave motion. Later, Vincent[9] was able with more refined apparatus to obtain beautiful photographs of the same phenomena. H. Schultze[10] devised an electrical method for producing ripples, modifications of which have been made by Pfund[1] and Palmer.[2] Waetzmann[3] developed a method in which the waves were generated by intermittent puffs of air, and were made visible by flashes of light isoperiodic with the puffs. Waetzmann's method, with extensions and modifications, has been used in the present investigation.

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[1] Phil. Mag. (4), Vol. 42, p. 375, 1871.

[2] Wied. Ann., Vol. 38, p. 118, 1889.

[3] Exner's Rep. der Physik, Vol. 24, p. 318, 1888.

[4] Exner's Rep. der Physik, Vol. 26, p. 102, 1890.

[5] Lord Rayleigh's Collected Works, Vol. III., p. 383.

[6] PHYS. REV., Vol. 5, p. 173, 1897.

[7] PHYS. REV., Vol. 12, p. 257, 1901.

[8] S. P. Thompson, "Light, Visible and Invisible," Chap. 1.

[9] Phil. Mag., Vol. 43, p. 417; 45, 191; 46, 290.

[10] Zeitsch. f. Instk., p. 151, 1907.

Fig. 1 is a diagram of the apparatus. Ripple waves were generated by puffs of air blown against the water surface in the glass-bottomed tank *A*. The puffs were secured by cutting a tube conveying compressed air and inserting in the gap a disc with a circular row of equally spaced holes. When the disc rotated, the current of air was periodically interrupted. The waves were made visible by the stroboscopic method. Light from an arc lamp was focused on the row of holes in the rotating disc, thus giving flashes isoperiodic with the puffs of air. By reflecting the light upward through the glass tank, a steady pattern of waves was revealed. Fig. 2 shows a photograph obtained with circular waves.

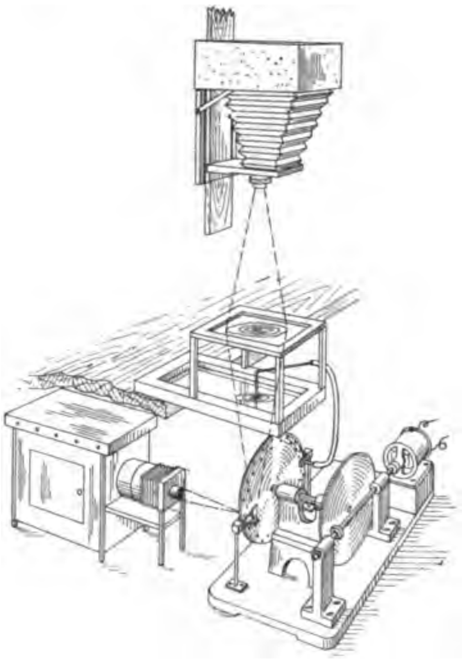

Fig. 1.

Diagram of apparatus showing how ripples are generated on a water surface by puff of air and made visible stroboscopically on the frosted glass plate above the water tank.

By using a second row of holes, which were fewer in number than those in the row already described, the flashes of light could be made to come a little slower than the puffs of air; the result being that the waves apparently moved forward slowly. This action allowed a leisurely study for the different phases of reflection, diffraction, etc., with slowly moving waves.

Trouble was experienced in getting steady patterns of waves, due to vibrations of the apparatus and fluctuations of the puffs of air. The vibrations were overcome by mounting the tank on a steady support. The fluctuations in the air puffs were almost entirely eliminated by using a cast iron disc 10 inches in diameter and one-fourth inch thick and facing it after it had been mounted on an axle so that it would run true. It was then mounted securely on a cast-iron base and rotated by a small wheel on an axle attached by a toggle joint to a 1/6-H.P. direct-current motor. Variation of the speed of the disc was obtained by changing the resistance in series with the motor and also by shifting the position of contact of the small wheel with the disc.

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[1] Phys. Rev., Vol. 32, p. 324, 1911.

[2] Phys. Rev., Vol. 33, p. 528, 1911.

[3] Physikalische Zeitschrift, Vol. 12, p. 866, 1911.

To make the flashes of light sharp and definite in position, an arrangement of two small screens, each pierced by a small hole, was placed over the edge of the disc so that light could pass only during the instant that a hole in the disc came into coincidence with the holes in the screens. The reflecting mirror, which was small in area, was made by polishing the end of a circular aluminum rod cut at an angle of 45° to its axis. The source of light was a carbon arc fed by a 110-volt direct current. It gave good illumination except for the colors explained by Mrs. Ayrton.[1] These proved to be troublesome when taking photographs. A pattern of waves that appeared well illuminated to the eye would show serious distortions on the photographic plate.

The position and form of the aperture for the puffs of air influenced the shape of the waves to a marked extent. Satisfactory results were obtained by using a small copper tube of about 1 mm. diameter placed near the water surface. It could easily be bent into any desired position.

The waves may be shown to an audience by mounting a mirror at an angle of 45° over the surface of the tank so that the light is reflected to a screen. For this purpose it is desirable to let more light through so that the pattern will be well illuminated. This makes the definition of the waves less sharp but still satisfactory enough for demonstration.

Figs. 2 to 7 show some of the patterns investigated. Fig. 4 shows a curious overlapping interference. The patterns in Figs. 5, 6, and 7 were obtained by placing metal forms on the bottom of the glass tank and allowing the water surface to barely cover them. The metal must be free from oil or grease. In Fig. 6 the diffraction waves were made more intense by shielding the central portion from the camera for part of the exposure. The time of exposure varied from 10 seconds to 20 minutes, depending on the amount of light. The length of the waves was measured to be nearly 0.4 cm.

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[1] "The Electric Arc," Chap. 1.

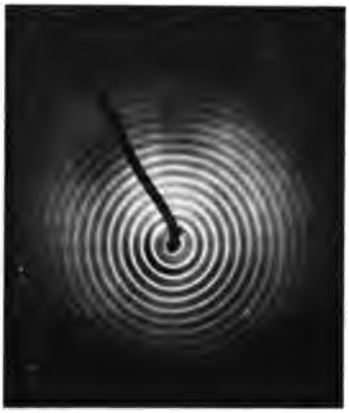

Fig. 2.
Circular waves.

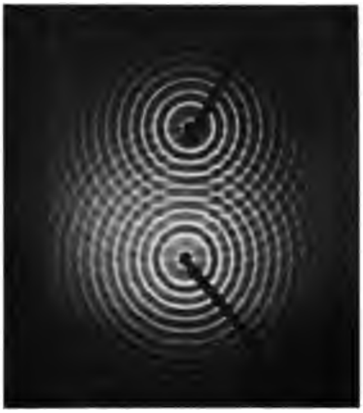

Fig. 3.
Interference of two sets of circular waves.

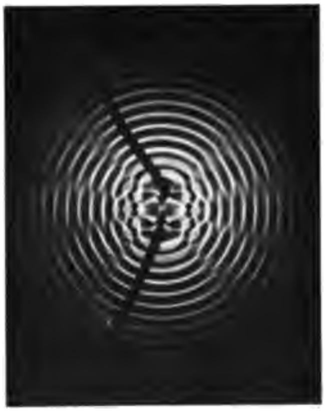

Fig. 4.
Interference of two sets of circular waves with sources close together.

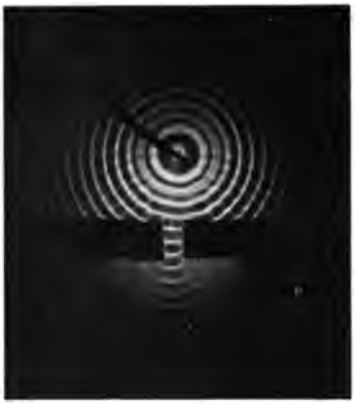

Fig. 5.
Diffraction through a narrow channel. Also reflection of waves.

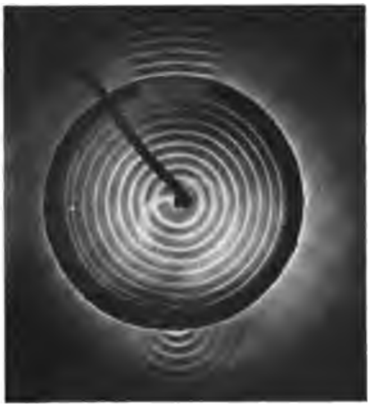

Fig. 6.
Diffraction of waves through narrow and wide aperture.

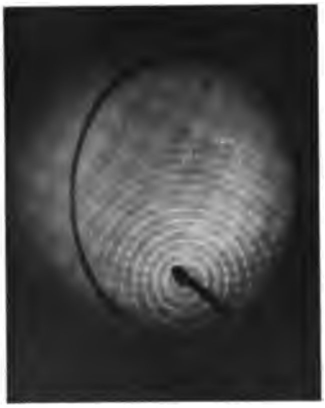

Fig. 7.
Reflection of waves in ellipse. Note the conjugate focus.

THE DISTRIBUTION OF ENERGY IN THE NORMAL RADIATION SPECTRUM.

By Leigh Page.

In their attempt to derive a radiation formula from classical dynamics Rayleigh and Jeans have been led to an expression which is in disagreement with experiment in so far as it demands that in the state of equilibrium the radiant energy in the ether should consist mainly of waves of infinitesimally short wave-length, and that the total energy per unit volume should be infinite. Lorentz has been led to the same expression by a consideration of the absorption and emission due to the free electrons contained in a piece of metal placed in a field of stationary radiation. In fact it has been asserted that this radiation formula is based only on the most general principles of classical dynamics and electrodynamics, and that any other expression for the distribution of energy in the normal radiation spectrum must be in contradiction to these principles.

The only radiation formula agreeing with experimental observation is that due to Planck. In obtaining his formula Planck considers the absorption and emission of ideal linear oscillators capable of executing only simple harmonic vibrations. His justification in using this special mechanism lies in the fact that Kirchhoff's law shows that the distribution of energy among the waves of different frequencies in a stationary field of radiation depends only upon the temperature of the material bodies with which the radiation is in equilibrium, and not at all upon their nature. However Planck finds it necessary, in order to obtain his formula, to make a number of revolutionary assumptions, at least one of which is in contradiction with classical electrodynamics. The following are the more important assumptions on which Planck bases his theory:

(*a*) While an oscillator absorbs energy continuously according to classical electromagnetic theory, the radiation of energy demanded by the electrodynamic equations is replaced by an emission by quanta. To be more specific, Planck assumes that an oscillator of frequency ν can emit only when its energy is an integral multiple of $h\nu$, h being Planck's constant, and that when emission does take place the oscillator loses all its energy and starts absorbing afresh. Such an assumption not only involves a form of emission of energy utterly outside that contained in the electromagnetic theory, but is in contradiction with classical electrodynamics in that it supposes the radiation of energy demanded by the electrodynamic equations to be non-existent.

(*b*) The ratio of the probability of no emission, when the energy accumulated by an oscillator is an integral multiple of $h\nu$, to the probability of an emission is proportional to the intensity of the vibrations exciting the oscillator. If we grant assumption (*a*) assumption (*b*) seems reasonable, although its only justification lies in the fact that, in addition to the other assumptions, it enables Planck to deduce a formula which is in agreement with experimental facts.

(*c*) The constant of proportionality contained in assumption (*b*) is determined in such a way that the formula derived shall reduce to the Rayleigh-Jeans expression for long wave-lengths.

These three assumptions enable Planck to find the energy density of radiation for any frequency in terms of the average intrinsic energy of the oscillators of the same frequency with which the radiation is in equilibrium. To determine the value of the average energy of the oscillators in terms of their temperature, another assumption is necessary:

(*d*) If w is the probability that any given oscillator lies in any particular element of extension-in-phase, then in passing from one element of extension-in-phase to the next $\Delta w/w$ is not sufficiently small to make it possible to replace the summation by an integration in the expression for the entropy of the oscillators. In fact the elements of extension-in-phase are supposed to be of a size equal to the action quantum h. Consequently Planck finds for the average energy of a linear oscillator of frequency ν the expression

$$\frac{h\nu}{2} + \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$$

instead of the expression

$$kT \tag{2}$$

given by classical dynamics.

From these four assumptions Planck deduces the expression

$$u_\nu = \frac{\frac{8\pi h\nu^3}{c^3}}{e^{\frac{h\nu}{kT}} - 1} \tag{3}$$

for the energy density per unit frequency of homogeneous radiation of frequency ν and temperature T, c being the velocity of light in vacuum.

The object of this paper is to show that the radiation formula (3) may be obtained from the expressions given by classical dynamics and electrodynamics for the absorption and radiation of energy, provided we make use of a single supplemental assumption in determining statistically the distribution of energy among the absorbing and emitting oscillators. We shall, like Planck, make use of Kirchhoff's law in that we shall consider a stationary field of radiation in equilibrium with ideal linear oscillators capable of executing simple harmonic vibrations. We shall imagine a large number of these oscillators of every frequency placed inside a rigid perfectly reflecting envelope; and allowed to come into equilibrium with one another and with the enclosed radiation. These oscillators will absorb and emit according to classical dynamics and electrodynamics. Hence if f is the electric moment of an oscillator in terms of Lorentz's unit of electric charge, the energy of its oscillatory motion at any instant will be given by

$$U = \tfrac{1}{2}L\dot{f}^2 + \tfrac{1}{2}Kf^2 = 2\pi^2\nu^2C^2L, \tag{4}$$

where L and K are constants, and where C is the maximum value of the electric moment, or the product of the amplitude of vibration by the charge. The rate of absorption of energy from the electromagnetic field will be given by[1]

$$\frac{dA}{dt} = \frac{u_\nu}{12L} \tag{5}$$

and the rate of radiation by[1]

$$\frac{dR}{dt} = \frac{2\pi\nu^2 U}{3c^3L}. \tag{6}$$

for [illegible] in energy

So far we have made use only of classical dynamical and electrodynamical theory. To determine statistically the average value $\overline{U}$ of the energy of oscillations of frequency ν we shall now introduce a supplemental assumption, to wit:

The motion of an emitting and absorbing linear oscillator of frequency ν is stable only for those amplitudes for which the energy of its oscillations is an integral multiple of $h\nu$.

Hence the energy associated with an oscillation of frequency ν must be equal to $nh\nu$ where n is an integer. However it is not necessary for the development of our theory that the energy of the oscillation should be *exactly* $nh\nu$ but merely that it should lie between $nh\nu - \delta$ and $nh\nu + \delta$, where δ is very small compared to $h\nu$. Moreover it is not necessary to assume that the energy is, at every instant, an integral multiple of $h\nu$

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[1] See "Planck's Heat Radiation" (translation by Masius) pages 160 and 165 for corresponding expressions in electrostatic units.

within the limits assigned above, but merely that the time during which the energy is *not* an integral multiple of $h\nu$ shall be very small compared to the time during which the energy is an integral multiple of $h\nu$.

In attempting to explain how this condition of the oscillator may be maintained—in view of the fact that the rate of absorption of energy is changing from instant to instant, and that even if the average absorption were just sufficient to balance the emission from an oscillator of energy $nh\nu$ it would not balance the emission from one of the energy $(n+1)h\nu$—it is necessary to remember that there are forms of restricted motion which involve neither emission nor absorption of energy. For example, a ring of evenly spaced electrons revolving in a circle about a positive charge placed at the center would neither emit nor absorb energy, so far as the motion in the plane of the circle is concerned. If such a ring, however, vibrated as a whole in a direction perpendicular to its plane, it would constitute one of the simplest types of linear oscillators consistent with Rutherford's model of the atom. So far as the degree of freedom involved in this vibration is concerned, there would be both absorption and emission of energy. However there might be some connection between the two degrees of freedom referred to, such that the energy associated with the absorbing and emitting degree of freedom might always tend toward an integral multiple of $h\nu$, any surplus or deficit of energy being taken care of by the other degree of freedom.

Again, there is no reason to suppose that an electron itself might not possess considerable energy of rotation. If the charge on the electron is distributed symmetrically with respect to its axis, there will be no emission of energy in consequence of its rotation. In addition to its rotation about a diameter, the electron might constitute the movable element of an ideal linear oscillator. Some connection between the vibratory motion and the rotation might result in a distribution of energy such that the energy of the oscillations would always tend toward an integral multiple of $h\nu$.

This speculation as to the method by which the vibratory energy of an oscillator is maintained at some integral multiple of $h\nu$ does not provide any mechanism by which the energy of the oscillations may pass from one integral multiple of $h\nu$ to another. Such a transition, however, would probably take place only during the encounters between different oscillators. In fact, if we are going to apply the theory of probability to the derivation of an expression for the average energy of an oscillator, we must suppose that such transfers of energy may occur. This does not limit the transfers of energy to cases where the ratio of the frequencies of the colliding oscillators has certain particular values. For if the collision results in any surplus or deficit of oscillatory energy above or below the nearest integral multiple of $h\nu$, such surplus or deficit will be rapidly extinguished by transfer of energy between the emitting and non-emitting degrees of freedom.

A slightly different interpretation of our fundamental assumption would consist in the supposition that only when the energy of an oscillator was between $nh\nu - \delta$ and $nh\nu + \delta$ could the oscillator exist in the emitting and absorbing form. For energies outside of these limits the electrons and positive nuclei constituting the oscillator might form new aggregations such that their energy would be associated with types of motion involving no absorption or emission. A change of the energy content, as a result of collision, to the proper value would cause the elements of the oscillator to so rearrange themselves as to again constitute an absorbing and emitting mechanism. From this point of view we would have particles of all energies—the distribution of energy following the Maxwell-Boltzmann law—but only those whose energies were integral multiples of $h\nu$ would count in determining the conditions of equilibrium between matter and ether.

AVERAGE ENERGY OF AN IDEAL LINEAR OSCILLATOR IN TERMS OF ITS FREQUENCY AND TEMPERATURE.

Suppose that we have inside a rigid adiabatic envelope N_1 linear oscillators of frequency ν_1, N_2 of frequency ν_2, etc. in thermal equilibrium with one another and with N_0 non-absorbing and non-emitting degrees of freedom. Since the energy of each oscillation is completely determined by its amplitude of vibration, the state of the system so far as the oscillations are concerned will be determined by the way in which N_1 oscillations of frequency ν_1, N_2 oscillations of frequency ν_2, etc., are distributed among the elements of extension-in-phase in a four-dimensional generalized space having as coördinates the three positional coördinates x, y, z, and q, where $q = C^2$. Since the energy of an oscillation of frequency ν is limited to values between $nh\nu - \delta$ and $nh\nu + \delta$, where n can assume integral values only, the elements of extension-in-phase will not fill the whole of this generalized space, and will have different positions for oscillations of different frequencies. For example, the projection on the x, y, q space of the elements of extension-in-phase for oscillations of frequency ν will be the solids bounded by the planes obtained by giving integral values to n in the equations

$$2\pi^2\nu^2qL = nh\nu - \delta,$$

$$2\pi^2\nu^2qL = nh\nu + \delta.$$

So far as the N_0 non-emitting degrees of freedom are concerned, the elements of extension-in-phase will fill the whole of the generalized space having as coördinates x, y, z and the variables determining the energy, since the energy associated with each of these degrees of freedom is in no way restricted to particular values.

Let us suppose that the state under consideration is one in which N_{00} non-emitting degrees of freedom are in element of extension-in-phase O_0, N_{01} in element of extension-in-phase 1_0, etc., N_{10} oscillations of frequency ν_1 are in element of extension-in-phase O_1, N_{11} in element of extension-in-phase 1_1, etc. Then the thermodynamic probability of the state, or the number of ways in which the state can be formed, is

$$W = \left\{\frac{N_0!}{N_{00}!N_{01}!N_{02}!\cdots}\right\}\left\{\frac{N_1!}{N_{10}!N_{11}!N_{12}!\cdots}\right\}\left\{\frac{N_2!}{N_{20}!N_{21}!N_{22}!\cdots}\right\}\{\quad\}\cdots,$$

where, of course, the second figure in the subscripts does *not* refer to the same element of extension-in-phase for oscillations of one frequency as for those of another, nor for oscillations as for non-emitting degrees of freedom, at least in so far as the coördinates specifying the energy are concerned.

Now the state of equilibrium is that state the probability of which is a maximum. Since the logarithm of a positive quantity is an increasing function of the quantity itself we may find the maximum of log W instead of that of W. If by w_{ij} we denote N_{ij}/N_i, *i. e.*, the mathematical probability of an oscillation of frequency ν_i being in element of extension-in-phase j, it may easily be shown that[1]

$$\log W = -\sum_i N_i \sum_j w_{ij} \log w_{ij}. \tag{7}$$

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[1] In getting (7) from the expression for W use is generally made of Sterling's formula. A simpler and more direct method which obviates the use of Sterling's formula is the following: If x is a large integer

$$\begin{aligned}\log x! &= \log 2 + \log 3 + \cdots + \log x\\ &\doteq \int_1^x \log x dx\\ &\doteq x \log x - x\end{aligned}$$

since the curve $y = \log x$ approaches parallelism to the x axis as x increases.

Hence

$$\begin{aligned}\log W &= \sum_i \{\log N_i! - \sum_j \log N_{ij}!\}\\ &= \sum_i \{N_i \log N_i - N_i - \sum_j N_{ij} \log N_{ij} + \sum_j N_{ij}\}\\ &= \sum_i N_i \{\sum_j w_{ij} \log N_i - \sum_j w_{ij} \log N_{ij}\}\end{aligned}$$

as $\sum_j w_{ij} = 1$.

Hence

$$\log W = -\sum_i N_i \sum_j w_{ij} \log w_{ij}.$$

For equilibrium log W must be a maximum subject to the conditions that the total volume and the total energy remain constant.

For maximum of log W we have

$$\sum_i N_i \sum (\log w_{ij} + 1)\delta w_{ij} = 0.$$

From the constant volume condition we have as many equations as there are values of i, of the form

$$N_i \sum_j \delta w_{ij} = 0.$$

From the condition of constant total energy we get

$$\sum_i N_i \sum_j U_{ij} \delta w_{ij} = 0,$$

where U_{ij} is the average energy of a degree of freedom of type i in element of extension-in-phase j.

Using Lagrange's method of undetermined multipliers we get

$$w_{ij} = \alpha_i e^{-\beta U_{ij}}, \tag{8}$$

where β is independent of the type i of the particular degree of freedom under consideration while α_i is not.

So far as the non-emitting degrees of freedom are concerned U_{0j} may have any value from zero to infinity, although for oscillations of frequency ν_i, U_{ij} is limited to values between $nh\nu_i - \delta$ and $nh\nu_i + \delta$, except possibly for inappreciable periods of time immediately following collisions. Consequently expression (8) does not lead to the principle of equipartition of energy, namely that the average kinetic energy associated with each degree of freedom is $1/2\beta$ quite irrespective of its type or frequency. In fact, for the emitting and absorbing oscillations we have

$$w_{ij} = \alpha_i e^{-\beta n h \nu_i}. \tag{9}$$

Hence the average energy of an oscillation of frequency ν will be

$$\begin{aligned} \overline{U}_\nu &= \frac{\sum_0^\infty N_\nu \alpha_\nu e^{-\beta n h \nu} n h \nu}{\sum_0^\infty N_\nu \alpha_\nu e^{-\beta n h \nu}} \\ &= h\nu \frac{\sum_0^\infty n e^{-\beta n h \nu}}{\sum_0^\infty e^{-\beta n h \nu}}. \end{aligned}$$

If we put $x = \beta h\nu$, we have

$$\overline{U}_\nu = h\nu \frac{e^{-x} + 2e^{-2x} + 3e^{-3x} + \cdots}{1 + e^{-x} + e^{-2x} + \cdots}$$

$$= h\nu \frac{e^{-x}}{(1 - e^{-x})^2}(1 - e^{-x})$$

$$= \frac{h\nu}{e^x - 1}. \tag{10}$$

In order to find x in terms of the temperature T we must first determine α_ν and β in terms of the total volume V and the total energy E_ν of all the oscillations of frequency ν. Let G_i denote the size of an element of extension-in-phase for oscillations of frequency ν_i. Then

$$G_i = \iiiint dxdydzdq$$

$$= g_i \iiint dxdydz,$$

where $g_i = \int dq$.

Hence, for oscillations of frequency ν_i

$$\sum_j w_{ij} = \frac{\alpha_i g_i}{G_i} \sum_n e^{-\beta n h\nu_i} \iiint dxdydz$$

$$= \frac{Vg_i}{G_i} \frac{\alpha_i}{1 - e^{-x}}$$

since the elements of extension-in-phase fill up the whole of the x, y, z space, even though there are unoccupied gaps in the q space.

But

$$\sum_j w_{ij} = 1.$$

Therefore

$$\alpha_i = \frac{G_1}{Vg_i}(1 - e^{-x}). \tag{11}$$

If E_i is the total energy of all the oscillations of frequency ν_i

$$E_i = N_i h\nu_i \frac{\alpha_i g_i}{G_i} \sum_n n e^{-\beta n h\nu_i} \iiint dxdydz$$

$$= \frac{N_i h\nu_i}{e^x - 1} \tag{12}$$

as can be seen at once from (10).

Hence

$$e^x = 1 + \frac{N_i h\nu_i}{E_i}. \tag{13}$$

So, if we put

$$y = \frac{E_i}{N_i h\nu_i} = \frac{\overline{U}_i}{h\nu_i},$$

$$\beta = \frac{1}{h\nu_i} \log\left(1 + \frac{h\nu_i}{\overline{U}_i}\right) = \frac{1}{h\nu_i} \log\left(\frac{1+y}{y}\right) \tag{14}$$

and substituting the value of e^x from (13) in (11)

$$\alpha_i = \frac{G_i}{Vg_i} \frac{1}{\left(1 + \frac{\overline{U}_i}{h\nu_i}\right)} = \frac{G_i}{Vg_i} \frac{1}{(1+y)}. \tag{15}$$

Hence the probability w_{in} of an oscillation of frequency ν_i having an intrinsic energy between $nh\nu_i - \delta$ and $nh\nu_i + \delta$ is, from (9)

$$w_{in} = \frac{y^n}{(1+y)^{1+n}}. \tag{16}$$

Now we are ready to write down the expression for the entropy of the oscillations. If W_i is the thermodynamic probability of the state of equilibrium of the N_i oscillations of frequency ν_i, then the entropy S_i of these N_i oscillations will be, by definition,

$$S_i = k \log W_i. \tag{17}$$

As we are dealing with oscillations of a single frequency we can drop the subscript i. Hence

$$\begin{aligned} S &= k \log W \\ &= -kN \sum_j w_j \log w_j \\ &= -kN\left[\log \alpha - \frac{x}{e^x - 1}\right] \\ &= kN \log\left\{\frac{Vg}{G} \frac{(1+y)^{1+\nu}}{y^{\nu}}\right\}. \end{aligned} \tag{18}$$

Now, by definition

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_V.$$

Hence

$$\frac{1}{T} = \frac{k}{h\nu} \log \frac{1+y}{y},$$

$$\frac{1}{y} = e^{\frac{h\nu}{kT}} - 1 = \frac{Nh\nu}{E} = \frac{h\nu}{\overline{U}}.$$

and

$$E = \frac{Nh\nu}{e^{\frac{h\nu}{kT}} - 1} \tag{19}$$

or[1]

$$\overline{U} = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}\,. \tag{20}$$

Also

$$\beta = \frac{1}{h\nu} \log \frac{1 + y}{y} = \frac{1}{kT}, \tag{21}$$

which is independent of the frequency or nature of the oscillation, as it should be.

Moreover, we have from general thermodynamic theory

$$\frac{p}{T} = \left(\frac{\partial S}{\partial V}\right)_E$$

giving the laws of Charles, Boyle, and Avogadro, namely:

$$pV = NkT.$$

THE RADIATION FORMULA.

The radiation formula follows from the results of the preceding analysis at once. In the state of equilibrium the rate of radiation must, on the average, be equal to the rate of absorption. Hence from (5) and (6)

$$u_\nu = \frac{8\pi\nu^2\overline{U}}{c^3}. \tag{22}$$

Substituting the value of $\overline{U}$ from (20) we get the radiation formula

$$u_\nu = \frac{\frac{8\pi h\nu^3}{c^3}}{e^{\frac{h\nu}{kT}} - 1}\,. \tag{23}$$

THE PARTITION OF ENERGY.

From the radiation formula we can find the way in which energy is distributed among the different degrees of freedom in the ether in the case of thermodynamic equilibrium. It is, however, necessary to emphasize one essential difference between the ether and material media.

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[1] This expression is the same as that obtained from the first form of Planck's theory. Indeed much of our formal analysis is similar to that by means of which Planck first obtained his radiation formula, although the interpretation given is quite different. See Vorlesungen über die Theorie der Wärmestrahlung, first edition, page 157.

Energy travelling in the form of waves of frequency ν through a material medium may be converted in part or in whole into energy of another frequency by the agency of the medium itself. Such is never the case in the ether. Energy travelling through the ether in the form of waves of frequency ν will remain unchanged forever unless transformed into waves of another frequency by being absorbed and reemitted by matter. Hence the distribution of energy among the different degrees of freedom in the ether is conditioned entirely by the characteristics of the matter with which it is in equilibrium. Consequently this distribution would be quite different for ether in equilibrium with matter so constituted that the energy per degree of freedom associated with each radiating and absorbing oscillation of frequency ν is an integral multiple of $h\nu$, from what it would be if the material oscillations were capable of containing any amounts of energy whatsoever.

Consider N_1 linear oscillators of frequency ν_1, N_2 of frequency ν_2, etc., inside a rigid adiabatic perfectly reflecting envelope in thermal equilibrium with each other and with N_1' ethereal modes of vibration of frequency ν_1, N_2' ethereal modes of vibration of frequency ν_2, etc. The state of equilibrium is the one having maximum probability. Following the same line of reasoning as before, the probability that a material oscillation of frequency ν has an energy between $nh\nu - \delta$ and $nh\nu + \delta$ is, as before,

$$w = \alpha e^{-\beta n h \nu} \tag{24}$$

while the probability that any one degree of freedom of vibration in the ether of frequency ν has an energy U' is

$$w' = \alpha' e^{-\beta U'},$$

where U' is not restricted in value in the same way as is the energy of the material oscillations. Now the elements of extension-in-phase must be all of equal probability. Hence U', which denotes the *average* energy represented by an element of extension-in-phase, must change in going from one to the next by a constant amount, say ϵ. So we have

$$w' = \alpha' e^{-\beta n \epsilon}. \tag{25}$$

Consequently we can show by exactly the same method as used in the case of the linear oscillations in matter that the average energy $\overline{U'}$ associated with each degree of freedom in the ether of frequency ν is given by

$$\overline{U'} = \frac{\epsilon}{e^{\frac{\epsilon}{kT}} - 1}. \tag{26}$$

Now the number of degrees of freedom per unit volume of the ethereal modes of vibration of frequencies between ν and $\nu + d\nu$ is given by the well known expression

$$Nd\nu = \frac{8\pi\nu^2 d\nu}{c^3}. \tag{27}$$

Consequently

$$u_\nu d\nu = \frac{\frac{8\pi\epsilon\nu^2}{c^3}\,d\nu}{e^{\frac{\epsilon}{kT}} - 1}$$

or

$$u_\nu = \frac{\frac{8\pi\epsilon\nu^2}{c^3}}{e^{\frac{\epsilon}{kT}} - 1}. \tag{28}$$

Comparing with (23) we see that

$$\epsilon = h\nu$$

and that *the average energy associated with each degree of freedom of a simple harmonic vibration of frequency ν, whether in matter or in ether, is given by*

$$\overline{U} = \overline{U}' = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}. \tag{29}$$

SUMMARY.

Planck's radiation formula has been derived from the expressions given by classical dynamics and electrodynamics for the absorption and radiation of energy with the aid of the supplemental assumption that the motion of an absorbing and emitting linear oscillator of frequency ν is stable only when the energy of its oscillations is an integral multiple of $h\nu$. This assumption avoids the necessity of replacing the emission of energy demanded by the electrodynamic equations by an emission that is discontinuous in time and difficult, if not impossible, to reconcile with the phenomena of diffraction and interference.

The partition of energy among the different degrees of freedom in the ether has been investigated, and found to be the same as in material oscillators.

SLOANE PHYSICS LABORATORY,
August, 1915.

CATHODE MEASUREMENTS WITH GLOW CURRENT IN HYDROGEN.

By W. L. Cheney.

PROFESSOR SKINNER[1] has recently developed a theory of the cathode fall in gases which suggests that with a plane cathode the "normal" current density should be proportional to the square of the gas pressure provided the number of mean free paths of the electrons from the cathode to the point of minimum gradient in the negative glow is the same for all pressures. In measurements made with aluminum and steel cathodes in hydrogen both of these conditions were found to exist.

He has also found that, provided the current density used is the same multiple of the "normal" current density, potential measurements in the region between the cathode and the negative glow, taken at various gas pressures and plotted against the number of mean free paths from the cathode, all lie on the same curve. For example, the potential difference obtained at "normal" current density at various gas pressures gave one curve, at twice "normal" current density a second curve, and so on.

The purpose of this research was to repeat some of Skinner's measurements with aluminum and steel and to extend the investigation to various other metals.

I. Apparatus.

This consisted of a discharge tube (Fig. 1) about three centimeters in diameter, having a movable disc cathode K, carried by the micrometer screw B. The graduated drum M, on the ground joint J, served to give the axial position of the cathode. The current was confined to the front face of the cathode by the glass hood H. A probe wire P (Al, .24 mm. in diam.) sheathed to very near its end with a fine glass tube S, served to give the poten-

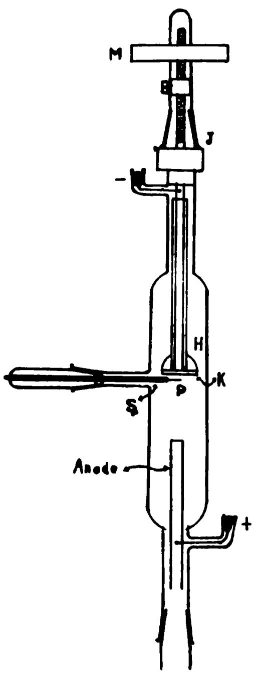

Fig. 1.

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[1] C. A. Skinner, Phys. Rev., June and August, 1915.

tial in the gas. This sheath was renewed from time to time to prevent its becoming conductive as a result of metal sprayed off from the cathode.

Potential measurements were made by a quadrant electrometer in which a definite P.D. (6 volts) was placed between the quadrants, the potential sought being placed between one pair of quadrants and the needle. The electrometer was, from time to time, compared on a battery circuit with a Weston voltmeter. Customary precautions were taken against the possible influence of static charges and the more important leakage effects avoided by using sealing wax insulators throughout. This leakage was tested by placing a charge on the electrometer, disconnecting, and noting the rate at which the deflection changed. The change was not appreciable over a considerable length of time. In order to work rapidly—quite necessary in view of the deterioration of the gas—the deflections of the electrometer (read by telescope and scale) were taken only in one direction.

The current was furnished by a battery of small storage cells, giving in all about 1,100 volts; measured by means of a mil-ammeter or, for smaller currents, a d'Arsonval galvanometer; and regulated by means of a liquid rheostat consisting of a solution of cadmium iodide in amyl alcohol.

A description of the modified Dolazalek electrometer used may not be out of place. The regular needle of the electrometer was replaced by one of aluminum and the ordinary suspension by a ribbon of phosphor-bronze. A damping arrangement consisting of a fork of platinum, dipping into concentrated H_2SO_4, was used.

Another factor demanding care was the cathode surface. To insure uniform results, each day before beginning a series of observations (and often during the series) the cathode was freshly polished by scouring with Tripoli composite, by rubbing on a pad of felt, and finally by wiping carefully on a new piece of cheese cloth.

The hydrogen was obtained by the action of KOH on aluminum. Great care was exercised to rid the gas of air by initial pumping and flushing with hydrogen; also to thoroughly dry it by passing it, first through a tube containing P_2O_5, and then allowing it to stand for about a half hour in a chamber containing the same material. The hydrogen was then stored in a bulb (freed from occluded gases) connected with the apparatus, from which it could be drawn off as desired.

II. Experimental Results.

1. *Current Density and Number of Mean Free Paths to the Negative Glow with 'Normal' Cathode Fall.*—Cathodes of aluminum, steel, nickel,

zinc, and platinum were used. The gas pressures (measured by a McLeod gauge) were necessarily confined to the range of about one to four millimeters. With pressures below one millimeter the negative glow became crowded so closely to the cathode as to make distance measurements too uncertain. With each pressure the "normal" current density was obtained by increasing the current until the glow just covered the cathode and ceased to curl away from it, or (measuring with the electrometer) when the cathode fall began to rise. The former method was used in most cases, though at the very low pressures the latter was often resorted to. The point of minimum gradient was found, by moving the cathode toward the probe wire, and observing where the P.D. began to drop appreciably below the normal value given by the probe in the main part of the negative glow.

Before each set of observations, the cathode was cleaned by passing a heavy current through the tube; the gas then pumped out; and fresh gas introduced. With each gas filling the "normal" current, the "normal" cathode fall, and the distances to the point of minimum gradient were measured.

Representative results are given in Tables I.–V. The first column gives the pressures—each representing a fresh filling of gas. Column 2 gives the "normal" fall in volts; Column 3, the current density in mill-amperes. Column 5 gives the mean free path (in mm.) of the electrons

TABLE I.

Aluminum Cathode in Hydrogen—Area = 3.0 sq. cm.

Gas Pressure (mm.).	Normal Fall (Volts.)	j_n (m.a.)	Dist. to Neg. Glow (mm.).	$\overline{\lambda}$ (mm.)	No. of m. f. p. to Neg. Glow.	$j_n \overline{\lambda}^2$
1.00	206	.067	10.80	.720	15.0	.0358
1.69	194	.193	6.60	.426	15.4	.0353
2.00	196	.287	5.33	.360	14.8	.0374
2.08	199	.307	5.36	.348	15.4	.0369
2.30	197	.400	4.82	.313	15.4	.0384
2.70	199	.550	3.99	.266	15.0	.0389
2.80	194	.607	3.98	.257	15.5	.0405
2.85	197	.613	3.98	.253	15.6	.0395
2.96	197	.683	3.72	.243	15.3	.0400
3.00	196	.653	3.75	.240	15.0	.0379
3.05	196	.713	3.66	.238	15.5	.0400
3.72	194	1.067	2.91	.194	15.0	.0400
3.90	196	1.167	2.78	.185	15.0	.0400
4.00	194	1.267	2.78	.180	15.4	.0410
Mean	197				15.3	.0387
Max. dev.	4.5%				3%	9%
Mean dev.	1%				1.5%	4%

TABLE II.

Steel Cathode in Hydrogen Area = 3.14 sq. cm.

Gas Pressure (mm. Hg.).	Normal Fall (Volts.)	j_n (m.a.).	Dist. to Neg. Glow (mm.).	$\bar{\lambda}$ (mm.).	No. m.f.p. to Neg. Glow.	$j_n\bar{\lambda}^2$
1.94	315	.191	7.70	.371	20.7	.0267
2.20	299	.270	6.70	.327	20.5	.0291
2.30	311	.245	6.42	.313	20.5	.0241
2.45	299	.314	5.88	.294	20.0	.0272
2.50	294	.350	5.76	.288	20.0	.0291
2.53	299	.350	5.70	.285	20.0	.0285
2.64	304	.334	5.70	.273	20.9	.0250
3.07	293	.540	4.70	.234	20.0	.0294
3.40	303	.620	4.24	.212	20.0	.0281
3.55	294	.700	4.15	.203	20.4	.0291
3.90	299	.780	3.85	.184	20.9	.0265
Mean	301				20.3	.0275
Max. dev.	4.5%				3%	12%
Mean dev.	2%				1.5%	5.5%

in hydrogen, at the corresponding gas pressure, as calculated from that of the molecule by using the ratio given by Franck and Hertz.[1] Column 6 gives the number of mean free paths to the minimum gradient, obtained by dividing the magnitudes in 4 by the corresponding ones in 5. Column 7, giving the product $j_n\bar{\lambda}^2$, is simply another way of expressing the relation

TABLE III.

Nickel Cathode in Hydrogen Area = 3.14 sq. cm.

Gas Pressure (mm. Hg).	Normal Fall (Volts).	j_n (m.a.).	Dist. to Neg. Glow (mm.).	$\bar{\lambda}$ (mm.).	No. of m.f.p. to Neg. Glow.	$j_n\bar{\lambda}^2$
1.72	325	.159	8.47	.419	20.2	.0281
2.50	328	.382	5.85	.288	20.3	.0317
2.81	319	.430	5.37	.256	21.0	.0283
2.90	322	.493	4.97	.248	20.0	.0307
3.06	323	.535	4.85	.235	20.6	.0296
3.54	316	.669	4.06	.203	20.0	.0278
3.60	325	.700	4.00	.200	20.0	.0281
3.73	320	.795	3.86	.193	20.0	.0296
3.80	324	.764	3.80	.190	20.0	.0275
4.03	320	.891	3.70	.178	20.8	.0286
Mean	322				20.3	.0290
Max. dev.	2%				3.5%	9%
Mean dev.	1%				1.5%	4%

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[1] J. Franck and G. Hertz (Deutsch. Phys. Gesell., No. 9, p. 373, 1913) found the mean free path of the electron to be $4\sqrt{2}$ times the mean free path of the molecule. For 1 mm. pressure then the m.f.p. of the electron in hydrogen is about .72 mm.

TABLE IV.

Platinum Cathode in Hydrogen Area = 1.76 sq. cm.

Gas Pressure (mm. Hg.).	Normal Fall (Volts.)	j_n (m.a.)	Dist. to Neg. Glow (mm.).	$\bar{\lambda}$ (mm.).	No. of m. f. p. to Neg. Glow.	$j_n\bar{\lambda}^2$
1.61	310	.114	8.50	.446	19.0	.0231
1.95	306	.176	7.01	.369	19.0	.0239
2.21	302	.233	6.30	.326	19.3	.0249
2.35	300	.256	5.82	.306	19.0	.0240
3.05	303	.466	4.70	.236	19.9	.0260
3.10	303	.482	4.61	.232	19.9	.0260
3.35	300	.545	4.30	.215	20.0	.0252
3.60	299	.636	4.10	.200	20.5	.0254
3.86	297	.739	3.70	.186	19.9	.0257
4.00	304	.767	3.20	.180	20.0	.0249
4.34	302	.938	3.30	.166	19.9	.0260
Mean	302.5				19.7	.0250
Max. dev.	2.5%				3.5%	7%
Mean dev.	1%				2%	3%

j_n/p^2, since $\bar{\lambda}$ varies inversely as the pressure, p. Columns 2, 6, and 7 show for all metals merely irregular fluctuations about a certain mean value, thereby proving the constancy of the relations already stated.

Comparing the different metals, aluminum gives a much higher current density than the others, the lowest cathode fall, and the least number of

TABLE V.

Zinc Cathode in Hydrogen Area = 3.14 sq. cm.

Gas Pressure (mm. Hg.).	Normal Fall (Volts).	j_n (m.a.).	Dist. to Neg. Glow (mm.).	$\bar{\lambda}$ (mm.).	No. of m.f.p. to Neg. Glow.	$j_n\bar{\lambda}^2$
1.40	284	.108	9.25	.515	18.0	.0285
1.50	278	.130	9.15	.480	19.1	.0301
1.85	273	.203	7.35	.389	18.9	.0309
2.33	280	.302	5.85	.309	18.9	.0290
2.45	277	.346	5.30	.294	18.0	.0301
2.45	276	.344	5.30	.294	18.0	.0290
2.80	269	.460	4.85	.257	18.9	.0306
2.97	275	.509	4.36	.242	18.0	.0301
3.02	270	.557	4.40	.238	18.5	.0316
3.25	272	.620	3.99	.222	18.0	.0306
3.33	272	.685	4.00	.217	18.4	.0322
3.48	265	.764	3.81	.207	18.4	.0327
4.00	267	.923	3.24	.180	18.0	.0301
4.18	271	1.08	3.22	.172	18.7	.0322
Mean	273.5				18.4	.0306
Max. dev.	4%				4%	7%
Mean dev.	1.5%				2%	3%

mean free paths to the negative glow. The mean values of the cathode fall given here for aluminum and steel agree very closely with the values given by both Warburg[1] and Skinner.[2] The other results from these metals are also in excellent agreement with Skinner's as shown by the following table.

	Cheney.		Skinner.	
	Al.	Steel.	Al.	Steel.
Cathode fall (volts)	197	301	197	303.5
No. of m.f.p. from Cathode to minimum gradient	15.3	20.3	15.3	20.5
$j_n\lambda^2$	.0387	.0275	.039	.0273

These comparisons, along with that of individual values given in the tables I–V., indicate that one is but slightly less certain of reproducing both $j_n\lambda^2$ and the distance of the minimum gradient from the cathode than one is of reproducing the cathode fall itself.

Zinc ranges next to aluminum in the series of metals, having the next lowest cathode fall, the next highest current density, and the next smallest number of m.f.p.'s to the negative glow. Both zinc and nickel were rather difficult to work with, owing to tarnish. This necessitated frequent polishing of the cathode and many observations. The values given for these (tables III. and V.) are only those results which were obtained immediately after polishing and before any tarnish became apparent.

It is interesting to note that in the case of nickel, both the current density and the cathode fall are higher than in the case of steel, while the number of mean free paths from the cathode to the minimum gradient are the same.

With platinum much trouble arose from the distintegration of the cathode. In a relatively short time the metal would spray off so badly that it necessitated renewing the sheath and cleaning both the probe wire and the walls of the tube. With this metal, the cathode fall (302.5 volts) runs slightly higher than the value given by Warburg[1] (300 volts) and Capstick[2] (298 volts). The " normal " current density runs a trifle lower than for steel or nickel.

2. *Potential Curves in the Gas with ' Normal ' Current Density.*—Before each series of observations the cathode was cleaned as usual by a heavy

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[1] E. Warburg, Wied. Annalen, 31, p. 542.

[2] C. A. Skinner, PHYS. REV., *l. c.*

[3] E. Warburg, Wied. Annalen, 31, p. 542.

[4] Capstick, Roy. Soc. Proc., 63, p. 356, 1898.

current, the tube evacuated, and fresh gas introduced. Starting at the negative glow, the P.D. between the cathode and points at various mean free paths from it (measured to the center of the probe) were obtained as rapidly as possible—the current being maintained at its normal value during the series of observations. Check values were always taken after each series to indicate if disturbing changes had arisen during the operation. Only those values are given where the change in the cathode fall during a series was less than 2.5 per cent.

The observations are incorporated in Tables VI. and VII. Table VI. gives the results in detail obtained from platinum. It shows that the magnitudes are practically the same at all gas pressures. In Table VII. are given therefore only the mean values obtained from Al 1, Al 2, Ni, steel, and Pt. These mean values were obtained in the same way as the values in Table VI.—Al 1 from 8 different gas pressures ranging from 1.7 to 5.0 mm.—Al 2 from 4 different pressures, from 1.1 up to 2.0 mm. only—steel from 11 different pressures, from 2.2 to 3.9 mm.—Ni from 8 different pressures from 1.7 to 4.0 mm.—and also Pt from those of Table VI.

Table VI.

Potential Curve—Platinum—"Normal" Current Density ($j_n\bar{\lambda}^2 = .0387$).

No. of m. f. p.	Gas Pressure in Mm.										Mean P. D.
	1.41	1.75	1.95	2.25	2.65	3.05	3.10	3.35	3.60	3.86	
3	175	178	175	170	173	174	176	—	—	173	174.5
4	—	190	190	182	185	189	191	183	183	187	187
5	200	202	204	193	196	200	203	198	197	201	199.5
6	—	—	215	204	207	214	215	210	208	213	211
7	222	224	228	215	218	227	228	222	219	224	223
9	245	244	248	237	238	246	249	241	237	244	243
13	274	278	281	265	274	276	269	270	265	269	272
17	293	293	301	290	286	294	295	289	290	288	292
20	302	302	306	299	294	303	303	300	299	297	301

Al 1 was the same cathode used in previous measurements. Comparing these results with those obtained by Skinner, the P.D. between the cathode and two mean free paths from it differed by about 7 per cent. To ascertain if this were due to a difference in the samples of the metal his cathode (Al 2) was also tried. The results are practically the same as given by Al 1 at the same gas pressure. The discrepancy in our results is somewhat larger than the conditions would ordinarily allow.

Skinner calls attention to the divergence of the potential curves at low gas pressures from the constant relation illustrated in Table VI. the difference shown here between Al 1 and Al 2 arises from this fact—the observations for Al 1 being simply those obtained below 2 mm., while for Al 2 they range from 1.7 up to 5 mm.

Observations were also made with zinc but the results were very inconsistent, indicating conductivity of the probe sheath.

Table VII.

Potential Curves "Normal" Current Density.

Number m.f.p. from Cathode.	Mean P.D.'s				
	Al 1.	Al 2.	Steel.	Nickel.	Platinum.
2	107	100	171	——	——
3	119.5	124	187	213	174.5
4	131	137	198	225	187
5	142	152	208	237.5	199.5
6	154	164	218	248	211
7	163	172	227	257	223
9	179	186	243	274	243
11	190	194	——	——	——
12	——	——	267	291	——
13	193	——	——	——	272
15	196	197	——	——	——
16	——	——	285.5	309	——
17	——	——	——	——	292
20	——	——	——	320	301
20.5	——	——	297	——	——

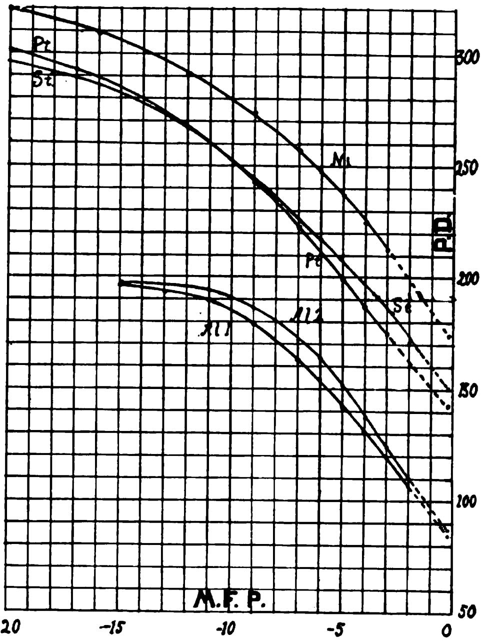

Fig. 2.

The results of the observations in Table VII. are plotted as curves in Fig. 2 with mean free paths as abscissæ and P.D.'s as ordinates. The broken line parts of these curves (practically straight) are their theoretical extension up to one m.f.p. of the positive ion from the cathode —over which the fall of potential between the cathode and the gas is conceived to occur. The method of obtaining these theoretical values follows later. A comparison of these values with the total cathode fall in the gas is given in the following table (VIII.):

Table VIII.

Cathode.	Total Cathode Fall (Volts).	Fall in Gas.	Fall at Cathode.
Al 1	196	111.5	84.5
Al 2	197	110	87
Steel	297	146	151
Nickel	320	145.5	174.5
Platinum	301	158	143

It is interesting to note that with Al and Pt the larger part of the fall is in the gas while with the other metals the larger part is between the cathode and gas.

III. Some Applications of the Results.

1. *Mobility of the Positive Ions.*—This is defined as their velocity in cm. per sec. in unit field. From the potential measurements given in Table VII., the upper and lower limits of the mobility of the positive ions can be calculated from expressions given by Skinner (*l. c.*). These are

$$\begin{aligned} V_0 - V_1 &< \left\{ \left(\frac{16\pi j}{9\overset{+}{\mu}} \right)^{\frac{1}{3}} (x_1 - x_0) \right\}^{\frac{3}{2}} \\ &> \left\{ \left(\frac{16\pi j}{9\overset{+}{\mu}} \right)^{\frac{1}{3}} (x_1 - x_0) - \epsilon^{\frac{2}{3}} \right\}^{\frac{3}{2}} + \epsilon \end{aligned} \qquad (1)$$

in which $V_0 - V_1$ is the difference of potential (in electrostatic units) between some point x_1 near the cathode and x_0 the point of minimum gradient; j is the current density in electrostatic units; $\overset{+}{\mu}$ is the mobility of the positive ions also in electrostatic units; and ϵ is the ionizing potential of the electron[1] (11 volts or .037 electrostatic unit).

From these expressions it follows that

$$\begin{aligned} \overset{+}{\mu} &< \frac{16\pi}{9} j \frac{(x_1 - x_0)^3}{(V_0 - V_1)^2} \\ &> \frac{16\pi}{9} \frac{j(x_1 - x_0)^3}{\{(V_0 - V_1 - \epsilon)^{\frac{2}{3}} + \epsilon^{\frac{2}{3}}\}^3} \end{aligned} \qquad (2)$$

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[1] J. Franck and G. Hertz, Deutsch. Phys. Gesell. Verh., 15, 2, p. 34, 1913.

To get the mobility in a field of one volt per cm. the values of $\overset{+}{\mu}$ given by (2) must be divided by 300.

Substituting from Table VII.—x_1 being taken as the point where measurements were made nearest the cathode and $(x_1 - x_0)$ obtained by multiplying the distance in m.f.p. by .072 cm. (the m.f.p. of the electron at 1 mm. pressure)—the limits of $\overset{+}{\mu}$ for a gas pressure of 1 mm. were calculated. These are given in Table IX. The values found here for the two samples of aluminum (2.3×10^4 and 3.7×10^4 cm./volt) agree closely with Skinner's values (2.2×10^4 and 3.6×10^4). There is also good agreement for Al, steel and Ni, while Pt gives a distinctly lower mobility—probably the effect of the presence of the metallic vapor arising from the disintegration of the cathode, the vapor hindering the progress of the positive ions toward the cathode.

TABLE IX.

Mobility of Positive Ions at One mm. Pressure.

Metal.	Lower Limit of $\overset{+}{\mu}$.		Upper Limit of $\overset{+}{\mu}$.	
	Electrostatic.	cm./volt.	Electrostatic.	cm./volt.
Al 1	$7.0\times10^6\frac{\text{cm.}}{\text{sec.}}$	$2.3\times10^4\frac{\text{cm.}}{\text{sec.}}$	$11.0\times10^6\frac{\text{cm.}}{\text{sec.}}$	$3.7\times10^4\frac{\text{cm.}}{\text{sec.}}$
Al 2	7.0×10^6	2.3×10^4	11.0×10^6	3.7×10^4
Steel	8.0×10^6	2.7×10^4	11.8×10^6	3.9×10^4
Nickel	7.0×10^6	2.3×10^4	10.5×10^6	3.5×10^4
Platinum	5.6×10^6	1.9×10^4	8.4×10^6	2.8×10^4

2. *Fall at the Cathode.*—By using the above values of $\overset{+}{\mu}$ in their respective expressions in (1) the potential curves may be extended to the cathode, or better to one mean free path of the positive ion from the cathode—this being considered as the region within which the positive ions causing the immediate fall between the gas and the cathode accumulate.

These calculations give the following values for the fall between the cathode and the adjacent gas. It appears surprising that the upper and lower limits of $\overset{+}{\mu}$ give so nearly the same values.

TABLE X.

Metal.	Fall at One m. f. p. of Positive Ion from the Cathode.		
	Upper Limit.	Lower Limit.	Mean.
Al 1	85	84	84.5
Al 2	88	86	87
Pt	143	143	143
Steel	152	150	151
Nickel	177	172	174.5

3. *The Magnitude of the Electron Current from the Cathode.*—Assuming that each electron leaving the cathode results in two electrons at one m.f.p., four at two m.f.p., and so on; and, that in the region of lower gradient the number of electrons doubles in moving through the ionizing potential, Skinner calculates that the ratio of the electron current density ($\bar{j}_k$) from the cathode to the total current (j) is

$$\frac{\bar{j}_k}{j} = \frac{\bar{\mu}}{\bar{\mu} + \overset{+}{\mu}} \cdot 2^{-\left\{\frac{x_k - x_a}{\bar{\lambda}} + \frac{V_0 - V_a}{\epsilon}\right\}} \tag{3}$$

where $[(x_k - x_a/\lambda)]$ is the number of mean free paths between the cathode and the point where ionization ceases to take place at every mean free path, $V_0 - V_a$ the fall of potential between this point and the negative glow, ϵ the ionizing potential, and $\bar{\mu}$ and $\overset{+}{\mu}$ the mobilities in the low gradient region of the negative glow. Assuming that $\bar{\mu}$ and $\bar{\mu}$ are practically equal the following values for $\bar{j}_k/j$ were obtained.

Table XIV.

Metal.	$\frac{x_k - x_a}{\bar{\lambda}}$	$\frac{V_0 - V_a}{\epsilon}$	$\frac{\bar{j}_k}{j}$
Al 1	6	3.8	57×10^{-5}
Al 2	6	3.0	98×10^{-5}
Ni	6	6.5	8.4×10^{-5}
Steel	6	7.2	5.3×10^{-5}
Pt	7	7.1	2.8×10^{-5}

Al 2 gives nearly twice the electron current of Al 1 (which is somewhat larger than the value obtained by Skinner). The effect of this is shown in the potential curve (Fig. 2)—Al 2 reaching a constant value at a somewhat shorter distance from the cathode. The difference does not arise from a difference in the samples of the metal but merely shows the effect of lowering the gas pressure, as already noted. The ratio $\bar{j}_k/j$, although very small for all metals, is about 7 times as large with Al 1 as with Ni, 10 times as large as with steel, and about 19 times as large as with Pt.

It is also interesting to note that with all metals, the point at which ionization at every collision takes place extends to about 6 m.f.p. from the cathode.

Summary.

1. The "normal" current density at the cathode has been measured in hydrogen with Al, steel, Ni, Zn, and Pt as cathodes, and found in all cases to be proportional to the square of the gas pressure.

2. The number of mean free paths of the electron from the cathode to the point of minimum gradient in the negative glow has been measured for these metals under the same conditions and found to be independent of the gas pressure.

3. Potential curves in the gas between the cathode and the negative glow have been obtained at "normal" current density with these metals as cathodes. The potentials plotted against the number of mean free paths of the electron from the cathode lie, for all pressures, practically on the same curve.

4. From these potential measurements, the mobility of the positive ions has been calculated and consistent values obtained for all metals except platinum where the mobility is somewhat low, probably arising from the presence of metal vapor.

5. The potential curves have been theoretically extended to the cathode, and the fall between the cathode and the gas obtained therefrom.

6. The ratio of the electron current from the cathode to the total current has been calculated for the various metals.

I wish to express my sincere thanks to Professor C. A. Skinner for suggesting this problem and directing its course, also to Dr. A. Q. Tool for many helpful suggestions.

BRACE LABORATORY OF PHYSICS,
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June, 1915.

CATHODE MEASUREMENTS WITH GLOW CURRENT IN OXYGEN AND NITROGEN.

By W. Neuswanger.

IN a recent paper, Prof. Skinner[1] has developed a theory of the so-called cathode fall in gases which gives a general relation between the fall, the current density, the mean free path of the electron, and the distance from the cathode to the point of minimum potential gradient in the negative glow. A series of experimental results with hydrogen substantially corroborated the theoretical deductions. The purpose of this investigation has been to make a similar experimental study of the cathode fall in oxygen and nitrogen.

Apparatus.

The apparatus consisted essentially of a discharge tube, Fig. 1, connected with a McLeod gauge, a gas generator and storage chamber, and a mercury vacuum pump. The tube, which was similar to the one used by Skinner, was provided with a circular disk cathode *K*, carried on a threaded shaft *S*, whereby it could be accurately adjusted, through the ground joint *J*, to any given distance from the aluminum probe wire *P*, which served to give the potentials in the gas. The distances were indicated by the graduated drum *D*. To insure a fixed position of the nut *N*, the air pressure forced *J* against a shoulder *Q*, cemented on the stem of the tube. A glass hood *H* confined the discharge to the front face of the cathode. The diameter of the probe wire (.125 mm.) was about one half that used by Skinner (.24 mm.) in hydrogen—making it about the same relative to the magnitude of the mean free paths of the electrons in oxygen and nitrogen, as his was to that in hydrogen. This probe was covered by a close fitting glass sheath which was frequently renewed since any metallic deposit would render it conductive and hence give spurious potentials.

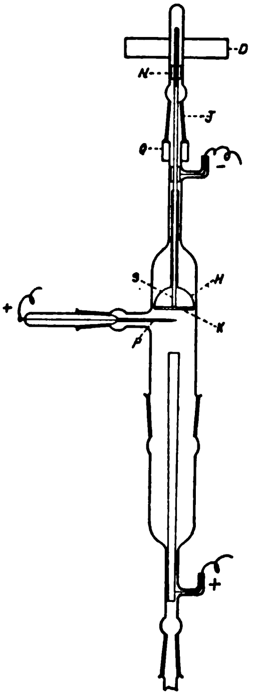

Fig. 1.

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[1] C. A. Skinner, Phys. Rev., June and August, 1915.

The electrometer used was a modified Dolazalek fitted with a phosphor-bronze suspension and an aluminum needle. A constant potential difference of 18 volts was placed between the two pairs of quadrants, while that to be measured was placed between the needle and one pair of quadrants. The electrometer was calibrated from time to time by comparison (on a storage battery circuit) with a Weston voltmeter. In order to avoid leakage, the electrometer itself, together with the batteries used to charge the quadrants, were placed on sealing wax—no ground to the electrometer being used. All connecting lines and switches were similarly insulated. The electric current was furnished by a 1,200-volt battery of accumulators and regulated by a liquid resistance of amyl alcohol and cadmium iodide. It was measured by a Weston milliammeter.

EXPERIMENTAL.

Oxygen.

Oxygen was generated by heating a mixture of potassium dichromate and concentrated sulphuric acid. The gas was allowed to stand for several hours in a bulb containing crushed potassium hydrate to remove any acid vapor, and then after being thoroughly dried over phosphorous pentoxide, was stored in a chamber connected to the discharge tube, to be drawn off as desired.

Current Density and Number of Mean Free Paths to the Negative Glow with Normal Cathode Fall.—The general plan followed was to clean and polish the cathode, mount it in the discharge tube, evacuate this and flush with the gas. With the gas reasonably pure, an electric current was then passed through the tube which always increased the pressure by freeing occluded gases from the cathode. This impure gas was then removed and fresh introduced in which the measurements were made, namely: of the gas pressure, the normal cathode fall, normal current density, and the number of mean free paths of the electron from the cathode to the point of zero gradient. This set was then followed by a similar one in fresh gas—the process being repeated until the normal cathode fall rose above what was considered a satisfactory magnitude. The cathode was then removed and again carefully polished with Tripoli composite, and a new clean piece of cheese cloth before further measurements were made.

Series of observations with each of the metals, aluminum, steel, and zinc as cathodes, are given in Tables I., II. and III. respectively. Each gas pressure, column 1 of each table, represents a new gas filling. At pressures below those given, the luminosity was too faint to allow accurate adjustment to normal current density. At higher pressures than those given, the other magnitudes became inaccurate. In column 2 is given the

TABLE I.

Aluminum Cathode in Oxygen. Area of Cathode: 1.35 sq. cm.

Gas Pressure (mm.).	Normal Cathode Fall (Volts).	Normal Current Density (j_n)	Distance to Neg. Glow (x_k-x_0).	Mean Free Path ($\bar{\lambda}^2$)	($j_n\bar{\lambda}^2$)	$\left(\frac{x_k-x_0}{\bar{\lambda}}\right)$
1.06	301	.44 ma.	4.20 mm.	.41 mm.	.074	10.2
1.45	306	1.04	3.30	.30	.090	11.0
1.58	312	1.33	2.50	.28	——	——
1.80	313	1.63	2.50	.24	.092	10.4
1.80	307	1.56	2.60	.24	.092	10.8
1.98	312	1.85	2.30	.22	.087	10.4
2.27	308	2.15	2.10	.19	.076	11.0
2.35	312	2.52	1.90	.19	.090	10.0
2.85	309	3.34	1.60	.15	.074	10.7
2.88	313	3.48	1.65	.15	.079	10.7
3.27	311	4.52	1.50	.13	.076	11.5
3.51	316	5.18	1.25	.125	.081	9.6
3.71	312	5.33	1.30	.12	.076	10.8
3.87	312	5.77	1.30	.11	.070	——
3.93	317	7.18	1.15	.11	.087	10.4
4.16	316	6.81	1.20	.105	.075	11.4
Mean	311			Mean	.081	10.6
Max. dev.	3%			Max. dev.	12%	10%
Mean dev.	1%			Mean dev.	8%	4%

normal cathode fall with its percentage deviation, in all cases surprisingly small. Column 3 gives the normal current density. This was found to decrease rather rapidly below that required to cover the cathode when first starting the current. However, consistent values could be obtained if the time interval required to make the reading was consistent. The method used here was to take the mean of the first two adjustments to the point at which the cathode glow just covered the surface of the electrode with uniform brightness. These were of course made as quickly as careful judgment would allow. The high value of the normal current density in oxygen as compared with hydrogen—six times as large as the same gas pressure—made it advisable to use cathodes of about 1.3 sq. cm. area instead of 3.0 sq. cm., as used by Skinner for hydrogen. The distance, in millimeters, from the cathode to the point of minimum potential gradient, given in column 4, was observed by noting where the potential began to decrease as the cathode was moved toward the probe wire, or, as described by Skinner, by setting the probe wire just in the outer face of the bright striation next to the cathode dark space. The values of the mean free paths of the electron, column 5, corresponding to the given pressure were obtained from the relation given by J. Franck and G. Hertz,[1] namely: $4\sqrt{2}$ times the mean free path of the molecule. Column 7 obtained by dividing the distances in column 4 by the corresponding $\bar{\lambda}$ gives the number of mean free paths from the cathode to the negative glow. This according to theory should be a constant under conditions giving a constant cathode fall. Column 6 is the product of the normal current density by the square of the mean free path of the electron.

TABLE II.

Steel Cathode in Oxygen. Area of Cathode: 1.36 sq. cm.

Gas Pressure (mm.).	Normal Cathode Fall (Volts).	Normal Current Density. j_n	Distance to Neg. Glow. x_k-x_0	Mean Free Path. $\bar{\lambda}$	$j_n\bar{\lambda}^2$	$\frac{x_k-x_0}{\bar{\lambda}}$
1.05	336	.38 ma.	4.30 mm.	.42 mm.	——	10.2
1.37	338	.59	3.70	.32	.060	11.5
1.55	340	.77	3.30	.28	.060	11.8
1.65	343	.81	——	.27	.059	——
1.83	340	1.07	2.80	.24	.063	11.7
2.23	342	1.54	2.20	.20	.060	11.0
2.27	341	1.62	1.91	.19	.058	10.0
2.62	344	2.06	1.77	.17	.061	10.6
2.67	342	2.20	1.72	.16	.056	10.6
2.80	341	2.42	1.65	.16	.061	10.3
3.02	343	2.86	1.50	.145	.061	10.3
3.03	342	2.79	1.56	.14	.055	11.4
3.21	343	3.02	1.52	.14	.059	10.7
3.23	344	3.09	1.47	.135	.057	11.1
3.37	345	3.31	1.52	.13	.056	11.5
3.58	345	3.53	1.37	.12	.050	11.7
3.81	346	3.75	——	.115	.049	——
3.83	345	4.11	1.26	.115	.054	11.3
3.96	344	4.11	1.30	.11	.050	11.8
4.08	344	4.41	1.22	.11	.053	10.9
4.14	346	4.71	1.19	.105	.052	11.4
4.25	347	4.64	1.17	.10	——	——
Mean	343			Mean	.057	11.0
Max. dev.	2%			Max. dev.	13%	10%
Mean dev.	1%			Mean dev.	5%	4%

This too should be constant. Inspection shows for all three metals that throughout the range of gas pressures used, the normal cathode fall, and the number of mean free paths of the electron to the point of minimum potential gradient, are all practically constant. In the case of aluminum, several different samples of metals were used, all giving similar results.

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[1] J. Franck and G. Hertz, Ber. d. Deutsch. Phys. Ges., 9, 1913, p. 373.

Comparing the different metals we find aluminum to give, as expected, the lowest cathode fall, 311 volts; next is steel with 343 volts; and then zinc with 354 volts. These values are from two to three per cent. higher than those found by Skinner.[1] The values of the other constants show a considerably larger variation than the cathode fall, as might be expected when the accuracy of the settings made for these is considered. The values of $\overline{j_n\lambda^2}$ for aluminum and steel are in about the same ratio, 3:2, as found by Skinner for hydrogen—both being about twice as large in oxygen as in hydrogen. Zinc gives about the same value as steel. As with the cathode fall the number of mean free paths to the negative glow differs but little with the different metals. As should be expected, also, the metals are in the same sequence here as with the cathode fall, the metal having the highest fall giving the greatest distance to the negative glow. These results show that the same general relation holds with oxygen as

TABLE III.

Zinc Cathode in Oxygen. Area of Cathode: 1.34 sq. cm.

Gas Pressure (mm.).	Normal Cathode Fall, (Volts).	Normal Current Density, j_n.	Distance to Neg. Glow, $x_k - x_0$.	Mean Free Path, λ.	$\overline{j_n\lambda^2}$	$\frac{x_k - x_0}{\lambda}$
1.31	353	.46 ma.	3.90 mm.	.33 mm.	.050	11.8
1.45	354	.67	3.14	.30	.060	10.3
1.57	353	.66	——	.28	.052	——
2.13	356	1.19	——	.21	.053	——
2.35	354	1.49	2.31	.19	.054	12.1
2.38	354	1.64	2.05	.18	.052	11.7
2.42	354	1.64	2.19	.18	.052	12.2
2.63	354	1.94	1.96	.17	.055	11.7
2.73	354	2.16	1.75	.16	.056	10.6
2.94	354	2.62	1.66	.15	.058	11.3
3.13	355	2.80	1.54	.14	.055	10.7
3.32	355	3.54	1.38	.13	.059	10.8
3.60	354	4.11	1.32	.12	.059	10.8
3.62	356	3.96	1.33	.12	.056	10.8
3.79	356	4.63	1.38	.115	.061	12.2
3.91	356	4.40	1.34	.11	.053	11.8
3.96	355	4.77	1.20	.11	.058	10.9
4.24	352	5.15	1.18	.10	.052	12.0
4.28	355	5.22	1.22	.10	.052	12.0
4.46	352	6.05	1.11	.10	.058	11.2
4.52	356	5.90	——	.095	.055	——
4.78	354	6.71	1.04	.090	.056	10.9
Mean	354			Mean	.055	11.4
Max. dev.	.6%			Max. dev.	11%	11%
Mean dev.	.3%			Mean dev.	5%	5%

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[1] C. A. Skinner, Phil. Mag., 6, VIII., p. 387.

found by Skinner with hydrogen, namely:—*all gas pressures giving the normal cathode fall have the same number of mean free paths to the point of minimum potential gradient in the negative glow, and, have a normal current density proportional to the square of the gas pressure, i. e., inversely proportional to the square of the mean free path of the electron.*

Potential Curves in the Region between the Cathode and the Negative Glow.—Skinner found with hydrogen that the difference of potential between the cathode and any point at the same number of mean free paths from it is practically the same at various gas pressures, if the current density is the same multiple of the normal current density. The following gives the results of a similar investigation for oxygen.

For obtaining these potential curves an aluminum cathode was used with normal, twice normal and four-normal current densities. The same precautions were observed here as in the other tests, the probe sheath being renewed more often, since any leakage with it in the less conductive space near the cathode would cause greater error than with it in the highly conductive negative glow.

TABLE IV.

Potential Difference in Volts between the Cathode and Points in the Gas at a given Number of Mean Free Paths from it. Aluminum Cathode in Oxygen. For Normal Current Density: $j\bar{\lambda}^2 = .081$

Gas Pressure (mm.).	Number of Mean Free Paths.						
	2	3	5	7	9	10	11
1.34	198	230	272	300	310	312	312
1.39	205	236	278	305	312	312	313
1.49	194	225	272	300	310	312	313
1.56	205	238	279	301	306	306	306
1.92	211	246	290	311	315	315	314
2.02	210	245	292	312	316	316	316
2.04	205	231	268	296	307	308	308
2.24	211	243	283	306	315	318	318
2.27	210	238	270	304	309	312	313
2.28	228	253	282	302	312	313	313
2.45	220	248	282	302	314	315	315
2.50	213	242	282	302	307	307	307
2.54	220	250	282	302	312	314	314
2.64	203	232	272	292	311	313	313
2.67	226	250	278	298	311	313	313
2.72	216	249	282	301	306	306	306
2.82	222	254	286	306	314	315	315
3.04	223	256	287	306	314	316	316
3.08	236	264	290	308	312	312	312
3.08	223	254	287	306	313	315	315
Means:	214	244	281	303	311	312	312

The procedure was to start with a fresh gas filling, and with the probe first in the negative glow, then at positions successively nearer the cathode, the difference of potential between it and the cathode was observed—all with a constant current maintained through the gas. These observations were taken as rapidly as possible to reduce the magnitude of the time changes—indicated by repeating the first observation at the close of each series. In the results given—incorporated in Tables IV., V. and VI.—the time change in the cathode fall was in no case greater than four per cent.—always an increase and greater for the larger current. An inspection of these tables warrants the conclusion that the potentials

Table V.

Potential Difference in Volts between the Cathode and Points in the Gas at a given Number of Mean Free Paths from it. Aluminum Cathode in Oxygen. For Twice Normal Current Density: $j\lambda^2 = .162$

Gas Pressure (mm.).	Number of Mean Free Paths.					
	2	3	5	6	7	8
1.52	256	292	322	324	325	325
1.84	240	276	318	323	323	323
2.06	252	286	320	325	325	325
2.25	256	287	317	322	323	323
2.26	256	287	318	—	326	326
2.35	260	292	320	324	326	326
2.39	262	290	320	324	325	324
2.46	260	291	320	324	324	324
2.48	264	294	320	326	327	326
2.94	253	288	320	325	327	327
3.02	258	286	320	325	326	327
3.05	259	291	320	326	328	328
Means:	259	288	320	324	325	325

Table VI.

Potential Difference in Volts between the Cathode and Points in the Gas at a given Number of Mean Free Paths from it. Aluminum Cathode in Oxygen. For Four Times Normal Current Density: $j\lambda^2 = .324$

Gas Pressure (mm.).	Number of Mean Free Paths.				
	2	3	4	5	6
1.43	306	338	346	348	348
1.97	302	338	351	356	356
2.07	300	345	359	367	366
2.23	312	343	352	361	362
2.24	312	348	360	361	361
Means:	306	342	353	359	359

are practically independent of the gas pressure; hence their means are taken as representative. These means are plotted in Fig. 2 (circles) using the potential differences as ordinates and the number of mean free paths from the cathode as abscissæ. They show, as in hydrogen, a marked decrease in distance to the point of minimum gradient (at x_0, see Fig. 2) as the current density is increased.

Mobility of the Positive Ions.—To the above data we may apply the theoretical deductions for calculating the upper and lower limits for the mobility of the positive ions. This is defined as the velocity in centimeters per second of the ion in unit field. Skinner gives[1] the two inequalities:

$$V_0 - V < \left[\left(\frac{16\pi j}{9\overset{+}{\mu}}\right)^{\frac{1}{2}} (x - x_0)\right]^{\frac{2}{3}}$$

$$> \left[\left(\frac{16\pi j}{9\overset{+}{\mu}}\right)^{\frac{1}{2}} (x - x_0) - \epsilon^{\frac{3}{2}}\right]^{\frac{2}{3}} + \epsilon$$

where $V_0 - V$ is the difference in potential between the point of minimum gradient at x_0 in the negative glow and any other point x nearer the cathode; j the current density, $\overset{+}{\mu}$ the mobility of the positive ions and ϵ the ionizing potential of the gas all expressed in c.g.s.-e.s. units. The upper and lower limits of $\overset{+}{\mu}$ may be calculated by making equations of the above inequalities and by substituting the observed values in their proper units. For the first we write:

$$V_0 - V_1 = \left(\frac{16\pi j}{9\overset{+}{\mu}}\right)^{\frac{1}{3}} (x_1 - x_0)^{\frac{2}{3}} \qquad (1)$$

The second may be transformed as follows:

$$V_0 - V_1 - \epsilon = \left[\left(\frac{16\pi j}{9\overset{+}{\mu}}\right)^{\frac{1}{2}} (x_1 - x_0) - \epsilon^{\frac{3}{2}}\right]^{\frac{2}{3}}$$

$$(V_0 - V_1 - \epsilon)^{\frac{3}{2}} + \epsilon^{\frac{3}{2}} = \left(\frac{16\pi j}{9\overset{+}{\mu}}\right)^{\frac{1}{2}} (x_1 - x_0) \qquad (2)$$

$$[(V_0 - V_1 - \epsilon)^{\frac{3}{2}} + \epsilon^{\frac{3}{2}}]^2 = \left(\frac{16\pi j}{9\overset{+}{\mu}}\right) (x_1 - x_0)^2.$$

In the plot (Fig. 2) are indicated the positions of minimum gradient x_0. These are 9.5 mean free paths for "normal"; 6.6 for "twice normal" and 5.0 for "four-normal." The corresponding values of the potential; V_0 were taken from Tables IV., V. and VI. The position x_1 was chosen at two mean free paths from the cathode which also determines V_1 from the tables. The value of ϵ for oxygen is given by Franck and

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[1] *L. c.*, June, p. 488.

Hertz[1] as 9 volts. Expressing these quantities and also the current in c.g.s.-e.s. units and substituting in (1) and (2) we have calculated the upper and lower values of $\overset{+}{\mu}$. They do not differ greatly. Their mean for a pressure of 1 mm. was found to be 1.47×10^6 cm. per sec. for a field of 1 c.g.s.-e.s. unit or, in a field of one volt per cm. 4.9×10^3 cm. per sec.—about five times the value customarily obtained. For hydrogen Skinner obtained 2.7×10^4 cm. per sec. for one volt per cm.—about six times the accepted magnitude under ordinary measurements.

Using the above mean for the value of $\overset{+}{\mu}$, the same values for x_0, and the potentials at two mean free paths from the cathode as a starting point, we have determined from (1) and (2) the theoretical curves between which the observed values should fall. The calculated and observed potentials are tabulated in Table VII.—starting points in italics. The corresponding curves with their extensions to the cathode in broken lines are shown in Fig. 2. Referring to this figure, we find the observed

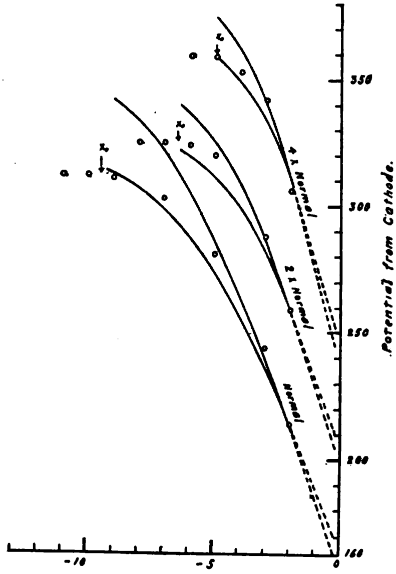

Fig. 2.

TABLE VII.

Calculated and Observed Potential Differences between Cathode and Points in the Gas at a given Number of Mean Free Paths from it. Aluminum in Oxygen.

Number of Mean Free Paths.	Calculated from Equation 1.	Calculated from Equation 2.	Observed.
For Normal Current Density: ($j = 1.33 \times 10^6$ c.g.s.-e.s.)			
0.2	163	170	—
2.	*214*	*214*	*214*
3.	240	236	244
4.	264	256	—
5.	285	274	281
6.	305	287	—
7.	322	299	303
9.3	344	314	312

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[1] J. Franck and G. Hertz, Ber. d. Deutsch. Phys. Ges., 2, 1913, p. 34.

TABLE VII.

Calculated and Observed Potential Differences between Cathode and Points in the Gas at a given Number of Mean Free Paths from it. Aluminum in Oxygen.

Number of mean Free Paths.	Calculated from Equation 1.	Calculated from Equation 2.	Observed.
For Twice Normal Current Density: ($j = 2.66 \times 10^6$ c.g.s.-e.s.)			
0.2	205	210	—
2.	*259*	*259*	*259*
3.	285	281	288
4.	308	299	—
5.	326	—	320
6.4	340	322	325
For Four Times Normal Current Density: ($j = 5.32 \times 10^6$ c.g.s.-e.s.)			
0.2	245	252	—
2.	*310*	*310*	306
3.	339	334	342
4.	362	349	353
5.	374	359	359

values falling fairly consistently between the theoretical limits; but with a gradient (or slope) near the negative glow distinctly lower than either.

The Immediate Cathode Fall.—From the fact that the difference of potential between the cathode and any definite number of mean free paths from it is simply a function of $j\bar{\lambda}^2$ it must follow that the difference of potential between the cathode and the gas immediately adjacent to it, is also a function of $j\bar{\lambda}^2$. The theory assumes this fall to take place over a distance equal to one mean free path of the positive ions from the cathode—approximately .2 mean free path of the electrons. By extending the theoretical curves up to this point, a calculated value for this " immediate fall " at the cathode may be obtained. The following table gives the mean values V_b of this immediate fall. They show that V_b

V_b	Ratios.	$(j\bar{\lambda}^2)^{.30}$	Ratios.
167	1.	.470	1.
208	1.24	.592	1.26
249	1.49	.713	1.52

is closely proportional to $(j\bar{\lambda}^2)^{.30}$. Skinner found with hydrogen that V_b is proportional to $(j\bar{\lambda}^2)^{.38}$.

The Electron Current from the Cathode.—For obtaining the ratio of the electron current to the total current, Skinner has derived the following equation.[1]

$$\frac{\bar{j}_k}{j} = \frac{\bar{\mu}}{\bar{\mu} + \overset{+}{\mu}} \cdot 2^{-\left[\frac{x_k - x_a}{\bar{\lambda}} + \frac{V_0 - V_a}{\epsilon}\right]} \tag{3}$$

in which $\bar{\mu}$ and $\overset{+}{\mu}$ are respectively the mobilities of the negative and positive ion in the negative glow; $\bar{j}_k$ the electron current density at the cathode; j the total current density; $(x_k - x_a/\bar{\lambda})$ the number of mean free paths between the cathode and the point where ionization ceases to take place at every mean free path: $V_0 - V_a$ the fall between this point and the negative glow; and ϵ the ionizing potential of the gas. This relation was obtained by assuming that each electron leaving the cathode resulted in two electrons at the end of the first mean free path, four at the end of the second, eight at the end of the third, and so on for the region in which the potential difference per mean free path was equal to or greater than the ionizing potential; and, in the region where the potential gradient was lower than this, ionization was assumed to occur each time an electron moved through a potential difference equal to ϵ. Assuming $\bar{\mu}$ and $\overset{+}{\mu}$ equal, the ratios of $\bar{j}_k/j$ for the three current densities are as follows:

Current Density.	$\left(\frac{x_k - x_a}{\bar{\lambda}}\right)$	$\left(\frac{V_0 - V_a}{\epsilon}\right)$	$\left(\frac{\bar{j}_k}{j}\right)$
Normal..........	7	1.0	2.0×10^{-3}
2×normal.......	5	.6	$10. \times 10^{-3}$
4×normal.......	4	.7	$19. \times 10^{-3}$

Observations on the extent of the cathode dark space in oxygen show that there is no perceptible change with large changes in current density.

Nitrogen.

Atmospheric nitrogen was obtained by passing air slowly over heated phosphorous. The gas was then allowed to stand over crushed potassium hydrate and phosphorous pentoxide to remove the carbon dioxide and moisture. Finally the last traces of oxygen were removed by heating the gas for a time in a tube coated on the inner surface with an electrolytic deposit of sodium which had been previously accumulated by sending an electric current from a sodium amalgam bath through the heated walls of the tube.[2] It is very important that no oxygen be present for as Warburg as shown and I also have observed, even a trace will vitiate results by giving an abnormally high cathode fall.

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[1] *L. c.*, August, p. 163.

[2] E. Warburg, Wied. Ann., 40, 1890, p. 1.

Current Density and Number of Mean Free Paths to the Negative Glow with Normal Cathode Fall.—A series of observations with an aluminum cathode in Nitrogen is given in Table VIII. As in the case of oxygen, each pressure represents a fresh gas filling. There was considerable difficulty in obtaining satisfactory observations in nitrogen, because with each new gas filling, the cathode fall was usually distinctly larger than with the preceding filling. At most, only three fillings could be used before it became necessary to remove the electrode and repolish it. Unlike the other gases the cathode fall showed, however, a decrease with duration of the discharge in the same filling—especially noticeable if the electrode had not been previously cleaned. This was attributed to the presence of freed hydrogen. The mean, 218 volts, of the values given in Table VIII., column 2 is about five per cent. higher than the fall as given by Warburg, 207 volts.[1] It agrees closely, however, with Skinner's value, 217 volts.[1] There was some hesitation in the glow covering the cathode when the current was first started. The normal current densities given in column 3, represent the minimum which would cover the cathode after the discharge had first been forced over the whole face. This minimum sometimes showed a tendency to rise, depending on how much the gas

Table VIII.

Aluminum Cathode in Nitrogen. Area of Cathode: 3.04 sq. cm.

Gas Pressure (mm.).	Normal Cathode Fall (Volts).	Normal Current Density, j_n.	Distance to Neg. Glow, $x_k - x_0$.	Mean Free Path, $\bar{\lambda}$.	$j_n\bar{\lambda}^2$	$\frac{x_k - x_0}{\bar{\lambda}}$
1.43	216	.29 ma.	3.30 mm.	.28 mm.	.023	11.8
1.78	——	.39	2.90	.23	.021	11.0
1.81	213	——	2.54	.225	——	11.1
2.30	220	.72	2.32	.18	.023	12.8
2.32	219	.76	2.08	.175	.023	12.0
2.41	219	.76	2.10	.17	.022	12.3
2.53	220	.92	1.80	.16	.024	11.2
2.69	214	.99	1.90	.15	.022	12.6
2.91	218	1.08	1.70	.14	.021	12.1
2.96	220	1.12	1.85	.14	.022	12.8
3.30	216	1.45	1.54	.12	.021	12.5
3.33	212	1.58	——	.12	.023	——
3.59	222	1.81	1.47	.115	.024	——
4.07	218	——	1.22	.10	——	12.0
4.49	223	2.46	1.15	.091	.021	12.1
Mean	218			Mean	.0223	12.0
Max. dev.	3%			Max. dev.	9%	8%
Mean dev.	1%			Mean dev.	5%	4%

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[1] C. A. Skinner, Phil. Mag., 6, VIII., p. 387.

pressure had been increased by the gases freed from the metal. This pressure increase during a set of readings was never more than four per cent. The normal current density for any given gas pressure of nitrogen is about twice that for hydrogen and about one third that for oxygen; $j_n\bar{\lambda}^2$ (column 6) is approximately one half that in hydrogen and one fourth that in oxygen. The number of mean free paths from the cathode to the negative glow (column 7) lies between the values for hydrogen and oxygen.

Potential Curves in the Region between the Cathode and the Negative Glow.—Table IX. gives observations on the differences of potential between the cathode and successive points in the gas, using an aluminum cathode in nitrogen with the three current densities " normal," " twice normal ", and " four-normal." Each series, except the last (at " four-

Table IX.

Potential Difference in Volts between the Cathode and Points in the Gas at a given Number of Mean Free Paths from it. Aluminum in Nitrogen.

Gas Pressure (mm.).	Number of Mean Free Paths.							
	2	3	5	7	9	10	11	12
For Normal Current Density: $j\bar{\lambda}^2 = .022$								
1.38	129	151	187	213	221	224	226	226
2.06	117	135	167	195	213	218	222	223
2.17	132	148	178	199	211	215	216	217
2.57	130	151	178	196	206	209	210	211
2.63	131	149	181	207	222	226	229	231
2.65	129	147	177	196	209	212	213	215
2.92	119	136	164	191	210	217	221	222
3.86	130	148	175	195	210	216	220	223
Means	127	146	176	199	213	217	220	221

Gas Pressure (mm.).	Number of Mean Free Paths.					
	2	4	6	7	8	9
For Twice Normal Current Density: $j\bar{\lambda}^2 = .044$						
2.22	155	194	219	229	235	236
2.23	148	184	211	220	224	224
2.24	142	180	204	212	217	218
2.55	156	195	222	229	234	234
2.70	153	190	214	223	228	229
Means	151	189	214	223	228	228
For Four Normal Current Density: $j\bar{\lambda}^2 = .088$						
2.46	173	214	238	242	242	—

normal ") was taken in a fresh gas filling. The large increase in the cathode fall with successive fillings, however, made it impossible to obtain comparative results with four-normal current density with fresh gas fillings as has been the plan followed with other gases. Comparing the different gases, it is found that oxygen and hydrogen both show a greater relative rise in the cathode fall with the same multiple of normal current density. Here as in the other gases, potentials are independent of the gas pressures, and the point of minimum gradient draws in toward the cathode as the current density is increased. The means are plotted as circles in Fig. 3.

Some years ago Skinner found[1] that after passing a heavy current through nitrogen the potential difference between cathode and probe was practically the same for all positions of the probe between the cathode and the negative glow. My observations have shown, however, in all cases a marked gradient in this region. It is very probable that the heavy preliminary current used by Skinner simply served to render (by cathode spray) the insulating sheath of his probe conductive, the potentials indicated being then practically those of the highly conductive negative glow through which the probe passed.

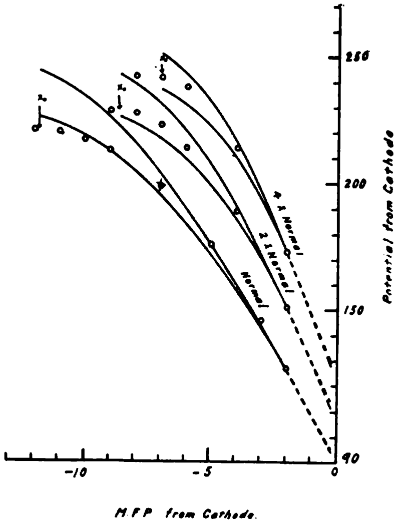

Fig. 3.

Mobility of the Positive Ions.—The mobility $\overset{+}{\mu}$ for the positive ions was calculated from equations (1) and (2) by the same method as used for oxygen. A mean value of 3.1×10^3 cm. per sec. in a field of one volt per cm. was thus obtained. It is rather surprising to find this only about six-tenths that of oxygen—when ordinary measurements show them to be about the same. Using this value, the theoretical curves shown in Fig. 3 were computed. The results are incorporated in Table X.

The Immediate Cathode Fall.—This obtained in the same way as for oxygen (at .2 mean free path from the cathode) is proportional to$(j\lambda^2)^{.25}$ approximately; as appears from the table on page 267.

Magnitude of the Electron Current from the Cathode.—The ionizing potential for nitrogen was used as 7.5 volts.[2] Substituting in equation

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[1] C. A. Skinner, Phil. Mag., [6], 2, 1901, p. 616.

[2] J. Franck and G. Hertz, Ber. d. Deutsch. Phys. Ges., 2, 1913, p. 34.

Table X.

Calculated and Observed Potential Differences between Cathode and Points in the Gas at a Given Number of Mean Free Paths from it. Aluminum in Nitrogen.

Number of Mean Free Paths.	Calculated from Equation 1.	Calculated from Equation 2.	Observed.
For Normal Current Density: (j = 4.02 × 10^5 c.g.s.-e.s.)			
0.2	94	95	—
2.	*127*	*127*	*127*
3.	144	143	146
5.	177	172	176
7.	204	196	199
9.	226	213	213
10.	235	218	217
11.8	244	226	220
For Twice Normal Current Density: (j = 8.04 × 10^5 c.g.s.-e.s.)			
0.2	111	114	—
2.	*151*	*151*	*151*
4.	190	185	189
6.	220	210	214
7.	232	217	223
8.6	243	226	228
For Four Times Normal Current Density: (j = 12.06 × 10^5 c.g.s.-e.s.)			
0.2	128	129	—
2.	*173*	*173*	*173*
4.	215	210	214
5.	232	221	230
6.	244	230	238
7.	251	237	242

V_b	Ratios.	$(j\bar{\lambda}^2)^{.25}$	Ratios.
94.5	1.	.387	1.
112.5	1.19	.460	1.19
128.5	1.36	.546	1.41

(3), as before, to obtain the ratio of the electron current at cathode to the total current the results are as follows:

Current Density.	$\left(\frac{x_k-x_a}{\bar{\lambda}}\right)$	$\left(\frac{V_0-V_a}{e}\right)$	$\left(\frac{\bar{j}_k}{j}\right)$
"Normal"......	7	2.8	6.×10^{-4}
2 "	7	.7	24.×10^{-4}
4 "	6	.5	55.×10^{-4}

These values are somewhat higher than those obtained by Skinner for hydrogen and very much less than those for oxygen.

Observations on the extent of the cathode dark space in nitrogen showed a marked decrease with increasing current density. For example if it extends 6 mean free paths from the cathode at normal current density, it decreases to about 4.5 mean free paths at four times normal current density.

SUMMARY.

To facilitate comparison the various magnitudes measured by Skinner in hydrogen with an aluminum cathode and those obtained in nitrogen and oxygen with the same metal are summarized in the following table:

Gas.	Cathode Fall.	$(\bar{j}\lambda^2)$	$\left(\frac{x_k - x_0}{\bar{\lambda}}\right)$	Immediate Cathode Fall.	Mobility of Positive Ions, at 1 mm. Gas Pres.	$\left(\frac{\bar{j}_k}{j}\right)$
			For Normal Current Density.			
H	197	.038	15.3	78.	27.0×10^3	$4. \times 10^{-4}$
N	218	.022	12.0	95.	3.1×10^3	$6. \times 10^{-4}$
O	311	.081	10.6	168.	4.9×10^3	$20. \times 10^{-4}$
			For Twice Normal Current Density.			
H	204	.076	12.	101.	———	$15. \times 10^{-4}$
N	228	.044	8.	112.5	———	$24. \times 10^{-4}$
O	325	.162	7.	207.5	———	$100. \times 10^{-4}$
			For Four Times Normal Current Density.			
H	227	.152	10.	130.	———	$42. \times 10^{-4}$
N	242	.088	7.	128.5	———	$55. \times 10^{-4}$
O	359	.324	5.	248.5	———	$190. \times 10^{-4}$

To conclude it is found for each of the different gases, hydrogen, nitrogen and oxygen, that all pressures giving the normal cathode fall have the same number of mean free paths to the point of minimum potential gradient in the negative glow, and have a normal current density proportional to the square of the gas pressure.

With normal current density the cathode fall is a minimum. As the current density is increased the immediate cathode fall also increases, but the fall over the remaining distance to the negative glow decreases. This decrease in the gas is due to the higher ratio of the electron current from the cathode to the total current, which reduces the distance from the cathode to where the negative charge in the gas equals the positive.

Finally, the difference in potential between the cathode and any point at a definite number of mean free paths from it is a function of the product of the current density into the square of the mean free path of the electron, *i. e.*, a function of the ratio of the current density to the square of the gas pressure.

I wish to express my indebtedness to Prof. Skinner, who suggested the problem, and under whose direction it has been my privilege to work, and also to Dr. J. T. Tate for valuable assistance in the experimental part.

THE BRACE LABORATORY OF PHYSICS,
UNIVERSITY OF NEBRASKA,
June, 1915.

TESTS OF THERMO-ELECTRIC FORMULÆ BASED ON BISMUTH AND BISMUTH-TIN ALLOYS.

By A. E. Caswell.

Introduction.

IN a previous paper[1] the writer pointed out the fact that the development of the electron theory made trustworthy determinations of the Peltier E.M.F. desirable. The experimental work outlined in this paper is a continuation of the previous work, but the present apparatus was designed to measure not only the Peltier E.M.F., but the Thomson E.M.F. and thermo E.M.F. as well. The metals used were bismuth and alloys of bismuth and tin containing less than ten per cent. tin. The latter part of this paper is devoted to a study of the electron theory of thermo-electricity, as developed by different individuals, in the light of experimental facts, from which it appears (1) that the formulæ proposed by Drude[2] are verified by experiment, and (2) that the electrons which take part in thermo-electric phenomena are identical with those which take part in the conduction of electricity and heat in metals.

The Peltier Effect.

The method here employed in the measurement of the Peltier E.M.F. was that devised by Barker and myself and referred to in my previous paper as the "First Method." The apparatus was modified, however, so as to adopt it to the measurement of the Thomson E.M.F. Since it is proposed to deal with the latter subject in a future paper, a description of the apparatus will not be given here further than to point out minor modifications tending to increase the accuracy of the work and to facilitate the operations. For a more detailed description of the apparatus the reader is referred to my previous paper.

The Dewar flasks, used as calorimeters, were mounted in a wooden frame, the top of which was flush with the tops of the flasks. A thick layer of felt was placed around the upper end of each flask between the flask and the frame so that it extended slightly above the top of the flask. In order to adapt the apparatus to the measurement of the Thomson E.M.F. two couples, connected in parallel, were used instead of one.

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[1] Phys. Rev., Ser. I., XXXIII., pp. 379–402, Nov., 1911.

[2] Drude, Ann. der Phys., 1, p. 590, 1900.

Each couple consisted of an inverted U-shaped casting of bismuth or one of the alloys, the sides and bottom of the U being about 19 cm. long, the cross-section about 20 sq. mm., connected to heavy copper leads by means of spring binding-posts. These facilitated the substitution of one specimen for another. The compensating resistance coils were single loops of No. 38 manganin wire about 7 cm. long, and having resistances of 3.529 and 3.655 ohms, respectively. These were soldered to heavy copper leads. The rotary stirrers previously used were replaced by ones having a vertical reciprocating motion. These have proven quite as satisfactory and have saved considerable time in the way of adjustment. Everything which went inside of the flasks was securely fastened to a board, the under side of which was completely covered with a heavy layer of felt. The vertical portions of the castings and the stirrer rods passed through holes in the board and a second layer of felt on the upper side of the board. A third layer of felt covered the horizontal portions of the castings, the only openings in this layer being for the stirrer rods. When this board was put in place and bolted to the frame the felt provided complete heat insulation except at the stirrer rods, which rods passed through snugly fitting holes in three layers of felt, each of which was about one cm. thick.

The ammeters used were carefully calibrated. The bismuth was C. A. F. Kahlbaum's reagent bismuth and the tin Kahlbaum's purified tin. These were fused together in an electric furnace and well stirred at a temperature not far above the melting-point of bismuth, care being taken to prevent the alloy becoming contaminated with an oxide of the metals. The first and second columns of Table I. give the percentages of tin, by weight, in three alloys and the corresponding Peltier E.M.F.'s between the alloys and copper, also the Peltier E.M.F. between the pure bismuth and copper. The latter value is somewhat greater than that given in my previous paper. The difference is probably accounted for by a slight impurity in the sample of bismuth previously used. The value given in the table is in each case the mean of several determinations, and the probable error varies between one and two per cent. The mean temperature of these experiments was 20.7° C. or 293.5° K.

In the third column of this table the values of the ratio, n_1/n_2, of the number of electrons taking part in thermo-electric phenomena in the case of an alloy to the corresponding number in the case of pure bismuth are given, these values being computed from Drude's formula. In the fourth column the values of the same ratio as computed from Thomson's formula are given.

THERMO E.M.F.

The thermo E.M.F. between these specimens and copper was determined in each case by heating the liquid in one of the calorimeters to a temperature about ten degrees above the other, which was kept at the temperature at which the Peltier experiments were carried out. The ratio of the thermo E.M.F. of the copper-specimen couple to that of the four copper-constantan couples in series, which were used to determine the equality of the temperatures in the two calorimeters during the Peltier experiments, was obtained with the aid of a potentiometer. The thermo-electric power of the copper-constantan couples was then determined absolutely, the value found agreeing within the limits of experimental error with that given in Kaye and Laby's tables, viz., 41.8 microvolts per Centigrade degree. The values of the thermo-electric power given in the fifth column of Table I. were obtained by multiplying this number by four times the ratio mentioned above.

The sixth and seventh columns of Table I. correspond, respectively, to the third and fourth columns, the equations used in these cases being those for thermo-electric power instead of Peltier E.M.F. The close agreement between the numbers in the third and sixth columns and between those in the fourth and seventh columns of this table depends upon the use of Lord Kelvin's equation $P = QT$, where P is the Peltier E.M.F., Q is the thermo-electric power, and T is the absolute temperature. In a way this relation serves to check the accuracy of the experimental work, provided we are willing to assume the validity of the second law of thermodynamics. If we are in doubt on this point the experimental facts should convince us of the validity of the law. I mention this point because of two facts. The first is that the validity of the law has been called in question recently. The second is that if this law is true, and if the quantities which are actually measured are those referred to in this equation based on the law, then it would seem unnecessary to carry out experiments on both Peltier E.M.F. and thermo E.M.F. since the second set of experiments would add no new information. Both of these conditions, I believe, are fulfilled.

TABLE I.

Per Cent. Tin.	Peltier E. M. F. in Millivolts.	n_1/n_2 Drude.	n_1/n_2 Thomson.	Thermo-electric Power in Microvolts per ° C.	n_1/n_2 Drude.	n_1/n_2 Thomson.
0	+19.35	1.000	1.000	+67.2	1.000	1.000
3.75	−10.87	0.550	0.303	−35.4	0.552	0.304
6.36	−11.67	0.542	0.293	−42.7	0.528	0.279
9.93	−10.45	0.555	0.308	−38.0	0.543	0.295

Curve I. in Fig. 1 is plotted from the data in the first and third columns of this table, curve II. from the data in the first and sixth columns.

It will be observed that the density of the electrons in copper lies between the density in bismuth and that in any of the alloys experimented upon.

THEORETICAL CONSIDERATIONS.

According to the electron theory as developed by Drude the Peltier E.M.F. between two metals should be given by the equation

$$P = \frac{4\alpha T}{3e} \log \left(\frac{n_1}{n_2}\right),$$

where P is the Peltier E.M.F., α is a universal constant, the ratio of the kinetic energy of a molecule to the corresponding absolute temperature T, e is the charge on an electron, n_1 is the number of electrons which take part in thermo-electric phenomena per unit volume of the first metal, and n_2 is the corresponding number for the second metal. If the current flows from the metal in which n is large to that in which n is small, then heat will be absorbed at the junction. The equation given above is equivalent to the equation.

$$P = \frac{2RT}{e} \log \left(\frac{n_1}{n_2}\right),$$

where R is the gas constant for a gram-molecule of a gas and is equal to 8.315 joules per Centigrade degree, and e is the electrochemical equivalent of hydrogen and is equal to 96,470 coulombs per gram.

Similarly, if Q is the thermo-electric power, then

$$Q = \frac{2R}{e} \log \left(\frac{n_1}{n_2}\right),$$

in conformity with the relation $P = QT$.

J. J. Thomson[1] and O. W. Richardson[2] have deduced formulæ identical with the above providing the factor 2 is omitted. Drude's calculation is based upon the assumption of a convective equilibrium between the diffusion of the electrons due to the difference in concentration and the opposing flow of electrons due to a difference in potential thus set up. Thomson assumes that the electrons are in equilibrium under the electric force and the opposing pressure gradient at the boundary. Richardson assumes that the electric current in metals is carried by electrons which are dynamically equivalent to the molecules of a gas, and that the potential energy of an electron is less when inside an uncharged conductor than when it is at a considerable distance outside. This difference in potential energy is due to a force at the interface which tends to hold the electrons in the conductor. The final test of the correctness of any of these assumptions is an experimental one.

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[1] "Corpuscular Theory of Matter," p. 74.

[2] Phil. Mag., [6], Vol. XXIII., p. 269, 1912.

At the beginning of this paper it was stated that the experimental facts seemed to agree with the formulæ of Drude rather than those of Thomson and Richardson. If such is the case, it appears that Drude's position is the correct one. It remains to justify that statement. It is obviously necessary to obtain from other sources reliable values of the ratio n_1/n_2. Such values can be obtained from the work of Schulze[1] on the electrical and thermal conductivities of bismuth and its alloys, providing we make two assumptions. The first is that the mean free path of the electrons, l, has practically the same value in a pure metal and in an alloy of that metal containing only a very small percentage of a second metal. The second assumption is that the electrons which take part in thermo-electric phenomena are identical with those which take part in the conduction of electricity and heat in metals. In most cases the assumption of the equality of the mean free paths of the electrons in two different metals would be unwarranted. In the case of a bismuth-tin alloy containing only a small percentage of tin, the density of the alloy not being very far different from that of pure bismuth, the average spacing of the molecules cannot be very far different from that in the pure metal, even though the number of molecules of tin present may suffice to destroy the crystalline structure characteristic of bismuth. The case is very similar to that of mixing a quantity of shot of one size with a very much larger quantity of shot of a different size. Neither the number of shot in a unit volume of the mixture, nor the average distance between centers, will differ to any marked extent from the corresponding values for the larger quantity of shot alone. It therefore follows that although we cannot assert the exact equality of l_1 and l_2, where the subscript 1 refers to the alloy and the subscript 2 to the pure bismuth, we feel confident that the assumption of this equality, doubtless slightly contrary to fact, cannot introduce into our calculations an error of more than a very few per cent. This point will be referred to again. It is not obvious that the electrons involved in electrical and thermal conductivity are identical with those involved in thermo-electricity. On the other hand, there is no *a priori* reason for denying such identity. The assumption is a simple one; it does no violence to the experimental facts, but instead serves to link together such facts in different portions

[1] Ann. der Phys., 9, pp. 555–589, 1902.

of the subject of physics; we are, therefore, justified in making it. An alternative would be to assume the validity of one of the proposed formulæ. If we assume the validity of Drude's formula, the calculated values of the ratio n_1/n_2 are such that one is led to the conclusion that the same electrons take part in the phenomena of conduction and thermo-electricity. If the validity of any other formula is assumed, no relation which we might reasonably expect is found.

If we assume that the specific electrical conductivity, σ, is given correctly by the equation

$$\sigma = \frac{ne^2lv}{6RT},$$

where n, e, l, R, and T, have the same meanings as before, and v is the mean square velocity of the electrons, which depends only upon the temperature; then the ratio of the electrical conductivities of two metals at the same temperature is given by the equation

$$\frac{\sigma_1}{\sigma_2} = \frac{n_1l_1}{n_2l_2}. \tag{1}$$

If, in addition, we assume that $l_1 = l_2$, then equation (1) becomes

$$\frac{\sigma_1}{\sigma_2} = \frac{n_1}{n_2}. \tag{2}$$

Similarly, if we assume that the specific thermal conductivity, k, is given correctly by the equation $k = \frac{1}{3}nlvR$, then it follows that

$$\frac{k_1}{k_2} = \frac{n_1}{n_2}, \tag{3}$$

The fifth and sixth columns of Table II. give the values of the ratio n_1/n_2 computed from Schulze's data on electrical and thermal conductivities, using equations (2) and (3), respectively. The fourth column gives the corresponding percentages, by weight, of tin. For the sake of comparison the data given in the first, third and sixth columns of Table I. is repeated in the first three columns of this table. In Fig. 1 the values of the ratio n_1/n_2 are plotted as ordinates and the corresponding percentages of tin as abscissae. The curves I., II., III. and IV are for Peltier E.M.F., thermo-electric power, electrical conductivity and thermal conductivity, respectively.

An examination of the data in Table I., or of the curves in Fig. 1, reveals the fact that the data obtained from the Peltier E.M.F. agree with those obtained from the thermo-electric power within the limits of

TABLE II.

Showing the values of the ratio n_1/n_2 obtained from different sources.

Per Cent. Tin.	Peltier E.M.F.	Thermo-electric Power.	Per Cent. Tin.	Electrical Conductivity.	Thermal Conductivity.
0	1.000	1.000	0	1.000	1.000
—	—	—	0.34	0.717	0.741
—	—	—	0.71	0.610	0.667
3.75	0.550	0.552	2.15	0.543	0.578
6.36	0.542	0.528	7.43	0.588	0.662
9.93	0.555	0.543	—	—	—

probable experimental error. Furthermore, there is substantial agreement between the data obtained from the above-mentioned sources and those obtained from either electrical or thermal conductivity, when the percentage of tin is less than, say, four per cent. My values of the ratio n_1/n_2 for 3.75 per cent. tin lie between the values deduced from the electrical and thermal conductivities. Yet the coincidence of the latter values is fundamental to the Wiedemann-Franz law. With larger percentages of tin curves III. and IV. diverge rather rapidly from curves I. and II. This is what we should expect since the value of l_1 must become greater than l_2 as the percentage of tin is increased. The conductivity of tin is greater than that of bismuth, but the density of the electrons is less in tin than in bismuth. The difference in conductivity must be accounted for by a greater mean free path of the electrons in tin than in bismuth. When the difference between l_1 and l_2 begins to be appreciable the curves III. and IV. begin to diverge from the curves I. and II., and this divergence might be used to determine the relative values of l_1

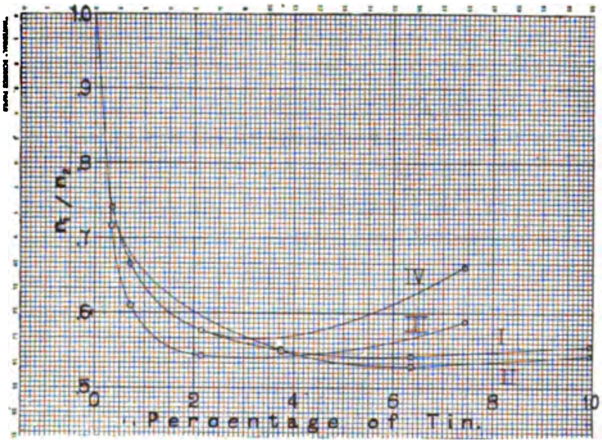

Fig. 1.

and l_2. The writer believes that the logical conclusions to which a study of the experimental facts leads are two, viz., (1) that Drude's thermoelectric formulæ agree with the experimental facts, and (2) that the same electrons take part in thermo-electric phenomena and in the conduction of electricity and heat in metals.

If we adopt the Thomson formulæ, we must deny either one or both of the assumptions made in the preceding discussion. Let us deny the equality of l_1 and l_2, and assume the identity of the electrons. A comparison of the data given in the fifth and sixth columns of Table II. with the data given in the fourth and seventh columns of Table I. shows that $(l_1n_1/l_2n_2)^2$ is approximately equal to N_1/N_2, where n is the number of electrons taking part in conduction and N is the number taking part in thermo-electric phenomena. But if $n = N$, then it follows that $(l_1/l_2)^2$ is approximately equal to n_2/n_1, or N_2/N_1. Since the value of l depends upon collisions with molecules rather than electrons, l should not vary with n except in so far as each varies with the concentration and kind of the molecules. A decrease in the value of n might correspond to a decrease in the concentration of the molecules and a corresponding increase in the mean free path of the electrons. From the values of the ratio N_1/N_2 and the relation stated above, it appears that the mean free path of an electron in one of these alloys would have to be about 1.8 times that in pure bismuth in order to satisfy Thomson's formulæ. Such a difference in the mean free path must correspond to a marked difference in the distance between the molecules and, consequently, to a marked difference in the density. Such a marked difference in density does not exist. If we assume $l_1 = l_2$, Thomson's formulæ lead to the relation

$$\frac{N_1}{N_2} = \left(\frac{n_1}{n_2}\right)^2.$$

This is a somewhat unique and, as it seems to me, an improbable relation. The third alternative is to deny both assumptions, in which case the agreement between Schulze's data and my own becomes merely an amazing coincidence.

The writer is aware that some may object to any line of reasoning based upon a metal such as bismuth which is difficult to obtain in the pure state, and perhaps on that account is abnormal in its behavior, and upon alloys which, generally speaking, have proven rather refractory when efforts have been made to reconcile their behavior with the simple assumptions of the electron theory. In all the work upon which our calculations are based unusual precautions were taken to insure the purity of the metals. And so far as the writer is aware such special assumptions as may be necessary to explain the unusual behavior of alloys, or of pure bismuth, would not affect the conclusions reached in this discussion.

CONCLUSIONS.

The conclusions which may be drawn from this paper are:

1. From the value of the thermo-electric power, or that of the Peltier E.M.F., between two metals and well-known physical constants the ratio of the concentrations of the electrons in the two metals which take part in the phenomena of thermo-electricity can be determined.

2. These electrons are identical with those which take part in the conduction of electricity and heat in metals.

3. The conductivity ratios of two metals are equal to the ratio of the products of the mean free path and the concentration of the electrons.

4. From the ratios mentioned in 1 and 3 it is possible to determine the ratio of the mean free paths of the electrons in the two metals.

This investigation is being continued and its scope extended.

In conclusion the writer wishes to acknowledge his indebtedness to Professor W. P. Boynton, who has been a constant source of inspiration, and to those students in the department who have assisted in the making of the apparatus.

Physical Laboratory,
University of Oregon,
July, 1915.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE COLUMBUS MEETING.

THE eighty-first meeting of the American Physical Society was held at Columbus, Ohio, December 27–30, 1915. It was the annual meeting and a joint meeting with Section B of the A. A. A. S. Six sessions were held for the reading of papers. President Merritt presided, except on Wednesday afternoon, which was devoted to a special program of invited addresses arranged by Section B. Vice-president E. P. Lewis was in charge of this session. At the other five sessions the following sixty-three papers were presented:

A Mechanical Device for the Rapid Evaluation of Certain Variable Exponential Functions. IRWIN G. PRIEST, National Bureau of Standards, Washington.

On the Value of $\gamma = C_p/C_v$ For Hydrogen. KARL K. DARROW.

Deviation of Natural Gas from Boyle's Law. R. F. EARHART and S. S. WYER.

Preliminary Report on the Diffusion of Solids. C. E. VAN OSTRAND AND F. P. DEWEY.

On the Properties of Matter at Low Temperatures. JAKOB KUNZ. (By title.)

Pressures and Critical Lengths in the Collapse of Short Tubes. A. P. CARMAN. (By title.)

A Photographic Study of the Relative Velocity of Sound Waves of Different Intensities. ARTHUR L. FOLEY.

A Preliminary Investigation of an Explosion Wave in a Gas. J. B. DUTCHER.

An Attempt to Detect a Change in the Specific Heat of Selenium with a Change in the Illumination, and also with the Application of an Electric Field. L. P. SIEG.

Wind Velocity and Elevation. W. J. HUMPHREYS.

A Proposed Physical Method for Reducing Radiant Power and its Luminous Value. IRWIN G. PRIEST AND CHAUNCY G. PETERS.

The Coefficient of Total Radiation of a Uniformly Heated Enclosure. W. W. COBLENTZ AND W. B. EMERSON.

A Luminosity Curve Equation and its Application to the Black Body. E. F. KINGSBURY.

Black Body Brightness and the Mechanical Equivalent of Light. HERBERT E. IVES AND E. F. KINGSBURY.

The True Temperature Scale for Tungsten and its Emissive Powers at Incandescent Temperatures. A. G. WORTHING.

Color Temperature Scales for Tungsten and Carbon. EDWARD P. HYDE, F. E. CADY AND W. E. FORSYTHE.

The Infra-red Spectra of the Alkaline Earth Group. H. M. RANDALL.

The Pole Effect in a Calcium Arc. HENRY G. GALE AND WALTER T. WHITNEY.

The Reflecting Power of Alkali Metals in Contact with Glass, as Determined by the Photo-electric Cell. J. B. NATHANSEN.

The Refractive Indices of Hydrogen and Helium on Bohr's Theory. C. DAVISSON.

A Photographic Study of the Diffraction Ring System in the Shadow of a Sphere. MASON E. HUFFORD.

The Ionization Produced by a β-particle. ALOIS F. KOVARIK AND L. W. MCKEEHAN.

The Rôle Played by Gases in Photo-Electric Discharge. R. A. MILLIKAN AND WILMER H. SOUDER.

The Relation between the Pressure Effect and the Light Effect in Selenium Crystals. E. O. DIETERICH.

On the So-called Magnetic Rays of Righi. JAMES E. IVES.

The Absorption Coefficients of various Metals for High Frequency X-Rays. S. J. M. ALLEN.

The Distribution of Energy in the X-Ray Spectrum of Tungsten and Molybdenum at Constant Potential. A. W. HULL.

The Emission Quanta of Characteristic X-Rays. DAVID L. WEBSTER.

The Tribo-luminescence of Manganese-Zinc Compounds. With Demonstration. C. W. WAGGONER.

A New Law relating Ionization Pressure and Current in the Corona of Constant Potentials. EARLE H. WARNER.

Arcs in Gases between Non-consuming Electrodes. G. M. J. MACKEY AND C. V. FERGUSON.

The Hall Effect and Allied Phenomena in Rare Metals and Alloys. ALPHEUS W. SMITH.

Mechanical and Acoustical Impedance, and the Theory of the Phonograph. A. G. WEBSTER.

Further Experiments on the Impedance of Conical Horns. A. G. WEBSTER.

The Effect of Surface Films on Contact E. M. F.'s. R. A. MILLIKAN AND WILMER H. SOUDER.

A Resonance Method for Measuring the Phase Difference of Condensers. H. L. DODGE.

The Magnetic Susceptibility of Oxygen. W. P. ROOP. (Read by E. P. LEWIS.)

An Unrecognized Error in the Measurement of Magnetic Flux. Arthur Whitmore Smith.

Unipolar Induction and Absolute Rotation. E. H. Kennard.

The Electrical Resistance of Vertically Suspended Wires. S. R. Williams.

A Simplified Apparatus for Measuring the Conductivity of Electrolytes. R. P. Hibbard.

Measurements of an Electric Current from its Heating Effect. S. Leroy Brown.

A Study of the Law of Response of the Silicon Detector. Louise S. McDowell and Frances G. Wick.

On the Free Vibrations of a Lecher System—IV. F. C. Blake and Charles Sheard.

Constant High Potential for X-Ray Work. Albert W. Hull.

Carbon Compression Rheostats. E. L. Clark.

On the Theorem that all Action Requires Action at a Distance Somewheres. E. H. Kennard.

The Black Body at the Melting Point of Platinum as a Fixed Point in Photometry. Herbert E. Ives.

The Luminous Efficiency of the Carbon Incandescent Lamp and the Mechanical Equivalent of Light. Herbert E. Ives and E. F. Kingsbury.

The Mobility of Positive Ions. Henry A. Erikson.

The Hall Effect and Allied Phenomena in Tellurium. P. I. Wold.

A Wehnelt Cathode Ray Tube Magnetometer. C. T. Knipp and L. A. Welo.

An Investigation of the Acoustical Properties of the Armory at the University of Illinois. F. R. Watson.

New Electromagnetic Phenomena demonstrated by Floating Sand, Pebbles and other Non-conductors in Acidulated Water. (By title.) J. C. Lincoln.

The Phonotrope, a New Instrument for finding the direction of an Acoustical Ray. A. G. Webster.

The Propagation of Transverse Waves in a Bar. Louis Thompson.

The Exceptions to the Law of Dulong and Petit. (By title.) J. E. Siebel.

Experiments with Slow Positive Rays. A. G. Dempster.

Theory of the Free Vibrations of a Lecher System. F. C. Blake.

Springs of Minimum Weight. (By title.) Henry C. Lord.

Spectra of Some Halogen Compounds and Phenomena Connected Therewith. Charles Sheard and C. S. Morris.

On a New Method of using the Reversible Pendulum for the Determination of "*g*." J. C. Shedd.

The Effect of Absorbed Gases in Photoelectric Emission. Robert J. Piersol. (Read by E. P. Lewis.)

At the special session in charge of Section B on Wednesday afternoon the following program was presented:

The Dependence of Progress in Science upon the Development of Instruments. (Vice-Presidential Address before Section B) Anthony Zeleny.

A General Survey of the Field of High Pressure. P. W. **Bridgman.** Discussion by A. G. Webster.

The Northrup Visible Molecules Apparatus was demonstrated at the close of the session by A. Zeleny.

At a short business session on Wednesday the result of the mail ballot for officers for 1916 was announced as follows: for president R. A. Millikan of Chicago; vice-president, H. A. Bumstead of New Haven; secretary, A. D. Cole of Columbus; treasurer, J. S. Ames of Baltimore; members of council, Irving Langmuir of Schenectady and G. B. Pegram of New York; members of editorial board, A. Trowbridge, E. P. Lewis and W. C. Sabine.

The amendments to the Constitution proposed at the Chicago meeting (designed to incorporate into the Constitution the more important features of the adoption of the Physical Review as the organ of the Society) were adopted. On recommendation of the Council that an effort to secure an endowment for the support of the Physical Review be made, the following motion was adopted: That the American Physical Society declares that the present advance of production in physical research in this country, as well as the actual conditions in Europe, render absolutely necessary the increase of means of publication; that to this end it is essential that an ample endowment be secured and that a committee be appointed by the President to inaugurate an effort to secure such endowment.

It was voted that a committee be appointed to consider how the Society can be made more useful to teachers in colleges and in secondary schools.

The report of the editor of the Physical Review was presented and on motion accepted and placed on file. It showed an excess of receipts over expenditures for the year of $984.28.

The report of the treasurer of the Society was received after adjournment. It shows receipts for the year of $4,295.28 and disbursements $4,248.34. Excess receipts over expenditures for the year $46.94. In bank December 1, 1915, $676.69. Number of members 703. It is probable that both of these reports will be printed in full and sent by mail to all members.

The registration for the meeting was 158. The attendance at every session exceeded one hundred. About one hundred were present at the Physicists' dinner on Wednesday evening. Whether judged by the number and character of the papers presented, by the attendance or by the amount and interest of the discussion following the papers, this meeting was one of the best that the American Physical Society has ever held.

A. D. Cole,
Secretary.

On a Supposed Allotropy of Copper.[1]

By G. K. Burgess and I. N. Kellberg.

MESSRS. E. COHEN and W. D. Helderman conclude from dilatometric observations that copper exists in two allotropic forms with a transition point at about 70° C. Several series of observations by three electrical resistance methods, sensitive to 1 part in 500,000 with copper wire of 0.05 mm. diameter, fail to give any indication of a discontinuity in the range 0 to 100° C. We therefore conclude the system copper is a stable one at ordinary temperatures and are able to find no evidences of allotropy.

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[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, November 26–27, 1915.

NATIONAL BUREAU OF STANDARDS.

SENSITIVE PHOTOELECTRIC CELLS AND A PHOTOELECTRIC RELAY.[1]

BY JAKOB KUNZ AND J. STEBBINS.

A DESCRIPTION of very sensitive photoelectric cells, containing the alkali metals and helium, argon and neon is given. The cells are exhibited at the same time. They are used in stellar photometry. The high sensitiveness of these cells suggested that the construction of a photoelectric relay which may be used for various scientific and technical purposes. In one part of the apparatus the current is of photoelectric origin, in the other the current is due to ionization by collision, which may assume comparatively large values.

1. It is possible to determine the amount of work necessary to the production of ions in alkalivapors by collision.
2. The reversed photoelectric effect can be studied.
3. It can be used for photometric measurements.
4. For the detection of electromagnetic waves.
5. As a thermostat for the regulation of temperature within a very small fraction of a degree.
6. As a telephone relay for weak electric currents.
7. Lecture experiments have been performed by means of this relay, in which a beam of light is made to ring a bell or to light a series of incandescent lamps.

UNIVERSITY OF ILLINOIS.

THE DISTRIBUTION OF ELECTRONS IN A SUBSTANCE THAT CAN EXIST IN BOTH THE CONDUCTING AND THE NON-CONDUCTING STATES.[1]

BY L. E. DODD.

METALLIC selenium in the crystalline form is an electrical conductor. In the amorphous form it is a good insulator. When crystals of the substance are melted, there is, therefore, a change from the conducting to the non-conducting state. By observation as to whether any appreciable quantity of electricity is absorbed or released by selenium during the melting of the crystals, some knowledge can be gained as to the distribution per atom, in both states of the element, of those electrons that have to do with electric conduction in the crystals.

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[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, November 26–27, 1915.

A simple experiment consisted in melting, by a focus of sun's rays, some selenium crystals that rested in a metallic cup attached to a delicate electroscope, previously charged. As the change from the conducting to the non-conducting state proceeded, no appreciable variation in the electroscope deflection was observed, save that which was experimentally accounted for by leak. The same results were observed regardless of the sign of charge on the electroscope. Crystals were also melted, in a similar manner, with the metallic cup connected to a quadrant electrometer. One pair of the quadrants was grounded and the other, connected with the cup, was initially at ground potential. No appreciable charging effect was observed under these circumstances. Obviously, the fact that the crystals were at a potential other than zero in the first case had no effect in preventing a transfer of charge. The same experiments, with electroscope and quadrant electrometer, were repeated, with a use of rays focused from an arc rather than the sun, with the same results.

Thus, there is clearly the experimental conclusion that no appreciable charge is either absorbed or liberated by the selenium during its change from the conducting to the non-conducting state. This means that the number of conducting electrons per atom in a crystal is the same as that of these same electrons, which have become non-conducting, in the amorphous, insulating mass of the melted crystal. The experiment favors the view that the conducting electrons in selenium crystals are not "free," like gas molecules, in which case one might expect their liberation from the substance when the crystal is melted, but are within reach of parent atoms, and thus belong to the atomic structures. One may imagine that, as the crystal melts, the conducting electrons so change their relations in the atomic structures as to become non-conducting.

State University of Iowa.

Talbot's Bands and the Resolving Power of Spectroscopes.[1]

By Thomas E. Doubt.

A BRIEF sketch of the history of Talbot's bands is given with bibliography. Diagrams show arrangements for forming bands with prism, grating, and echelon spectroscopes. Photographs illustrate both high and low resolving power of spectroscopes.

Plane parallel glass plates from 3 mm. to 92 mm. in thickness were used to retard half the beam in the spectroscopes of high resolving power. Instruments from the Bunsen-Kirchoff type to Michelson's 8 in. gratings were tested.

The usual statement in introduction to Talbot's bands is that they are formed "when half the aperture of the pupil is covered with a thin plate of mica or glass," can be generalized and not limited if we have sufficient resolving power.

The fact that 1 mm. of glass produces one interference fringe for about six Ångström units in the region near the sodium lines suggests that we have here an objective standard of resolving power for spectroscopes. A discussion of the usual definition of resolving power and its connection with this objective standard follows. It is pointed out that if we are to understand the phenomena of light, we must coordinate the ether pulse theory of radiation with the undulatory theory of interference.

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[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, November 26–27, 1915.

ARMOUR INSTITUTE OF TECHNOLOGY.

THE MEASUREMENT OF OPTICAL ROTATION IN THE INFRA-RED SPECTRUM.[1]

BY L. R. INGERSOLL.

BY a modification of the method already developed and used by the writer for measuring magnetic rotation of the plane of polarization of infra-red radiation it has been found possible to measure the rotation of sugar solutions and other naturally active liquids over a region of the spectrum extending from the D lines to about wave-length 2.2μ, with an accuracy about that of visual measurements.

Preliminary results for three sugar solutions and two organic liquids show a marked effect of an absorption band beyond 3μ on the rotation dispersion. It is planned to make a comparative study of the natural and magnetic rotation dispersion for a number of substances.

THE UNIVERSITY OF WISCONSIN.

THE CAUSE OF THE VARIATION OF THE EMANATION CONTENT OF SPRING WATER.[1]

BY R. R. RAMSEY.

LAST year at the Chicago meeting of the Physical Society I presented a paper in which I called attention to the fact that the emanation content of Indiana and Ohio springs varied from time to time. During nine months of the past year I have measured the radioactivity of two springs once every week. At the same time the flow of one of the springs was measured. These measurements show that the emanation content of the water increases and decreases as the flow increases and decreases. The flow of all springs in this locality varies with the rainfall. These facts and the fact that some of the highest values of the emanation content of spring water were obtained from " wet weather " springs a short time after very heavy rains, point to the conclusion that the rainwater in percolating through the soil dissolves and carries down with it some of the emanation which is continually moving upwards from the center of the earth to the surface. During dry weather when the flow is not rapid a large per cent. of the emanation which was dissolved in the water is transformed into radium A, B, C, and D, before the water issues from the ground. This conclusion is in accord with the observation of Wright and Smith[2] in which they show that the amount of emanation which issues from the soil is decreased as much as 50 per cent. at times after heavy rains.

INDIANA UNIVERSITY.

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[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, November 26–27, 1915.

[2] PHYS. REV., Vol. 5, p. 459, 1915.

A Theory of the Initial Conditions of the Corona.[1]

By Jakob Kunz.

A DESCRIPTION of the numerous phenomena is given which are connected with the corona discharge. The difference between positive and negative electricity appears in electrical, optical, mechanical and probably chemical effects. The thickness of the luminous layer has been shown to be independent of the radius of the wire. A certain amount of energy will have to be accumulated before the luminous discharge appears; this assumption leads directly to the law that the critical electric force at the surface of the wire is inversely proportional to the square root of the radius of the wire. The constants in this relation are different for the positive and the negative corona. By means of the theory of ionization by collision the influence of the pressure can be accounted for.

University of Illinois.

On the Optical Properties of an Isolated Crystal of Selenium.[1]

By Charles H. Skinner.

A SELENIUM crystal, having a surface about 2 mm. by 5 mm. has been employed in a determination of its optical constants. The method of the Babinet's compensator was used. It was found that when the crystalline axis of the hexagonal crystal was held in a vertical position, the results obtained agreed well with the best published results for crystalline selenium in mass, but when the axis was held in a horizontal position, quite different results were obtained. This points to the possibility that all such crystalline substances that reflect metallically have in general three sets of principal optical constants (two in any one plane), and that when one determines the optical constants for a crystalline substance, used in mass, one really obtains only a certain mean set of values.

State University of Iowa.

Atomic Models in Whose Behavior a Magnetic Field is an Important Factor.[1]

By Ernest Merritt.

WHEN an electron moves under the influence of a central force $F(r)$ in a magnetic field H normal to the plane of motion the chief effect of the magnetic field is to produce a uniform rotation of the axis of the orbit. This is perhaps most clearly indicated by the fact that the equation of motion may be put into the form

$$m(\ddot{r} - r\dot{\varphi}^2) = F(r) - mr\omega,^2$$

$$mr^2\dot{\varphi} = \text{const.},$$

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[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, November 26–27, 1915.

where φ is the angle between the radius vector to the electron and a reference axis through the center which rotates with the constant angular velocity

$$\omega = \frac{He}{2m}.$$

If there is no magnetic field the equations of motion are

$$m(\ddot{r} - r\dot{\theta}^2) = f(r) \quad mr^2\dot{\theta} = \text{constant}.$$

where θ measures the angular displacement of the radius vector from a fixed axis. If

$$f(r) = F(r) - mr\omega^2$$

the equations are of the same form. It appears therefore that the effect of a magnetic field is to produce the same motion relative to the rotating axis as would be produced, without a magnetic field and without rotation of the axis of reference, by a central force

$$F(r) - mr\omega^2.$$

An interesting special case is that in which the force $F(r)$ is a repulsion. With suitable starting conditions the motion with reference to the rotating axis will consist of a vibration about the point for which $F(r) = mr\omega^2$. The vibrations will be simple harmonic for infinitely small amplitudes, but in general "overtones" will be present. In such cases

$$r = r_0 + A \cos (pt + \varphi_1) + B \cos (2pt + \varphi_2) + \cdots$$

and the displacement of the electron in the direction of some *fixed* axis OX is

$$x = r \cos \omega t = r_0 \cos \omega t + A \cos \omega t \cos (pt + \varphi_1) + B \cos \omega t \cos (2pt + \varphi_2) + \cdots.$$

Each term of the form $\cos \omega t \cos (npt + \varphi_n)$ can be broken up into the sum of two terms of the form

$$\cos \{(\omega + np)t + \psi\} \text{ and } \cos \{(\omega - np)t + \psi\}.$$

The radiation from an electron moving in this manner would therefore contain lines having frequencies proportional to ω, $\omega + p$, $\omega - p$, $\omega + 2p$, $\omega - 2p$, etc. In other words, the spectrum would consist of a series of lines equally spaced on the frequency scale. Since the amplitude will ordinarily be less for the higher harmonics we should expect the central line, of frequency ω, to be brightest, while the brightness will fall off progressively on both sides of the central line. Such a spectrum would have the general characteristics of the fluorescence spectrum of the uranyl salts, although the numerical relations would be different.

While this simple case can scarcely be expected to correspond to any actual atomic structure it serves to illustrate a general principle. The same method of analysis can, for example, be employed in any case where the orbit described under the action of the force $F(r) - mr\omega^2$ is a closed curve. In many cases where the orbit is not closed it may be regarded as at least approximately equivalent to a closed orbit whose principal axes are rotating, and these cases also may be treated in much the same way. It should be noted also that the same general method may be employed in cases where the force on the electron is in part due to the action of other moving electrons. It will be recalled that the question of stability introduces difficulties in the case of the Saturnian type of atomic model, especially in the Nagaoka or Rutherford form in which the positive charge is concentrated. These difficulties are made less by the assumption of a magnetic field.

The existence of a strong magnetic field in the neighborhood of each molecule of a crystal like uranium nitrate seems highly probable, for the molecules would not be in stable equilibrium under the action of electrical forces alone. That the magnetic field should be uniform over the whole orbit of an electron is scarcely to be expected, however, since it is reasonable to expect that the field due to the molecule itself, which probably varies rapidly with the distance, will be an important part of the whole. If the fluorescence spectrum of the uranium salts is really produced in some such manner as that here suggested we must therefore assume special conditions, which are made possible perhaps by the molecular structure of the uranyl radical. The magnetic field required is of the same order of magnitude as the internal field predicted by the electron theories of magnetism.

CORNELL UNIVERSITY.

THE REFLECTION OF SOUND.[1]

BY F. R. WATSON.

THIS work is a continuation of the investigation on the transmission of sound.[2] Sound from a steadily blown whistle mounted at the focus of a parabolic mirror is directed toward an open doorway. This is assumed to give zero reflection. Layers of material are then placed over the doorway and the intensity of reflected sound measured by a Rayleigh resonator. The data obtained furnishes an explanation for the anomalous transmitting action of some materials.

UNIVERSITY OF ILLINOIS.

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[1] Abstract of paper presented at the Chicago meeting of the Physical Society, November 26, 27.

[2] PHYSICAL REVIEW, April, 1915.

THE EXCEPTIONS TO THE LAW OF DULONG AND PETIT.[1]

BY J. E. SIEBEL.

IN order to mitigate the apparent exceptions to the law of Dulong and Petit, the author undertakes to show a simple way to calculate the specific heat at low temperatures in accordance with the kinetic heat theory. For this purpose, instead of comparing specific heats, the total heat energy which a body should contain according to the kinetic heat theory, is compared with the energy which it contains in accordance with the molecular heat observed by Nernst and others at given temperatures. The resulting difference of energy is found to be almost a constant function of the different temperatures under observation, characteristic for every individual substance. According to the author, this is due to a nearly constant amount of potential molecular energy, probably chiefly of gravitation and expressible by formulæ in harmony with observed data, characterizing them as functions of the specific heat derived from the Dulong and Petit proposition in accordance with the kinetic heat theory.

CHICAGO, ILL.

[1] Abstract of a paper presented at the Columbus meeting of the Physical Society, December 28–30, 1915.

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THE

PHYSICAL REVIEW.

THE LIGHT SENSITIVENESS OF COPPER OXIDE.

By A. H. Pfund.

Of the various substances which experience an increase in conductivity under the influence of light, selenium and stibnite show the effect most markedly. A comparative study of these substances has shown an unmistakable parallelism of behavior and this has pointed toward the conclusion that the mechanism involved in the production of light-sensibility is probably of the same general character. It is needless to state that hardly a start has been made toward explaining the complicated behavior of these substances. With the idea in mind that something might be learned from a study of other light-sensitive substances, yet unknown, a search was begun. The first, new light sensitive substance discovered was cuprous oxide (Cu_2O). While indications of light-sensitiveness were also found in other substances, the following discussion is to be confined to the consideration of cuprous oxide.

Mode of Preparation of Cu_2O.

Cuprous oxide was prepared from a band of pure copper having the following dimensions: 0.75 × 10 × 150 mm. This band was heated in air by means of an electric furnace at a temperature of about 900° C. for 20 hours. At the end of that time, the copper was entirely oxidized but for a short portion near the ends of the band. The entire mass appeared grayish-black and was quite opaque to light. Upon breaking up the original long strip into lengths suitable for making cells, these shorter pieces were dropped into aqua-regia of moderate strength. This was done in order to dissolve off the outer coating of cupric oxide (CuO). The remaining mass of Cu_2O showed a beautiful crystalline structure and marked translucency toward red light even in layers exceeding 1 mm. in thickness. In order to prepare thinner layers several modes of procedure were followed: (1) the original, thick layer was ground down on an emery-wheel, (2) the thickness was reduced by means of aqua-regia, (3) pieces of oxide were scaled off the ends of the original band. These scales were removed by bending the partially oxidized strip, whereupon they were dropped into aqua-regia to remove the CuO. As a result of this procedure, the scales had a thickness of 0.1 to 0.3 mm. and appeared a brilliant ruby-red in transmitted light.

COPPER OXIDE CELLS.

The types of cells most frequently used are shown in Fig. 1, *A* and *B*. Here *a* represents a strip of tinfoil about 0.5 mm. wide, either straight as in 1*A* or hairpin shaped, as in 1*B*, in close contact with the surface of cuprous oxide. The two electrodes *bb′* consist of flexible tinsel which is held in contact with the Cu_2O by shallow pools of celluloid dissolved in amyl-acetate (albilene). After the acetate has evaporated and the firm celluloid is left, the entire upper surface of the cell receives an opaque, cathodic film of gold. Upon removing the strip of tinfoil the cell is mounted in a glass tube and is then ready for use.

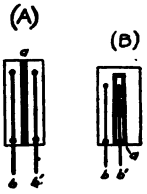

Fig. 1.

In order to study the behavior of such cells at temperatures ranging from +20° to −180° C., the mounting shown in Fig. 2 was used. Here *AA* represents a block of brass which is split—the two halves being separated by a thick layer of mica. A disc of mica *M* is fastened to the brass block and the Cu_2O cell *C* is mounted on the mica. The mica plate and cell are held in place with albilene which, unlike sealing wax, etc., does not release its hold at liquid air temperatures. The two strips of tinsel, coming from the cell, are soldered to the brass block as also are the two wires leading to the outside of the test-tube *T*. A small hole is drilled into the brass block so as to receive the junction of a german-silver-copper thermocouple by means of which the temperature of the system may be determined. A glass or quartz plate covers the end of the testtube and this is finally exhausted and sealed off.

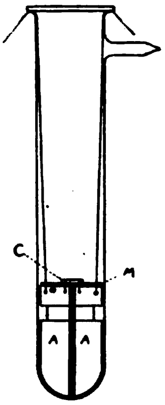

Fig. 2.

The resistance and light-sensitiveness could be made to vary over a wide range—depending upon the design of the cell. One of the best cells thus far made is of the type 1*B*. The width of the bare strip of Cu_2O is about 0.5 mm. and the thickness of layer is 0.8 mm. The resistance at 17° C. for 1 volt is 15,200 ohms in the dark. A 40 watt tungsten lamp at 20 cm. produces an increase in conductivity of about 15 per cent.

By connecting this cell to a 2 volt battery and a simple potentiometer, the sensibility is such that exposing the cell to diffuse daylight, too faint to read by, throws the galvanometer spot of light off the scale.

ACCESSORY APPARATUS.

In order to study the effect of monochromatic radiations in the spectral interval 3μ to 0.4μ a Fuess monochromatic illuminator was used. The energy carried by the various bundles of radiation was measured by a two-junction vacuum thermopile[1] whose receiving surface had an area 1 × 6 mm. The thermopile was connected to a D'Arsonval galvanometer of 15 ohms' resistance and of a working sensibility of 1.5×10^{-9} amp. per mm. deflection for a scale distance of two meters. The sensibility of the thermopile-galvanometer system was such that the total radiation from a candle at 1 meter distance (after passing through the quartz-window of the thermopile) produced a deflection of 400 mm. As in my previous work on selenium, the copper-oxide cell was connected electrically to form part of a simple potentiometer system in which the above-described galvanometer was also employed.

ULTRA-VIOLET MONOCHROMATIC ILLUMINATOR.

The quartz-spectrometer used for studying the ultra-violet region of the spectrum has been in constant use for the past three years with very satisfactory results. As the instrument has not as yet been described, a few words as to its design may not be out of place. The arrangement of the various parts is shown in Fig. 3. Here the light from an iron-arc A is collected by the condensor C and is focused on the primary slit S_1 from which it passes through the collimating lens L_1 thence through the Cornu prism P and is finally focused near the plane of the secondary slit S_2 by means of the lens L_2. The distinctive feature of this design is that the prism, the lens L_2 and the slit S_2 are fixed, hence the monochromatic radiation leaving S_2 always has the same direction and, as a result, any optical system, no matter how complex, may be mounted in a suitable and fixed position behind this slit. The lens L_1 is mounted rigidly on an iron beam B which is pivoted under the center of the first prism-face at x. The slit S_1, the condenser C and the arc A are likewise connected rigidly. The slit and its connections may slide along the beam B but are guided in their motion by a pin which lies under and in line with the slit and which is forced to move in a groove cut into the track T. This track is so inclined to the direction of B that the chromatic aberration of the lenses L_1 and L_2 is counteracted—and as a result, the spectrum on the slit S_2 is always in focus. To change from one spectral region to another, the arc-condensor-slit combination is shifted bodily along the beam B and the track T. While the chromatic aberration can not be eliminated perfectly by sliding the slit along a straight line, the adjustment is, nevertheless, surprisingly good. In order to fix, properly, the position of the track T a position of the slit is found which allows radiations of the wave-length 3,500 A.U. to pass through S_2 (which is made several millimeters wide and behind which a piece of uranium glass is mounted in close contact). Next S_1 is made very narrow and the lenses L_1 and L_2 are adjusted in such positions that their respective distances from S_1 and S_2 are the same when the spectrum is in sharp focus on the uranium glass. The next step is to shift S_1 into such a position that blue light (λ = 4,500 A.U.) is sharply in focus on S_2. That portion of the track T lying immediately under the engaging pin is now held fixed and a new position of S_1 is found so as to bring λ = 2,700 A.U. to a sharp focus on S_2. If the track be fastened down rigidly in this position it will be found that, upon shifting S_1, only the two spectral regions λ = 4,500 and 2,700 A.U. will be in sharp focus. While a compromise is effected for the other spectral regions, the deviations from perfect focus in the interval λ = 5,500 to 2,400 A.U. are very slight. It is, of course, always possible to improve the apparatus by constructing a properly curved track T, by keeping the prism always at minimum deviation and by keeping the lenses always at equal distances from their respective slits. In view of the fact that such improvements would have added complications to the construction the simpler form of apparatus was constructed.

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[1] Phys. Ztschr., 13, 870, 1912.

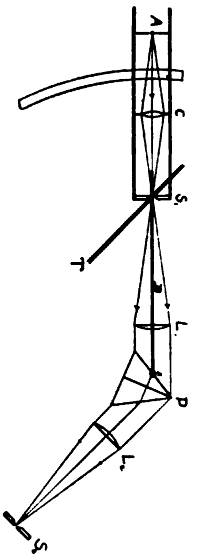

Fig. 3.

The iron-arc used has already been described elsewhere.[1] Suffice it to say that a current no greater than 5 amperes was used and that the upper electrode was positive. Before beginning a series of measurements a bead of iron-oxide, about 7 mm. high, was formed on the lower electrode and a smaller, hemispherical bead was formed on the upper. The arc burned with great steadiness when the two beads were separated by 1.5 to 2.0 mm. Increasing the separation adds but little to the energy radiated in the region of shorter wave-lengths since these come from the regions of the arc very close to the beads. By means of the fluorescence excited on a strip of glass from a broken incandescent lamp bulb, the condensor is adjusted to focus the ultra-violet radiation on the slit S_1. Many tests have shown that energy measurements, taken at intervals during 2 or 3 minutes rarely differ by more than 2 or 3 per cent. if the arc is burning properly. The amount of energy delivered by this spectrometer is large. By mounting the thermopile behind S_2, deflections greater than 500 mm. are obtained in the region $\lambda = 2{,}500$ A.U. with slit-widths no greater than 0.5 mm. It has been found advisable to insert a quartz tank containing a 2 cm. layer of water in front of the slit S_1. A piece of (copper) ruby glass was used as a shutter in the interval 5,500 to 2,900 A.U. and a piece of clear glass was used for the region below 2,900 A.U.

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[1] Astrophysical Journal, 27, p. 296, 1908.

The Voltage Effect.

In order to study the change in conductivity with applied voltage, a Cu_2O cell was connected to a Paul millivoltmeter (used as a galvanometer) and a variable source of potential. The resistance of the millivoltmeter was but 50 ohms which quantity is negligible in comparison with the resistance of the cell. The readings were taken as shortly as possible after closing the circuit in order to avoid the slow creep which sets in when the current is allowed to flow for some time. As soon as a reading had been taken, the circuit was broken and two minutes were allowed to elapse before the process was repeated. The glass tube containing the cell was mounted in a water-bath at 17° C. The results obtained are shown in the following table and in Fig. 4.

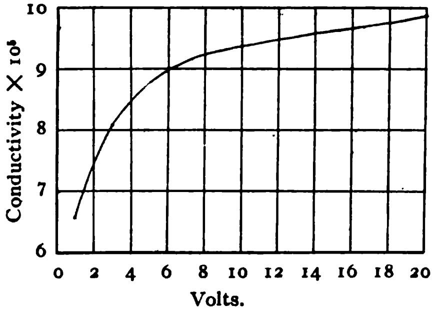

Fig. 4.

E.M.F.	Resistance.	Conductivity.
1 volt.	15,200 ohms.	6.6×10^{-5}
3	12,300	8.1
6	11,100	9.0
10	10,600	9.4
20	10,100	9.9

The fact, that the conductivity increases markedly with the voltage applied, shows that cuprous oxide, like selenium and stibnite, does not obey Ohm's law. That the " voltage effect " is not due to temperature changes has been demonstrated repeatedly.[1]

LIGHT EFFECT *vs.* HEAT EFFECT.

In the present work the question has arisen whether Cu_2O, with its large, negative temperature coefficient of resistance (0.024) owes its increase in conductivity to a heating effect or to a specific action of the light. While all of the experiments on the color-sensibility curves make it highly improbable that we are dealing with a heating-effect, the following crucial experiment was carried out. A cell, similar in design to that shown in Fig. 1*A* was prepared and the central portion was covered with a very thin, non-conducting layer of albilene (celluloid). Upon connecting this cell to the potentiometer system and projecting monochromatic beams of light upon the central strip, a galvanometer deflection of more than 100 mm. was observed. This deflection did not increase upon prolonged illumination, but remained steady. Since the galvanometer required 7 seconds in order to reach its maximum deflection, it is evident that the change in conductivity occasioned by light reached a steady state in an interval of time at least no greater than 7 seconds. That a temperature equilibrium could have been reached in the rather massive piece of Cu_2O seems improbable. Next, the central strip of oxide was painted over with a thin but opaque coating of lamp-black and shellac in alcohol. This produced an absorbing surface which would heat up the cuprous oxide upon exposure to radiation. Electrical conduction through the lamp-black was prevented by the coating of celluloid underneath. As a matter of fact the resistance of the cell was not affected by putting on the layer of lamp-black. Upon exposing the cell to the same radiations as before, no galvanometer deflection as great as 1 mm. was observed. The conclusion to be drawn is that, while there may be a slight effect due to heating, the observed effect is preponderatingly due to a specific action of the light.

COLOR SENSIBILITY CURVES.

The color-sensibility curves were obtained in the usual manner by throwing monochromatic slit-images on the copper-oxide cell and observing the resultant change in conductivity. The energy carried by each monochromatic bundle was made the same by projecting the radiation on the thermopile and varying the intensity so that the same galvanometer deflection was obtained. A typical curve for a cell, similar in design to that shown in Fig. 1*A* is plotted in Fig. 5. This cell had a thickness of layer of 1.2 mm. and a central, bare strip of Cu_2O 1.5 mm. wide. In all cases the equilibrium change in conductivity was recorded. Curve *A* applies to a temperature of +19° C. while curve *B* applies to a temperature of −127° C. The last-named curve was obtained by lowering the test-tube, containing the cell, into a Dewar flask containing liquid-air. By raising or lowering the liquid-air surface and by plugging up the top of the Dewar flask with cotton-batting, the temperature of the cell could be kept constant to within less than 5° C. for hours.

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[1] Luterbacher, Ann. d. Phys., 33, p. 1392, 1910.

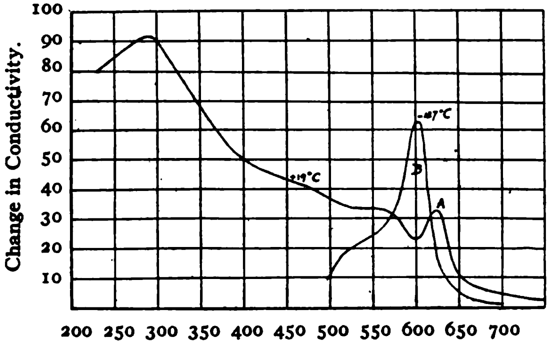

Fig. 5.

It will be noted that the sensibility maximum in curve *B* is shifted toward the shorter wave-lengths, as in the case of selenium and stibnite. The region of greatest sensibility lies far in the ultra-violet near $\lambda = 2800$ A.U. Later on it will be shown that such curves as the above do not represent *the* sensibility curves for cuprous oxide but that they apply only to the cell in question. This will be established by showing that the form of the sensibility curve may be changed by modifying the design of the cell.

RELATION BETWEEN ENERGY AND CONDUCTIVITY CHANGE.

In order to establish the law which connects the equilibrium change in conductivity (ΔC) with the amount of energy (E) absorbed, a cell was first exposed to a beam of monochromatic radiation which was subsequently cut down to half value by means of a rotating sector. The last column in the following table shows the value of the deflection, obtained by calculation, for half energy on the assumption that the change in conductivity is proportional to the square-root of the energy.

Results were obtained only in the spectral region entirely absorbed by cuprous oxide. From the above table it is evident that, within the limits

Wave-Length.	Deflection for Full Energy (E).	Deflection for Half Energy ($\frac{1}{2}E$).	Calculated Deflection for $\frac{1}{2}$ E.
4,820 A.U.	115 mm.	79 mm.	80 mm.
5,220	152	106	108
5,470	175	122.5	122
6,040	118	85	83
6,180	133	91	93

of accuracy of measurement, the law connecting ΔC (which is proportional to the deflections) and E is of the form:

$$\Delta C = KE^{\frac{1}{2}}$$

This same law also applies to selenium and stibnite.

FRONT AND BACK ILLUMINATION.

If electrodes be applied to the upper surface of a disc of selenium and if the lower surface be illuminated, it was shown by Gripenberg[1] that a change in conductivity is produced and that this change is of the same order as that obtained by illuminating the upper surface. Since the selenium was entirely too opaque to permit light from below to penetrate to the electrodes on the upper surface, it is evident that changes in conductivity are also brought about in portions of the selenium not directly illuminated. This phenomenon, which has been termed the "Transmitted Effect" has been studied in great detail by Brown[2] on isolated crystals of selenium. In order to determine whether or not a similar effect was also shown by Cu_2O, many experiments similar to those of Brown and Gripenberg were carried out. Without discussing in detail the ten or more experiments which were carried through, it may be stated that if cuprous oxide shows the effect at all, it is infinitesimal in comparison to that shown by selenium. As a matter of fact, the experimental evidence indicates that, in the case of Cu_2O, the change in conductivity is practically confined to the portion of the material penetrated by the light.

But one experiment will be discussed in this connection. A cell, similar in design to that shown in Fig. 1*A* was mounted in an evacuated glass tube. The disc of Cu_2O was but 0.4 mm. thick and, as a result, showed marked translucency to red light. The tube containing the cell was mounted vertically on a block of wood which could be rotated, between stops, through 180°. In this manner a monochromatic slit-image could first be thrown on that side of the cell bearing the electrodes (front illumination) and then, on the back (back illumination). It goes without saying that the light was focused on the bare, central strip of Cu_2O and on the portion immediately behind it. While the various beams were not reduced to equal energies, a water-cell containing cuprammonia was inserted in front of the primary slit so as to cut down the tremendous intensity of the red end of the spectrum. The results are shown, graphically, in Fig. 6.

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[1] Gripenberg, Phys. Ztschr., 15, p. 462, 1914.

[2] Brown, Phil. Mag., 28, p. 497, 1914.

It is here seen that, for wave-lengths below 6,000 A.U. the change in conductivity for "back-illumination" is small while for wave-lengths greater than 6400 A.U. the change in conductivity is even greater than in the case of front illumination.[1] If now we consider the transmission of a thin scale of cuprous oxide (as shown by arbitrary numbers in the dotted curve of Fig. 6) we see that the increasing deflections, due to "back-illumination" occur in the spectral region in which the cuprous oxide becomes increasingly transparent. Without carrying through the argument in detail, it would appear that the light from the back gives rise to large changes in conductivity as soon as it is able to penetrate into the region where the current-density is comparatively great. Whether the small, but definite, changes in conductivity, occasioned by the shorter wave-lengths, are in the nature of a true transmitted effect is still an open question.

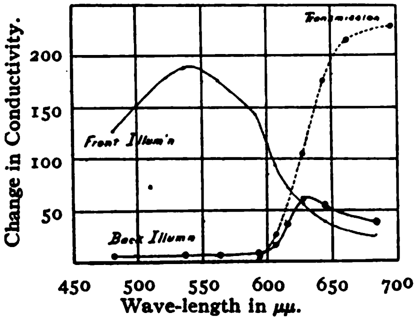

Fig. 6.

In this connection is was thought of interest to compare the "front" and "back" illumination curves for selenium and cuprous oxide. A selenium cell, similar to the Cu_2O cell in design and dimensions was prepared. The selenium layer was entirely opaque in the spectral region studied and the usual sensibility-curve showed a marked maximum at the wave-length 7,000 A.V. The ratio of the change in conductivity brought about by "front" ΔCF and "back" ΔCB illumination is shown in Fig. 7.

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[1] Just what this higher value is due to has not been determined. In part, at least, it is due to the fact that the gold electrodes cut off some of the light—furthermore, internal scattering of the light may also contribute toward the effect observed.

Since the scale of Cu_2O was only translucent, no true values for the transmission could be found. The values recorded were obtained by mounting the scale on the first slit of the monochromatic illuminator and measuring by means of the thermopile, the relative amounts of energy for the various wave-lengths. This, of course, yields only a qualitative determination.

The great disparity in the form of the two curves is apparent. In selenium we have a true "transmitted effect" which, instead of being greatest in the deep red where selenium is known to be the most transparent, is really the greatest in the violet. While these results were obtained with discs of selenium and cuprous oxide consisting of a conglomerate of small crystals, it would be premature to state that the "transmitted effect" does not exist in isolated crystals of Cu_2O. Recently the writer has found that such crystals (cuprite) may be obtained —therefore the above-mentioned uncertainty will be removed shortly. At any rate it has been established that cuprous-oxide, in the form used, shows no definite "transmitted effect" and in this one respect, differs decidedly from selenium.

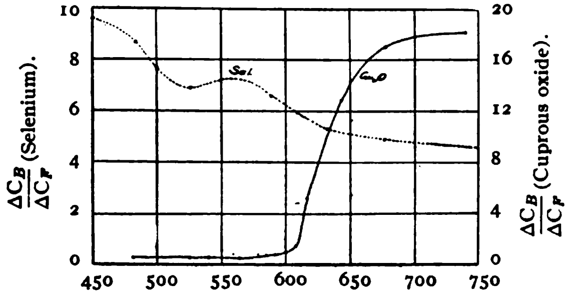

Fig. 7.

SENSIBILITY CURVES FOR CELLS OF DIFFERENT DESIGN.

If the change in conductivity is confined to the illuminated portion of cuprous oxide, then, by varying the design of the cell, different sensibility curves ought to result: This inference was tested by constructing two cells out of a single strip of Cu_2O. This strip had the dimensions 1.2 × 5 × 12 mm. In Fig. 8 (*a*) the two gold electrodes *A* and *B* are separated by a bare strip of Cu_2O about 0.75 mm. wide on the flat side of the strip while *A* and *C* are two identical electrodes on opposite sides of the strip. The electrode *B* is about 0.75 mm. wide. Upon connecting the electrodes *A* and *B* to a source of potential, the variation of current-

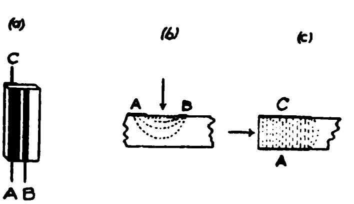

Fig. 8.

density (lines of flow) through the mass of material underneath is shown diagrammatically in Fig. 8(*b*). Here, the region of greatest current-density lies very near the surface, whereas, if the electrodes *A* and *C* be used, the current-density will be practically constant as shown in Fig. 8 (*c*). (Fig. 8 (*b*) and (*c*) are much magnified.) If a beam of red light be allowed to fall on these two cells, cell (*c*) ought to be, relatively, the more sensitive since the light penetrates almost the entire current-carrying layer before being absorbed. On the other hand, cell (*b*) ought to be the less sensitive since but little energy is absorbed in the region of greatest current-density. It is evident that exactly the reverse ought to be true for blue light which is absorbed very near the surface. When these two cells were tested for their sensibility-curves it was found that, while they had almost the same resistance, cell (*b*) was considerably the more sensitive. The actual sensibility curves are shown in Fig. 9, where curve I. applies to cell (*b*) and curve II. to cell (*c*).

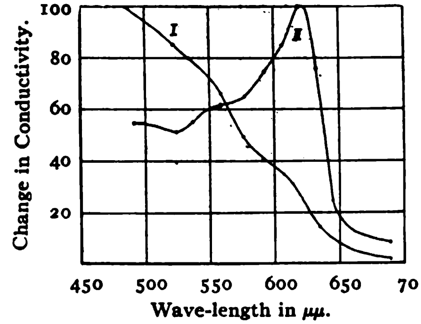

Fig. 9.

It is here apparent that the two curves, obtained from the same material are markedly different and that they differ in the predicted manner. The question here arises as to whether or not the sensibility maximum in the red (near $\lambda = 6{,}200$ A.U.) is really indicative of a high intrinsic light-sensibility of the cuprous oxide. In terms of electron-theory, the " intrinsic-light-sensibility " involves the idea of the number of electrons made available for carrying the current per second, per unit cube, per watt of absorbed energy. The energy is supposed to be absorbed uniformly, *i. e.*, each small element of volume of the unit cube is to absorb the same amount of energy per second. In actual experiment it would be necessary to work with films of the light-sensitive material so thin as to be transparent to all wave-lengths. This would make possible a measurement of the absorbed energy and would fulfill to a fair degree of approximation the condition of uniform current density and light-absorption, imposed on the experiment.

DISCUSSION.

Several years ago the writer suggested the idea that light-sensibility might be due to resonance in consequence of which the number of electrons, made available for carrying the current, was increased. The large amount of work since then carried out by Brown[1] and his co-workers, Nicholson,[2] Elliott,[3] Ries[4] and the writer[5] goes to show that the above suggestion well serves the purpose of a working hypothesis. That the final theory will be established on an electronic basis, seems certain. Without discussing the various points which have been established definitely for the other light sensitive substances, the contribution of the present work to the theory will be taken up briefly. According to the "free-electron" theory of Drude and Riecke, the light-sensibility of cuprous oxide would be explained as an actual liberation of electrons which, subsequently, float about freely between the molecules. As diffusion of electrons from the regions of higher concentration is bound to set in, it would follow that the conductivity of the entire mass of material would eventually be increased even though only a small portion of the surface be illuminated. This means that the "transmitted effect" would necessarily exist. On the "doublet" theory of J. J. Thomson, the effect of light would be to decrease the stability of electrons to such an extent that they might be drawn out of their original "doublet" system into the adjacent one. It is here assumed that the potential applied serves merely to orient the doublets—the actual transference of electrons from one doublet to the next is due to the electric field between doublets. According to this view the region of increased conductivity ought to be confined to the region illuminated and hence, a "transmitted" effect ought not to exist. The fact that a true "transmitted" effect is not shown by copper-oxide goes to show that the results obtained are best accounted for on the basis of the "doublet" theory of metallic conduction.

Similar considerations have been brought forth by Brown who has shown that the "voltage" and "pressure" effects in selenium are not transmitted. In selenium, however, the evidence gained from a study of light-sensibility is not clear-cut since matters are complicated by the "transmitted" effect. The true nature of this effect has not as yet been established. In the case of cuprous oxide conditions are much simplified for it has been shown that, qualitatively at least, the behavior of the substance can be accounted for on the assumption that there is no true transmitted effect.

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[1] PHYS. REV., 5, p. 395, 1915.

[2] PHYS. REV., 3, p. 1, 1914.

[3] PHYS. REV., 5, p. 53.

[4] Ann. d. Phys., 36, p. 1055, 1911.

[5] PHYS. REV., 34, p. 370, 1912.

Summary.

The results of the foregoing work may be stated briefly as follows:

1. A convenient monochromatic illuminator for the ultra-violet has been described.

2. It has been proved that cuprous oxide is light-sensitive and that the effect is not due to heat.

3. The failure of Ohm's law for Cu_2O has been demonstrated.

4. The region of greatest light-sensibility lies in the ultra-violet near $\lambda = 2{,}800$ A.U.

5. Lowering the temperature from 19° C. to $-127°$ C. causes a shift of the red sensibility maximum toward shorter wave-lengths.

6. The equilibrium change in conductivity of Cu_2O is proportional to the square-root of the light-energy absorbed per unit time.

7. "Front" and "back" illumination produces effects in cuprous oxide which are entirely different from those produced in selenium.

8. The change in conductivity in cuprous oxide is limited to the portions penetrated by radiation.

9. The absence of a true "transmitted" effect in cuprous oxide shows that this effect is not essential to light-sensibility.

10. With the same material, the light-sensibility curve of cuprous oxide may be changed markedly by modifying the design of the cell.

11. The results obtained are most readily explained on the basis of Thomson's "doublet" theory of metallic conduction.

Johns Hopkins University,
November, 1915.

THE CHARACTERISTICS OF TUNGSTEN FILAMENTS AS FUNCTIONS OF TEMPERATURE.

BY IRVING LANGMUIR.

THE tungsten filament offers the simplest means of producing very high temperatures under conditions suitable for accurate measurement. There have been, however, considerable difficulties in determining the true temperature of such filaments, so that the results published by different investigators have differed by several hundred degrees.

The characteristics of tungsten lamps have been investigated very extensively, but in nearly all instances only the relations between the variables—volts, amperes, and candle power, or functions of these—have been studied. These variables, in fact, are the only ones with which the illuminating engineer is directly concerned. Probably the most complete investigation of this kind is that of Middlekauff and Skogland[1] who give by equations and tables the changes in candle power and amperes for any given change in the voltage applied to tungsten lamps.

The physicist or chemist, however, is primarily interested in the relation between these variables (volts, amperes, and candle power) and the dimensions and temperature of the filaments. Pirani published some data on the relation between temperature and watts per candle and later gave data on the temperature coefficient of the resistance as a function of the temperature.

Several years ago a detailed study of the characteristics of tungsten filaments as functions of the dimensions and temperature, was undertaken in this laboratory. The data thus obtained have been in continual use ever since and have served as the basis for the calculation of temperatures in several investigations which have been published.[2]

More recently, in connection with a redetermination of the melting-point of tungsten,[3] an optical pyrometer was set up, suitable for measuring the true temperatures of filaments with considerable accuracy. With this instrument a new study of the characteristics of filaments has been made and the results are now felt to be of sufficient accuracy to warrant their publication.

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[1] Bull. Bur. Stand., 11, 483 (1915), and Trans. Ill. Eng. Soc., *9*, 734 (1914).

[2] For example: "Convection of Heat in Gases," PHYS. REV., *34*, 401 (1912); "Vapor Pressure of Tungsten," PHYS. REV, *2*, 329 (1913); "Thermionic Currents," PHYS. REV., *2*, 450 (1913); "Dissociation of Hydrogen," Jour. Amer. Chem. Soc., *34*, 860 (1912), *36*, 1708 (1914), *37*, 417 (1915).

[3] PHYS. REV., *6*, 138 (1915).

General Theoretical Considerations.

Any property of a filament which varies with the temperature may be used, after proper calibration, as a means of estimating the filament temperature. There are a large number of such properties and the choice of the one most suited to serve as the basis of a temperature scale, will depend entirely upon the experimental conditions. For example, the energy radiated per unit area might be adopted for a filament in vacuum, but would be very unsatisfactory if the filament were surrounded by a gas.

The various properties (or methods) that may be utilized to estimate the temperature of filaments may be divided into four groups according to the knowledge of filament dimensions required, as follows:[1]

1. Requiring no dimensions:
   (*a*) Intrinsic brilliancy.
   (*b*) Color of light emitted.
   (*c*) Ratio of "hot" to "cold" resistance.
   (*d*) Watts per candle (W/C).
2. Requiring both length and diameter:
   (*e*) Resistance, $R' = \frac{Rd^2}{l}$.
   (*f*) Power radiated $W' = \frac{W}{ld}$.
   (*g*) Candle power $C' = \frac{C}{ld}$.
   (*h*) Voltage $V' = \frac{V\sqrt{d}}{l} = \sqrt{W'R'}$.
   (*i*) Electron emission $i' = \frac{i}{ld}$.
3. Requiring diameter only:
   (*j*) Current $A' = \frac{A}{d^{\frac{3}{2}}} = \sqrt{\frac{W'}{R'}}$.

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[1] l = length, d = diameter,
V = volts, A = amperes,
W = watts, R = resistance (ohms),
C = candle-power (measured perpendicular to axis of filament); C is thus equal to the total lumens divided by π^2 or to the mean spherical candle-power multiplied by $4/\pi$;
V', A', etc., specific properties defined below.

(*k*) $\frac{C}{Vd^{3/2}} = \frac{C'}{V'}$.

(*l*) $\frac{1}{d}\sqrt[3]{\frac{C}{R}} = \sqrt[3]{\frac{C'}{R'}}$.

(*m*) Candle power (using slit) $\frac{C}{d}$.

4. Requiring length only:

(*n*) $\frac{V\sqrt[3]{A}}{l} = \frac{\sqrt[3]{W^2R}}{l} = \sqrt[3]{(W')^2R'} =$

(*o*) $\frac{\sqrt[3]{CV^2}}{l} = \sqrt[3]{C'(V')^2}$.

(*p*) $\frac{C}{A^{2/3}l} = \frac{C'}{(A')^{2/3}}$.

(*q*) $\frac{\sqrt[3]{C^2R}}{l} = \sqrt[3]{(C')^2R}$.

(*r*) Thermal expansion $\frac{\Delta l}{l}$.

(*a*) *Intrinsic Brilliancy.*—This is the property utilized in the Holborn-Kurlbaum pyrometer, and most other optical pyrometers, for the measurement of temperature. The use of intrinsic brilliancy for this purpose has the particular advantages

1st. That measurement can be made at any point on a surface.

2d. Extremely rapid increase with temperature.

3d. Variation with temperature follows known laws.

Only relative measurements of intrinsic brilliancy are involved in the use of a pyrometer. Since it has been shown[1] that the emissivity of tungsten does not change appreciably with the temperature, the variation of the intrinsic brilliancy of tungsten with the temperature is given accurately by Wien's law ($C_2 = 1.4392$):

$$\text{(1)} \qquad \log_{10}\frac{E}{E_0} = \frac{-.62503}{\lambda}\left(\frac{1}{T_0} - \frac{1}{T}\right).$$

(*b*) *Color of Light Emitted.*—The color of the light emitted by an incandescent body may be used to estimate its temperature. This method is a very old one but has only recently been used for accurate pyrometry. Coblentz[1] and Hyde[2] showed that by properly adjusting the temperature, filaments of platinum, carbon, tungsten, or tantalum could be color-matched against a black body. This color match is made by setting up the filament on one side of a Lummer-Brodhun photometer, and a black body on the other side. By adjustment of the temperature of the filament and its distance from the photometer the color and intensity of the light from the two sources can be made identical within the limits of observation. Hyde[1] pointed out that when filaments of the above named materials were brought to a color match, the distribution of energy over the whole visible spectrum was the same for all filaments. Hyde states in fact that the color match method "is perhaps more sensitive than the spectrophotometric method and yields results in close agreement with the latter. It is conceivable, of course, that two lamps might have the same color, and yet show different spectral distributions. Such a case has not yet been observed. On the other hand it has been found that in certain cases, *e. g.*, with the osmium *vs.* the carbon lamp, it is impossible to obtain exact color matches, although the spectrophotometric curves differ by an amount so small that the differences might well be ascribed to experimental error."

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[1] Phys. Rev., *6*, 138 (1915).

[1] Bull. Bur. Stand., *5*, 359 (1909).

[2] Trans. Ill. Eng. Soc., 1909.

Hyde showed that a filament could be set at definite temperature as accurately by the color-match method as the usual pyrometric methods.

The color-match method has been used for several years in this laboratory for estimating the temperatures of filaments.[2] It has proven to be of great practical value and appears to be of more universal applicability than any other pyrometric method.

The relation between the true temperature of a filament and that of a black body which matches it in color has only recently been the subject of study. Hyde considers that "there is much reason to believe that under this condition (of color-match) they are not operating at the same true temperature" and reasons that the temperature of the filament should in general be lower than that of the black body.

Lorenz,[3] by comparing the expansion of gases around similar filaments of platinum and tungsten which are alternately heated, concludes that filaments of these two metals are at the same true temperature when they are at a color-match.

Paterson and Dudding,[4] in an extended investigation with carbon and tungsten lamps, determine the relation between the watts per candle of these lamps and the temperature of a black body giving color-match. They also endeavor to find the difference in temperature between a platinum filament and that of a black body which color-matches it. They find as the average of about 30 determinations (varying from 1727° to 1789° C.) that the temperature of the black body which matches a platinum filament at its melting-point (1753° C.) is 1762° C. They also compare their data for the relation between watts per candle and temperature with published data (Forsyth, Pirani, Langmuir) on this relation. As a result they conclude that the differences in temperature between platinum or tungsten and a black body at color-match are small and probably do not exceed 1 or 2 per cent.

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[1] Jour. Frank. Inst., *169*, 439 (1910); and ibid., *170*, 26 (1910).

[2] Langmuir & Orange, Trans. Amer. Inst. E. E., *32*, 1944 (1913.)

[3] Phys. Rev., *1*, 332 (1913).

[4] Proc. Phys. Soc. of London, *27*, 230 (1915).

We shall see, from experiments to be described later, that there is a distinct difference in temperature between tungsten and a black body which matches it in color, and that this difference is in the direction predicted by Hyde. The difference however is small compared to the difference between true temperature and black body temperature.

(*c*) *Ratio of "Hot" to "Cold" Resistance.*—The ratio of the resistance at the temperature T to that at 0° C. or at room temperature has often been used for estimating temperature (v. Pirani, Corbino, Somerville).

This ratio affords the simplest and most convenient estimation of the temperature of filaments, and has the advantage that it can be used when the filament is surrounded by gas, or even when its *surface* is tarnished by oxidation. In practice, however, the method proves to be one of the least accurate methods of estimating temperatures for the following reasons:

1. The resistance increases relatively slowly with the temperature as compared with most other properties used for temperature estimation.
2. The resistance and its temperature coefficient are very sensitive to traces of impurities (carbon).
3. At room temperature the resistance of the filament is often so low that uncertainties in the lead and contact resistances are apt to play a large part.

(*d*) *Watts per Candle.*—This function has been used more than any other for the rating of tungsten lamps and is thus used as the basis of an arbitrary temperature scale.

Pirani has published[1] tables giving watts per candle as a function of the temperature.

Considerable confusion arises from the fact that the candle power may be measured in several ways: mean horizontal, maximum horizontal or mean spherical. For our purpose we shall measure the candle power of straight filaments in a direction perpendicular to their lengths. This corresponds most nearly to watts per mean horizontal candle power.

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[1] Verh. d. D. Phys. Ges., *14*, 213 (1912), and ibid., *14*, 681 (1912).

The advantages in the use of watts per candle are:

1. It requires no knowledge of the dimensions of the filament.

2. The function increases much more rapidly with temperature than does resistance, and is not so greatly affected by impurities in the metal.

The disadvantages are:

1. Unless the mean spherical candle power is measured the results will depend on the geometrical configuration of the filament.

2. The watts per candle do not increase nearly as rapidly with the temperature as the intrinsic brilliancy or the candle power.

3. Watts per candle cannot be used for estimating temperature when the filament is surrounded by a gas.

(*e*) *Resistance.*—If ρ be the resitivity of tungsten at the temperature T, then the resistance of a filament heated electrically to T will be

$$R = \frac{4\rho l}{\pi d^2}. \tag{2}$$

For convenience let us place

$$R' = \frac{4\rho}{\pi}.$$

Equation (2) thus becomes

$$R' = \frac{Rd^2}{l}, \tag{3}$$

where R' is a function of the temperature only and is numerically equal to the resistance of a filament of unit length and unit diameter.

(*f*) *Power Radiated.*—Since the energy radiated is proportional to the surface we may place for a filament in a vacuum

$$W' = \frac{W}{ld}, \tag{4}$$

where W' is numerically equal to the power (watts) radiated from a filament of unit diameter and unit length.

According to the Stefan-Boltzman law the power radiated per sq. cm. from a black body is σT^4 watts per cm.^{2}

From a tungsten surface the power radiated per cm.^{2} will be $E\sigma T^4$ where E, the "total emissivity," is also a function of the temperature. We may thus place

$$W' = \pi E\sigma T^4. \tag{5}$$

(*g*) *Candle Power.*—If C is the candle-power of a filament measured in a direction perpendicular to the axis of the filament, then we may place

$$C' = \frac{C}{ld}, \tag{6}$$

where C' (a function of the temperature only) is the total intrinsic brilliancy of the filament. This property differs, however, from (a), the intrinsic brilliancy, in being the calculated (mean) intrinsic brilliancy for the entire filament rather than the observed brilliancy of a small part of the filament.

The quantity C' can be theoretically calculated from the Planck equation, visibility function, emissivity of tungsten, and mechanical equivalent of light.

The power radiated from a black body per sq. cm. is

$$J_\lambda = \frac{C_1\lambda^{-5}}{e^{\frac{c_2}{\lambda T}} - 1} \tag{7}$$

If E_λ is the emissivity of tungsten for the wave-length λ then $E_\lambda J_\lambda$ will be the energy of wave-length λ, radiated from tungsten per sq. cm. The total light expressed in watts[1] is then

$$L_W = \int_0^\infty V_\lambda E_\lambda J_\lambda d\lambda. \tag{7a}$$

By dividing this by M, the mechanical equivalent of light (in watts per lumen) we obtain the total lumens per sq. cm. radiated by tungsten. To convert this to candles per sq. cm. the value in lumens per sq. cm. should be divided by π. Thus

$$C' = \frac{L_W}{\pi M}. \tag{8}$$

The function C' serves as a very convenient measure of temperature. It has the advantage over the measurement of intrinsic brilliancy by a pyrometer that it can be applied to filaments of very small diameter. The use of a photometer is often much more convenient than that of a pyrometer. In common with the pyrometer method it possesses the advantage of being applicable to filaments surrounded by gas.

(h) *Voltage.*—If we take the square root of the product of W' and R' we obtain for a filament in vacuum

$$V' = \sqrt{W'R'} = \frac{V\sqrt{d}}{l}. \tag{9}$$

Thus we see that the voltage drop per cm. along a heated filament in vacuum is inversely proportional to the square root of the diameter.

This relation is useful in determining what the length of a filament should be in order that it may be heated to a given temperature by a given applied voltage.

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[1] Ives, Astrophys. Jour., 36, 322 (1912).

(*i*) *Electron Emission.*—The use of electron emission to measure temperature was suggested by Richardson[1] but has apparently not been practically applied. Although under proper conditions[2] the electron emission from pure tungsten in a high vacuum is reproducible and could serve as a very accurate measure of temperature, it is found in practice that the electron emission is so enormously affected by minute traces of such substances as thorium in the filament or oxygen or water-vapor in the surrounding space, that this method is less reliable than almost any of the others considered in this paper.

(*j*) *Current.*—By taking the square root of the quotient of W' by R' we obtain

$$A' = \sqrt{\frac{W'}{R'}} = \frac{A}{d^{3/2}}. \tag{10}$$

In other words, the current necessary to heat a filament in vacuum to a given temperature varies with the 3/2 power of the diameter.

This function is especially convenient for estimating the temperature of filaments in experimental work. The diameter is readily measured (weight per unit length) and the length need not be known. This method proves in practice to be much more accurate than that based on resistance measurements (R') but less accurate than those based on optical measurements (C') or on total radiation (W'). Unfortunately it cannot be used when the filament is surrounded by gas.

(*k*) *and* (*l*) The two functions $C/(Vd^{3/2})$ and $\sqrt[3]{C/R}/d$ can be used to measure the temperature of filaments of unknown length but in most cases (*j*) will be found more convenient. However, when the filament is surrounded by a gas (*j*) is not applicable and in this case $\sqrt[3]{C/R}/d$ could be profitably used to measure temperature.

(*n*) $(V\sqrt[3]{A})/l$. This function proves very useful in estimating the temperature of filaments of varying diameter. For example, if it is desired to measure the rate of evaporation of a filament, or the rate of attack by a gas at very low pressure, the filament can be maintained at constant temperature by maintaining $V\sqrt[3]{A}$ constant, even when the diameter changes considerably. It also proves useful in estimating (by purely electrical measurements) the temperature of a filament in a sealed bulb. In this case the diameter often cannot be determined accurately but the length of the filament can be determined by cathetometer measurements.

This function is not applicable to filaments surrounded by gas.

(*o*) *and* (*p*) $(\sqrt[3]{CV^2})/l$, $C/(A^{2/3}l)$. These functions have as yet not found any application.

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[1] Phys. Rev., *27*, 183 (1908).

[2] Langmuir, Physik. Zeitsch., *15*, 516 (1914).

(*q*) $(\sqrt[3]{C^2R})/l$. This function can be used in place of $(V\sqrt[3]{A})/l$ for filaments of varying diameter when surrounded by a gas at relatively high pressure. With sufficient gas pressure convection currents carry away the evaporated material to a part of the bulb where it does not interfere with candle-power measurements. It may also be used for measuring the temperature of filaments (not helically wound) of unknown diameter in gases.

(*r*) *Thermal Expansion.*—This function can be used as an approximate measure of temperature. The change of length of a single loop filament can be determined fairly accurately by a cathetometer, so that the temperature may be readily found within 30–50° at 2500°.

EXPERIMENTAL MEASUREMENT OF CHARACTERISTICS.

The following general plan was adopted in the experimental study of the characteristics of tungsten filaments.

Specially constructed lamps with carefully measured long single loop filaments were thoroughly aged by running the filaments at 2400° K. for 24 hours after which they were set up in front of a Holborn-Kurlbaum pyrometer. The volts and amperes were thus obtained as functions of the temperature. The lamps were then measured on the photometer, and the candle power per cm. of length was determined by photometering through a measured horizontal slit.

From these data V', A' and C' were calculated as functions of the temperature and from these R', W' and W/C were obtained.

Lamps Used.—About 20 special lamps were constructed having cylindrical bulbs 7–9 cm. in diameter. The leads were of very large size (.05 cm. platinum for the smaller filaments, and .08 cm. tungsten for the larger sizes) so that the total lead resistance, obtained by subsequently welding together the two leads, averaged only .017 ohm. The filaments, in the form of single hair-pin loops, were electrically welded to the leads in such a way that the filament close to the lead was straight, and the junction of the lead and the filament was sharply defined. The lengths of the filaments were carefully measured after they were welded to the leads, but before they were sealed into the bulbs.

Tungsten Wires.—Twelve different samples of tungsten wire were used. Of these, six were regular samples of commercial wire, used in the manufacture of lamps. Four out of these six contained thorium (thoriated tungsten).

The remaining six samples were special experimental lots of wire drawn down from three different rods. Two of the rods were made by the Pacz process, and were of an unusually high degree of purity. The other rod was made of pure tungsten to which thorium oxide had been added.

The diameter of the wires ranged from .0028 to .0252 cm.

Measurement of Diameters.—Experience with many ways of measuring diameter has finally resulted in the adoption of calculation from the density and weight per unit length. The density of drawn tungsten wire (as taken from the spool) ranges from 18.4 to 19.0, but recent measurements on the filaments of lamps which have run 24 hrs. or more shows that the density nearly always lies between 18.9 and 19.1, with an average of 19.0. These measurements were made by weighing the filaments in air and in bromoform. Since it is the diameter of the wire after running in the lamp that is wanted, the value 19.0 has been chosen for the density in the calculation of the diameter.

The diameter in cm. is thus given by the formula (w = mg. per cm.).

$$d = .008186\sqrt{w}. \tag{11}$$

The diameters obtained this way for wires which have been run in lamps agree within the experimental error with measurements made with a micrometer.

Pyrometer.—The pyrometer was the same as that described in connection with the determination of the melting-point of tungsten (*l. c.*).

Before undertaking the study of the characteristics of tungsten filaments the pyrometer was carefully recalibrated.

For this purpose Hyde's value $\lambda = .664\ \mu$ for the effective wave-length of the red screen (double thickness of Schott & Gnossen's red glass No. 4512) was taken instead of the value[1] $\lambda = .667$ previously used. Slight errors of 0–12° at high temperatures were found in the calibration curve giving the relation between pyrometer current and true temperature. Although the new calibration curve differed very little from the old, yet by its aid the results obtained with different screens (red or green) or sectors agreed among themselves better than before.[2]

In calibrating this pyrometer for true temperatures it was assumed: First, that the melting-point of gold is 1335.4° K. Second, the emissivity of tungsten (for $\lambda = .664\ \mu$) is 0.46. Third, the bulb absorption was 9 per cent. and fourth, the constant C_2 of the Wien equation is 1.4392.

Photometer Measurements.—A Lummer-Brodhun constant illumination photometer was used. In most of the work an illumination of 57 candle meters was used, but with filament temperatures below 1900° K. it was necessary to use lower illuminations. In photometering the filaments at very high temperatures, the large difference in color between the filament and the standard lamp was avoided by inserting a blue glass screen between the standard lamp and the photometer head. The transmission coefficient for this screen was determined in this laboratory by the use of a flicker photometer. It was also calibrated for us through the kindness of Dr. Ives, by the use of his physical photometer.[1] The results by the two methods were practically identical.

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[1] Phys. Rev., *6*, 70 (1915).

[2] The temperatures obtained with the new calibration are on the whole slightly higher than with the old. The values for the melting-point of tungsten previously obtained ranged from 3530–3566°. The present recalibration would indicate that the higher values are probably correct, so that the most probable value for the melting-point would seem to be 3570° K.

The object of the photometer measurements was primarily to determine the intrinsic brilliancy of the filaments in candles per sq. cm. To avoid uncertain corrections due to the cooling effects of the leads, a slit, usually 2.5 cm. wide, was placed horizontally in front of and close to the lamp to be photometered. The intrinsic brilliancy was then found by dividing the observed candle power by the effective area of the filament. This effective area was calculated as follows: The effective length of the filament was taken as twice the width of the slit multiplied by the ratio of the distance of the filament from the photometer head to that of the slit from the photometer head. The effective area was then obtained as the product of the effective length by the diameter. Of course care was always taken that the two portions of the filament seen through the slit were as nearly straight and parallel as possible. Correction was made, when necessary, for any lack of parallelism.

Electrical Measurements.—The volts and amperes taken by the lamps were measured by " laboratory standard " instruments which were calibrated both before and after the measurements. Corrections were made for the current taken by the voltmeter and for the resistance of the leads of the lamps (usually about .017 ohm).

Correction for the Cooling by Leads.—As indicated above, the candle-power measurements were made in a way which eliminated any effect due to leads. The voltage measurements, however, were influenced by this cause, since the cooling of the ends of the filaments lowered the resistance. To determine the necessary correction, the temperature of the filament at two or three points near the leads was determined. From these data, by methods which will be described in a subsequent paper,[2] the correction was found which should be *added* to the observed voltmeter reading in order to eliminate the effect of the leads. In all cases this correction was approximately equal to

$$.00026\ (T - 300)\ \text{volts},$$

where T is the temperature (° K.) of the central portion of the filament.

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[1] PHYS. REV., *6*, 319 (1915).

[2] See also Worthing, PHYS. REV., 4, 538 (1914).

This correction is independent of the diameter of the filament in case the size of the lead bears a fixed ratio to that of the filament.

Preliminary Treatment of Filaments.—While the lamps were being exhausted the filaments were heated for one minute to a temperature of about 1500°. This treatment causes the evolution of an amount of gas (mostly carbon monoxide) which measured at atmospheric pressure is usually about 3 to 6 times the volume of the filament. There is also a simultaneous change in the filament whereby its cold resistance is lowered 15 to 20 per cent., and the temperature coefficient of its resistance increased by a like amount. Longer or more severe heat treatment causes the evolution of only insignificant quantities of gas.[1]

After sealing off from the pump, the characteristics of the lamps were studied and the temperatures were determined by the pyrometer. The observations were made at gradually increasing temperatures up to about 2600° and then a series of measurements at descending temperatures was made. When the observations with the descending temperatures are compared with the initial measurements at 1100 to 1500° it is found that at a given temperature the *wattage* decreased 30 to 50 per cent., whereas the resistance suffered little if any change. The effect of the first heating to 2600° is thus to decrease the total emissivity.

The filaments were then heated to 2400° K. for twenty-four hours before studying the characteristics. This heating produced a slight further decrease in emissivity and a slight (2 to 4 per cent.) decrease in resistance. More prolonged heat treatment than twenty-four hours produces (with drawn wire filaments) only very gradual changes. These changes may be of three kinds: First a roughening of the surface with slight increase in emissivity; second, a decrease in cross-section by evaporation and consequent increase in resistance of the filament, and third, a gradual elimination of traces of impurities by distillation, accompanied by a corresponding decrease in specific resistance.

With filaments of very large diameter the last of these processes takes place rather slowly, but with filaments up to 0.1 or 0.2 mm. diameter the change is nearly complete in twenty-four hours. The time of twenty-four hours was chosen for the heat treatment in these experiments because, if the time is extended beyond this, the changes due to the evaporation of tungsten and the resulting darkening of the bulb, are liable to more than offset the changes in resistance due to the elimination of minute traces of impurities.

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[1] If the bulb has not been well baked out, or if stop-cocks (grease!) are used in the system the continued heating of the filament will, by the decomposition of water-vapor or hydrocarbons, cause the slow evolution of apparently unlimited quantities of gas. (See J. Amer. Chem. Soc., *35*, 105 (1913).

EXPERIMENTAL RESULTS.

Volt-Ampere Characteristics.—With each lamp tested the readings of volts and amperes for about 30 to 50 pyrometer settings from 1050 up to 3540° K. were obtained. Up to temperatures of about 3000° K. the changes in the filament and the blackening of the bulb occurred so slowly that readings could be taken without haste. For higher temperatures than this the readings were taken at greater intervals (usually about 100° apart) and were made as quickly as possible, and after each reading the volts and amperes were redetermined at one or two lower temperatures (about 2400 and 2600° K.). From these data it was possible to calculate the change in diameter of the filament (due to evaporation) and the amount of absorption of light by the bulb. This calculation was made by the aid of the function $V\sqrt[3]{A}/l$. We have seen that this function for any given filament temperature is independent of the diameter of the filament. The relation between $V\sqrt[3]{A}$ and temperature was plotted from the data obtained before the lamp was run up to 3000°. This curve could be used to determine the true temperatures even after the filament had lost material by evaporation. By then comparing the observed pyrometer reading with the calculated true temperature, the bulb absorption was determined. Knowing the bulb absorption it was then possible to determine the true temperature when the filament was subsequently raised again to a temperature above 3000° K.

In this way it was found practicable to study the characteristics of the filaments in vacuum right up to the melting-point of the filaments. Although in some cases corrections as large as 250° (at 3500°) needed to be applied for the effect of bulb absorption, yet it is felt that the accuracy with which such corrections could be made was so high that the uncertainty in the final result was not over 30 to 40° (at 3540° K.). As a matter of fact the agreement between the results obtained with different lamps was considerably better than this.

From the observed values of volts and amperes (corrected for lead resistance, etc.) the values of the functions V' and A' were found from the known lengths and diameters. The logarithms of these functions were then plotted on a large scale against log T. The points from all the lamps tested were found to lie very closely along a smooth curve. The average deviation of the points from the smooth curve was found to be about 0.2 per cent. in temperature (both for V' and A'). For filament temperatures above 1200° K. the maximum departure of any of the observations was 1.2 per cent. in temperature. The small deviations which did occur were not all irregular, for certain lamps tended to give low values while others gave consistently high values (within the above

Table I.

Characteristics of Tungsten Filaments.

$T.$	V'	A'	W'	R'	C'	W/C	$V'\sqrt[3]{A'}$	E	Log V'	Log A'	Log C'
°K.	$\frac{V\sqrt{d}}{l}$	$\frac{A}{d^{3/2}}$	$\frac{W}{ld}$	$\frac{Rd^2}{l}$	$\frac{C}{ld}$		$\frac{V\sqrt[3]{A}}{l}$				
				$\times 10^{-6}$					−10		−10
273				6.37							
300	.000050	6.9	.00034	7.24			00009	.0237	5.6959	0.8362	
400	.000341	32.7	.0112	10.43			.00109	.0330	6.5331	1.5148	
500	.000764	55.5	.0424	13.76			.00291	.0421	6.8828	1.7442	
600	.001396	81.0	.1131	17.23			.00604	.0514	7.1448	1.9085	
700	.002330	111.8	.2606	20.83			.01123	.0626	7.3673	2.0486	
800	.003648	148.6	.5420	24.55			.01932	.0755	7.5620	2.1720	
900	.005439	191.8	1.043	28.36			.03136	.0904	7.7355	2.2828	
1000	.007795	241.8	1.885	32.24	.00014	13,460.	.04856	.1069	7.8918	2.3834	6.1455
1100	.01080	298.5	3.225	36.20	.00120	2,680.	.07221	.1247	8.0336	2.4749	7.0803
1200	.01454	361.5	5.258	40.23	.00738	712.5	.1036	.1435	8.1627	2.5581	7.8680
1300	.01908	430.2	8.207	44.34	.03461	237.1	.1440	.1625	8.2805	2.6337	8.5392
1400	.02445	504.0	12.32	48.52	.1325	93.0	.1945	.1813	8.3883	2.7024	9.1222
1500	.03071	582.0	17.87	52.77	.4243	42.1	.2564	.1995	8.4873	2.7649	9.6277
1600	.03792	663.7	25.17	57.13	1.179	21.35	.3309	.2170	8.5789	2.8220	0.0715
1700	.04613	748.9	34.55	61.61	2.928	11.80	.4189	.2338	8.6640	2.8744	0.4666
1800	.05539	836.8	46.34	66.19	6.552	7.074	.5219	.2494	8.7434	2.9226	0.8163
1900	.06575	927.5	60.98	70.89	13.46	4.530	.6412	.2643	8.8179	2.9673	1.1291
2000	.07725	1,021.	78.87	75.67	25.90	3.045	.7778	.2785	8.8879	3.0090	1.4133
2100	.08995	1,117.	100.5	80.52	46.8	2.147	.9333	.2920	8.9540	3.0481	1.6702
2200	.1039	1,216.	126.3	85.41	80.6	1.568	1.109	.3046	9.0165	3.0850	1.9063
2300	.1192	1,318.	157.1	90.41	133.3	1.179	1.307	.3174	9.0762	3.1200	2.1249
2400	.1357	1,423.	193.2	95.39	209.8	0.9207	1.527	.3290	9.1327	3.1532	2.3218

TABLE I.—*Continued.*

T	V'	A'	W'	R'	C'	W/C	$V'\sqrt[3]{A'}$	E	Log V'	Log A'	Log C'
°K.	$\frac{V\sqrt{d}}{l}$	$\frac{A}{d^{3/2}}$	$\frac{W}{ld}$	$\frac{Rd^2}{l}$	$\frac{C}{ld}$		$\frac{V\sqrt[3]{A}}{l}$				
				$\times 10^{-6}$					-10		
2500	.1538	1,531.	235.5	100.48	319.6	0.7367	1.772	.3407	9.1870	3.1849	2.5046
2600	.1733	1,642.	284.5	105.56	471.0	0.6041	2.045	.3517	9.2388	3.2153	2.6730
2700	.1943	1,756.	341.1	110.69	674.9	0.5054	2.344	.3625	9.2885	3.2444	2.8293
2800	.2169	1,873	406.3	115.83	944.0	0.4303	2.674	.3733	9.3363	3.2725	2.9750
2900	.2410	1,993.	480.5	120.9	1,290.	0.3725	3.034	.3839	9.3821	3.2996	3.1106
3000	.2669	2,117.	565.2	126.1	1,729.	0.3270	3.428	.3941	9.4264	3.3258	3.2377
3100	.2944	2,244.	660.7	131.2	2,272.	0.2908	3.854	.4041	9.4689	3.3511	3.3564
3200	.3236	2,376.	768.8	136.2	2,941.	0.2615	4.318	.4142	9.5100	3.3758	3.4684
3300	.3543	2,511	889.6	141.1	3,763.	0.2364	4.816	.4239	9.5494	3.3998	3.5755
3400	.3863	2,649.	1,025.	146.0	4,725.	0.2169	5.352	.4334	9.5875	3.4231	3.6744
3500	.4213	2,792.	1,176.	150.9	5,869.	0.2004	5.932	.4428	9.6246	3.4459	3.7685
3540	.4355	2,850.	1,241.	152.8	6,373.	0.1948	6.175	.4467	9.6390	3.4549	3.8043

mentioned limits). No relationships, however, could be traced between these deviations from the mean and the diameter or other characteristics of the wire.

The values of log V' and log A' obtained from the smooth curve described above from 1300° up to 3540° have been tabulated in Table I., together with the corresponding values of V' and A'. For temperatures below 1300° the uncertainty in the cooling effect of the leads and the difficulties in the accurate temperature measurements have made the results of the direct experiments less trustworthy, so that the characteristics below 1300° have been determined by another method which will be described later.

The values of W', R' and $V'\sqrt[3]{A'}$ given in Table I. have been obtained directly from V' and A'.

Candle-Power Data.—To determine the relation between candle power per sq. cm. and temperature, three lamps were studied. Two of these were nitrogen-filled lamps with large (0.6 mm. diam.) single loop, straight (not helically wound) filaments, and the third was a vacuum lamp with a single loop filament, 0.25 mm. diameter. By the use of nitrogen-filled lamps it was possible to determine the relation between temperature and candle power up to temperatures of 3100° K. without any blackening of the bulbs or any permanent change in the characteristics of the filament.

TABLE II

Experimental Values of C'

Temp.	C' Candles per Sq. Cm.	Ratio C'/L_W.
1400	.109	216.2
1500	.381	235.8
1600	1.101	245.2
1700	2.824	253.4
1800	6.493	260.4
1900	13.43	262.0
2000	25.67	260.4
2100	46.85	262.8
2200	81.33	265.2
2300	133.3	262.6
2400	208.	260.8
2500	318.0	261.4
2600	470.5	262.4
2700	677.	263.4
2800	949.	264.2
2900	1295.	263.8
3000	1733.	263.4
3100	2273.	262.8
	Average.....	262.7

At low filament temperatures irregularities occur in the convection losses from the filaments of gas-filled lamps. It was to avoid these, and to find whether the nitrogen exerted any effect on the emissivity that the vacuum lamp was studied.

The values of log C' obtained from the three lamps were plotted against $1/T$. A smooth curve (not far from a straight line) was passed as nearly as possible through the points. The average departure of the points from the smooth curve was about 0.15 per cent. in temperature, and the maximum departure of any of the points was 0.8 per cent. in temperature.

The values of C' taken from the smooth curve for every hundred degrees from 1400 to 3100° are given in the second column of Table II.

DISCUSSION OF EXPERIMENTAL DATA.

It will be of interest to compare the results of the above experiments with the data of other investigators. This will also have the advantage that it will enable us to extend our data on the characteristics of tungsten filaments down to temperatures below 1300° K., where our experiments gave unreliable results because of the cooling effects of leads and the difficulty of measuring temperatures by the optical pyrometer.

The ratio of hot to cold resistance for tungsten filaments has been determined by Somerville,[1] Pirani,[2] and Corbino.[3]

From these data it is possible to calculate R' provided we know the specific resistance of tungsten at room temperature.

Specific Resistance of Tungsten.—Fink,[4] has given the specific resistance of tungsten at 25° C. as 5.0×10^{-6} ohm-cm. units, and its temperature coefficient between 0° and 170° as .0051. Thus at 0° C. according to Fink the specific resistance would be 4.43×10^{-6} ohm-cm. More recent measurements made in this laboratory of the specific resistance of well-aged tungsten filaments have given at 0°, 5.0×10^{-6} ohm-cm.

The temperature coefficient from 0 to 300° is found to agree well with Fink's value, *i. e.*, .0051.

We have seen that the function R' is equal to $4\rho/\pi$ so that we may place at 0° C. $R' = 6.37 \times 10^{-6}$.

At higher temperatures we may now calculate R' from Somerville's, Pirani's and Corbino's data by simply multiplying their values of the ratio of hot to cold (0° C.) resistance by the above value for R' at 0° C. In this way the results given in the second, third and fourth columns of Table III. have been obtained.

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[1] PHYS. REV., *30*, 433 (1910).

[2] Verh. deutsch. Phys. Ges., *12*, 301 (1910), and Phys. Zeit., *13*, 753 (1912).

[3] Phys. Zeit., *13*, 375 (1912).

[4] Trans. Amer. Electrochem. Soc., *17*, 229 (1910).

In our own experiments on the characteristics of tungsten filaments we have measured R' from 1300° K. up to 3540° K. and from 273° K. to 600° K. The agreement with Somerville's results at low temperatures is excellent, while at high temperatures the results agree with Pirani's. At lower temperatures Pirani's values appear to be too low.

To fill in the gap in our measurements between 600° and 1300° the values have been chosen by interpolation between the higher and lower values, laying due weight on the results of Somerville and Pirani.

Corbino's results above 1700° are too low, probably due, as Pirani has pointed out[1] to failure to correct for the cooling effect of the leads.

Table III.

Comparison of Values of R^1.

T	Somerville.	Pirani.	Corbino.	Langmuir.
273	6.37	6.37	——	6.37
300	7.24	7.24	——	7.24
400	10.45	10.06	——	10.43
500	13.76	13.19	——	13.76
600	17.4	16.5	——	17.23
700	20.8	20.0	——	20.83
800	24.5	23.6	——	24.55
900	28.2	27.4	——	28.36
1000	31.7	31.2	——	32.24
1100	35.6	35.2	——	36.20
1200	40.1	39.2	——	40.23
1300	45.4	43.3	——	44.34
1400	——	47.3	——	48.52
1500	——	51.5	——	52.77
1600	——	56.1	——	57.13
1700	——	60.8	60.2	61.61
1800	——	65.6	64.5	66.19
1900	——	70.6	68.5	70.89
2000	——	75.5	72.8	75.67
2100	——	80.5	76.9	80.52
2200	——	85.5	81.2	85.41
2300	——	90.5	85.4	90.34

The values of R' obtained as above (from 300 to 1300°) have been inserted in Table I.

The specific resistance of tungsten at any temperature may be obtained by multiplying the corresponding value of R' by $\pi/4$. A table of specific resistances calculated as above has been published.[2]

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[1] Phys. Zeitsch., *13*, 753 (1912).

[2] Phys. Rev., *7*, 154 (1916).

Total Emissivity.—Planck's law (Equation 7) gives J_λ the energy of wave-length λ radiated from a black body. The total energy W' radiated from a tungsten filament one cm. in diam. and one cm. long is then

$$W' = \pi \int_0^{\infty} E_\lambda J_\lambda d\lambda. \tag{12}$$

On the other hand, by the Stephen-Boltzman law we may place

$$W' = \pi E \sigma T^4, \tag{13}$$

where E is the *total emissivity* and the constant σ is equal to 5.633×10^{-12} watts per sq. cm.

The values of E, above 1200°, given in Table I. have been calculated from the corresponding values of W' by means of this equation.

For the lower temperatures (up to 600° K.) the value of E has been calculated by a formula given by Foote,[1] namely,

$$E = 0.5736\sqrt{\rho T}, \tag{14}$$

where ρ is the resistivity of ohm-cm.

The results obtained from this equation for the higher temperatures do not agree at all well with the values found from our experiments, as may be seen from the following comparison.

TABLE IV

T.	By Foote's Formula.	From Table I.
1200	.1118	.1435
1600	.1537	.2170
2000	.1978	.2785
3000	.3125	.3941

The failure of Foote's formula at the higher temperatures is to be expected if we take into account the work of Rubens and others on the temperature coefficient of the emissivity of metals. Rubens[2] by a study of the emissivity of platinum, platinum-rhodium, nickel and constantan at different temperatures, concludes that for wave-lengths below 2 μ the temperature coefficient of the emissivity is negligible or at least very small, whereas for wave-lengths greater than 6 μ the emissivity may be calculated accurately by the Hagen-Rubens formula

$$E_\lambda = .365\sqrt{\frac{\rho}{\lambda}}. \tag{15}$$

For these longer wave-lengths the temperature coefficient of the emissivity is thus one half that of the electrical resistivity.

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[1] Bull. Bur. Stand., *11*, 607 (1915).

[2] Verh. deutsch. Phys. Ges., *12*, 172 (1910).

Between the limits of 2 and 6 μ Rubens finds that the transition from one type of temperature coefficient to the other occurs gradually.

Investigations on many different metals during the last few years have shown that the Hagen-Rubens formula is universally applicable for the longer heat rays, whereas in the visible spectrum the emissivity seems to be determined by totally different causes and is, in general, very nearly independent of the temperature.

The Foote formula for the total emissivity is derived from the Hagen-Rubens equation and should therefore be applicable only where most of the energy radiated is limited to wave-lengths for which the emissivity is given by the Hagen-Rubens equation. Since the wave-length of maximum energy emission is about 5 μ for a black body at 600° K. and is about 2 μ at 1500° K. we should expect the Foote equation to apply quite accurately up to about 500 or 600° K. but above this temperature the Foote equation should give low values, and at temperatures as high as 1500° K. it should be nothing more than a rough approximation. This seems to be borne out by the experimental data.

In order to obtain values for the emissivity at temperatures between 600° and 1200° recourse has been taken to interpolation between Foote's formula at lower temperatures and the experimental data at the higher temperatures. The results obtained in this way have been entered in Table I.

The uncertainty involved in this interpolation probably does not exceed five per cent. in any single value given for the emissivity.

From the data on the total emissivity the values of W' for temperatures below 1200° were calculted by means of equation (13). At the lower temperatures, however, " back radiation " was taken into account so that W' was actually calculated from the equation

$$W' = \pi\sigma(ET^4 - E_0T_0^4). \tag{16}$$

Here T_0 is room temperature which is taken to be 20° C. ($T_0 = 293$).

E_0 is the total absorption coefficient of a filament at a temperature T for energy radiated by a black body at T_0, that is, at 293°. Since the wave-length of the energy radiated at room temperature is longer than 6 μ Hagen and Rubens's formula should apply and we may therefore consider that E_0 increases in proportion to $\sqrt{\rho}$. For $T = 293$ we may place $E_0 = E$ and may thus calculate E_0 from Foote's formula.

$$E_0 = 0.0230\sqrt{\rho/\rho_0}, \tag{17}$$

where ρ_0 is the resistivity at 293°.

The values of W' from room temperature up to 1200° have been calculated from equations (16) and (17). The functions A', V', etc., for these temperatures have been calculated from W' and R'.

Total Intrinsic Brilliancy.—It will be of interest to compare the experimentally determined candles per sq. cm. (Table II.) with the results of the calculation of this quantity from the Planck equation and visibility function according to equation 8.

For this purpose we take the constants of the Planck equation to be

$$C_1 = 3.721 \times 10^{-12} \text{ watts per sq. cm.,}$$

$$C_2 = 1.4392 \text{ cm.}$$

These values, according to Planck's theory, correspond to the following values of the fundamental constants h, N, and σ[1]

$$h = 6.580 \times 10^{-27} \text{ ergs. sec.,}$$

$$N = 6.062 \times 10^{23},$$

$$\sigma = 5.633 \times 10^{-12} \text{ watts per sq. cm.}$$

Nutting's[2] visibility data has been chosen since Ives[3] considers this more accurate than his own.

From these data the light (in watts per sq. cm.) radiated from a black body has been calculated by the equation

$$L_B = \int_0^\infty V_\lambda J_\lambda d\lambda. \tag{18}$$

The integration was performed by the aid of Simpson's one third rule using 35 ordinates at each temperature. The calculations were carried out to four significant figures and the results should be accurate to 0.2 per cent. The values of L_B obtained in this way are given in Table V.

Similar calculations of the light radiated from tungsten were then made by the same method. For this purpose the emissivity of tungsten

TABLE V

Light Radiated from a Black Body and from Tungsten in Watts/Cm.[2] *Calculated from the Planck Equation and Nutting's Visibility Data.*

T °K.	L_B Black Body.	L_W Tungsten.
1000	.00000109	.000000532
1300	.000269	.000132
1500	.00329	.001615
1750	.03429	.01687
2000	.2000	.0986
2500	2.458	1.216
3000	13.26	6.580
3500	44.98	22.34

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[1] Dushman, Gen. Electric Rev., December, 1915.

[2] Trans. I. E. S., *9*, 633, (1914).

[3] PHYS. REV., *5*, 287 (1915).

E_λ was taken to be 0.46 for $\lambda = .66\,\mu$ and 0.50 for $\lambda = 0.55$ while for other wave-lengths in the visible spectrum the emissivity was obtained by linear interpolation or extrapolation from these values.[1]

Thus the light radiated from tungsten was obtained by the equation

$$L_W = \int_0^\infty E_\lambda V_\lambda J_\lambda d\lambda. \tag{19}$$

The values of L_W have been recorded in Table V.

The quantity L_W should be proportional to C' the candles per sq. cm. found by experiment. In Table II. the ratio of C' to L_W has been tabulated.[2] It is seen that at temperatures above 1900° the ratio is very nearly constant. The absence of any systematic error is an excellent confirmation of the correctness of the pyrometric measurements. At temperatures below 1900° the ratio of C' to L_W decreases considerably. This, however, is to be expected, for it was necessary to work with very low field illuminations (even as low as 0.2 meter candles) at the lower temperatures, and under these conditions it is well known that the Purkinje effect tends to give low values in the measurement of red light. We are therefore justified in discarding the measurements made below 1900°. The average of the remaining values of the ratio gives

$$\frac{C'}{L_W} = 262.7. \tag{20}$$

The mechanical equivalent of light may be obtained from this ratio by equation 8:

$$M = \frac{L_W}{\pi C'} = .00121 \text{ watt per lumen.} \tag{21}$$

This value is considerably lower than the latest value of this constant given by Ives and Kingsbury,[3] namely .00159. Nutting[4] gives the value 66.2 candles per watt which corresponds to .00120 lumen per watt, and this he considers is uncertain within about five per cent.

Ives[5] has previously calculated the mechanical equivalent by the method given above, using Nernst's[6] data for the intrinsic brilliancy of a black body. This result, recalculated according to more recent visibility data[1] leads to the value .00125 watt per lumen. Ives suggests that the discrepancy between this value and .00159 may be due to a failure of the Planck equation for short wave-lengths.

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[1] The value 0.46 is taken from the author's paper on the melting-point of tungsten (l. c.). The slope of the curve E_λ (λ) was taken from Coblentz (Bull. Bur. Stand., *7*, 198 (1911)).

[2] For this purpose the values of L_W for temperatures other than those given in Table V. were found by interpolation. This could be done with high accuracy since log L_W is very nearly a linear function of $1/T$.

[3] Phys. Rev., *6*, 319 (1915).

[4] Trans. I. E. S., *9*, 633 (1914).

[5] Electrical World, 1911, p. 1565.

[6] Phys. Zeit., *7*, 380 (1906).

It should be noted that the value .00121 obtained from the experiments described in this paper does not depend to any great extent upon assumptions as to the emissivity of tungsten. This is clear when we consider that an error in emissivity would affect the temperatures obtained by the pyrometer in such a way as to offset the change in the brilliancy calculated by equation 19.

The ratio $C' : L_W$ in Table II. is so nearly constant that we may use it to calculate C' from L_W with a higher accuracy than that of the original values of C' given in the table.

The values of C' given in Table I. have therefore been obtained from the corresponding values of L_W by multiplying them by 262.7, the mean value of C'/L_W.

In a similar way we may obtain the intrinsic brilliancy of a black body in candles per sq. cm. from the corresponding value of L_B by multiplying these by 262.7. Results obtained this way are given in the second column of Table VI.

There have been several other determinations of the total intrinsic brilliancy of the black body, and it will be of interest to compare some of these with the above results.

In a recent publication Pirani[2] calculates the intrinsic brilliancy by a similar method to that which we have used. He chooses for the constants of the Planck equation

$$C_1 = 3.5 \times 10^{-12} \text{ watts per sq. cm.},$$

$$C_2 = 1.440 \text{ cm.},$$

and he uses Ives's visibility data. In this way he finds the slope of the curve (log C against $1/T$), and then in order to fix the position of the curve takes the value 1 Hefner candle per sq. cm. as the brilliancy of a black body at 2090° K. This value he obtains by extrapolation over a range of 400° from some data given by Lummer and Pringsheim in 1901.

In the third column of Table VI. are given Pirani's results converted to international candles per sq. cm.

At low temperatures Pirani's values for the brilliancy are 10 to 20 per cent. *higher*, and at high temperatures as much as 15 per cent. *lower* than those we have calculated. Both sets of values are calculated from the Planck Equation and visibility data. The discrepancy cannot be accounted

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[1] PHYS. REV., *5*, 273 (1915).

[2] Verh. d. D. Phys. Ges., *13*, 219 (1915).

TABLE VI

Brilliancy of Black Body (C_B').
International candles per sq. cm.

T	Langmuir.	Pirani.	Nernst.	$\Delta T.$ T_L-T_N
1300	.0706	.086	.0482	−26.
1400	.270	.306	.199	−24.
1500	.864	.936	.648	−26.
1600	2.40	2.52	2.01	−18.
1700	5.95	6.03	5.19	−16.
1800	13.3	13.23	12.1	−13.
1900	27.3	27.9	25.7	− 9.
2000	52.5	53.1	50.8	− 5.
2100	94.9	94.5	94.1	− 2.
2200	163.	153.	164.7	+ 2.
2300	270.	252.	275.0	+ 4.
2400	424.	414.	439.	+ 8.
2500	646.	630.	674.	+10.
2600	951.	900.	1,000.	+13.
2700	1,362.	1,260.	1,450.	+17.
2800	1,904.	1,800.	2,040.	+21.
2900	2,600.	2,430.	2,810.	+25.
3000	3,485	3,330.	3,780.	+29.
3100	4,579.	———	5,000.	+33.
3200	5,920.	5,040.	6,480.	+36.
3300	7,580.	———	8,280.	+37.
3400	9,515.	8,280.	10,430.	+41.
3500	11,820.	———	12,960.	+43.

for by the slightly different values chosen for the constant C_2 nor does the fact that Pirani chose the Ives instead of the Nutting visibility curve account for the difference.[1]

The difference between the Pirani and Langmuir values for C_B' would seem to be caused by insufficient accuracy in the graphical methods used by Pirani in determining the area of his curves.

Nernst[2] has given the following relation between the temperature and the brilliancy of a black body:

$$T = \frac{11230}{5.367 - \log_{10} K} \tag{22}$$

where K is the brilliancy in Hefnerkerzen per sq. mm. If we express the brilliancy in international candles per sq. cm. this becomes

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[1] In order to determine how much the values of C_B' would be changed by using Ives's visibility data in place of Nutting's, the values of L_B (Table V.) were recalculated from Ives's data. The new values were found to be uniformly 5.8 per cent. higher than those given in Table V. Thus the use of Ives's data cannot account for the fact that the Pirani and Langmuir curves (Table VI.) *cross each other.*

[2] Phys. Zeit., 7, 380 (1906).

$$T = \frac{11230}{7.321 - \log_{10} C_B'}. \tag{23}$$

The values given in the fourth column of Table VI. are calculated by means of this equation.

Until recently the temperature scale used in this laboratory has been based on this Nernst equation. The corrections that should be applied to this temperature scale to reduce it to the new scale are given in the fifth column. These figures may be used to correct temperature data which have been given in various publications from this laboratory. In most cases the errors involved are within the experimental error.

COLOR OF LIGHT RADIATED BY TUNGSTEN.

A series of experiments was undertaken to determine whether the light radiated from tungsten differs materially in color from that radiated by a black body. A helix was formed by winding a 20 mil tungsten wire on a 40 mil mandrel using a 20 mil spacer. This filament was sealed into a bulb which was then filled with nitrogen. A very much enlarged image of the heated filament was thrown on a white screen and by means of a Weber portable photometer the intensity and color of various parts of the image were determined.

The intensity of the light coming from the interior of the helix was from 1.5 to 1.8 times as great as that coming from the outer portions. The color of the light from the interior was distinctly redder than that from the outside. By first color-matching the interior of the helix against the photometer lamp and then increasing the current through the helix until the outer portions gave a color-match with the photometer, it was possible to determine the difference in color quantitatively.

If we take the emissivity of tungsten for white light to be 0.50 and assume that the inner portion radiates like a black body, then the ratio of the brilliancy of the inner and outer portions should be 2.0. Actually the observed ratio as found by the photometer was less than this (1.5 to 1.8) showing that strictly black-body conditions did not obtain. The observed difference in color between the inner and outer portions was therefore corrected for this lack of blackness as measured by the ratio of brilliances.

Two methods may be used in making this correction: First, it may be assumed that the light from the interior is the light emitted from a tungsten surface which has been reflected n times before escaping. On this assumption

$$\frac{\Delta_2}{\Delta_1} = \frac{-\log E}{\log R}, \tag{24}$$

where Δ_1 is the observed difference in color between the inner and outer parts (n reflections), Δ_2 is the difference in color which would exist if the inner parts radiated like a black body ($n = \infty$), R is the observed ratio of brilliancy and E is the emissivity of tungsten.

Second, it may be assumed that the light from the inner parts is a mixture of light from some parts which radiate as a black body, with light from other parts which radiate like an exposed tungsten surface. On this basis we find:

$$\frac{\Delta_2}{\Delta_1} = \frac{1 - E}{1 - 1/R}, \tag{25}$$

where the symbols have the same meaning as before. The corrections actually used were obtained by averaging those obtained by the two methods. The corrections in most cases amounted to 20 to 40 per cent. The results obtained in this way are given in Table VII.

Table VII.

T_W	T_B	$T_B - T_W$	$\frac{d \ln E_\lambda}{d\lambda}$
1800	1841	41	−5,900
2000	2050	50	−5,800
2200	2257	57	−5,500
2400	2464	64	−5,100
2600	2672	72	−4,900
2800	2877	77	−4,500
3000	3082	82	−4,200
3200	3285	85	−3,800

In this table T_B is the temperature of a black body which will give light of the same color as that emitted by a tungsten filament at the temperature T_W.

No great accuracy is claimed for these results; the differences in temperature, $T_B - T_W$, may be in error by 20 or 30 per cent. But, in contrast to the conclusions of Paterson and Dudding[1] these results show definitely that there is a distinct difference in color between the light from a tungsten filament and from a black body of the same temperature.

This difference in color is evidence that tungsten is not a gray body. Tungsten even in the visible region of the spectrum is a selective radiator giving a larger proportion of blue light than a black body. The energy distribution in the visible spectrum of a black body is given by Wien's equation. If E_λ be the emissivity of tungsten for the wave length λ, the energy distribution of the light emitted by a tungsten filament will therefore be

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[1] Loc. cit.

$$J_W = C_1 E_\lambda \lambda^{-5} e^{-\frac{C_2}{\lambda T_W}}, \tag{26}$$

where T_W is the true temperature of the tungsten.

Experimentally it has been found that the color of the light emitted by tungsten is the same as that emitted by a black body at a slightly different temperature T_B. The energy distribution from the black body is

$$J_B = C_1 \lambda^{-5} e^{-\frac{C_2}{\lambda T_B}}. \tag{27}$$

Since the tungsten filament and the black body are of the same color, although not of the same brilliancy, the ratio of J_W to J_B must be constant for all wave lengths in the visible spectrum. It is thus possible to obtain from (26) and (27) the relation

$$\frac{d(lnE_\lambda)}{d\lambda} = \frac{C_2}{\lambda^2}\left(\frac{1}{T_B} - \frac{1}{T_W}\right). \tag{28}$$

We have seen that $T_B - T_W$ is small compared to T_W so that we may place approximately

$$\frac{dlnE_\lambda}{d\lambda} = \frac{C_2(T_W - T_B)}{\lambda^2 T^2}. \tag{29}$$

From this equation, with the data of Table VII., we may calculate the emissivity of tungsten for one wave-length if we know it for another wave-length. Let us place $C_2 = 1.439$, $\lambda = 550 \times 10^{-7}$ cm. and take $T_W - T_B$ from Table VII.

The values of $(d/d\lambda)lnE_\lambda$ thus found by (29) are given in the fourth column of Table VII. Since this quantity varies with the temperature it must follow that the emissivity of tungsten varies with the temperature. How large must this variation be in order to account for the figures obtained? If we assume that $E_\lambda = 0.46$ for $\lambda = 0.664\,\mu$ at 2400° K. then we may calculate the value of E_λ at 2400° K. for other wave-lengths by integrating (29). The resulting values are given in the third column of Table VIII.

It is probable from Rubens's work that the variation of the emissivity with temperature occurs mostly in the region of longer wave-lengths; so that we may safely consider that the temperature coefficient of E_λ for $\lambda = 0.4$ micron is negligible. We thus obtain $E_\lambda = 0.525$ at $\lambda = 0.4$ for all temperatures. Then by (29) we may calculate E_λ at these temperatures for other wave-lengths. The results at 1800° and 3000° are given in Table VIII.

The change in emissivity with wave-length is in fairly good accord

TABLE VIII.

Emissivity E_λ of tungsten as a function of wave-length and temperature as calculated from the color of the emitted light.

λ (Microns).	1800° K.	2400° K.	3000° K.	Coblentz 300° K.
0.7	0.440	.450	.463	.460
0.664	0.449	(.460)	.470	
0.60	0.465	.475	.483	.487
0.55	0.480	.486	.494	
0.5	0.495	.498	.503	.507
0.4	(0.525)	.525	(.525)	.530

with the measurements of Coblentz,[1] which are given in the fifth column of Table VIII.

The variation of E_λ with temperature indicated in the table is so slight that it does not materially affect our temperature scale which has been based on the assumption of constant emissivity. The accuracy of the results given in Table VIII. is probably not high, so that it is not desirable at present to correct the temperature scale for these rather small variations. The changes are in such a direction as to make the true temperature scale lie somewhere between the Nernst and the Langmuir scales given in Table VI. The corrections in the last column of this table should therefore probably be reduced to about one half the values given.

THERMAL EXPANSION OF TUNGSTEN.

By cathetometer measurements of the length of a tungsten filament at temperatures from 1200° to 2500°, the following relation has been found, l being the length at 300° K.

$$\frac{\Delta l}{l} = .00245\left(\frac{T-300}{1000}\right) + .000567\left(\frac{T-300}{1000}\right)^2. \tag{30}$$

This equation is probably accurate within two or three per cent.

SUMMARY.

The contents of this paper are briefly as follows:

1. A discussion of the various functions that may be used in estimating the temperature of tungsten filaments.

2. Experimental data (see Table I.) on the volt-ampere-candle-power characteristics of tungsten filaments as functions of temperature and the dimensions of the filament. Derived functions such as watts, ohms and watts per candle are also given. These data cover the range of temperature from room temperature up to the melting point of tungsten. All the data are corrected for the cooling effects of the leads.

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[1] Bull. Bur. Stand., 7, 197 (1911).

3. Data on the specific resistance of tungsten from 300° K. to 3540° K. The specific resistance may be obtained by multiplying the values of R' in Table I. by $\pi/4$.

4. The total emissivity of tungsten (black body taken as unit) has been calculated from these data for temperatures up to 3540° K. and the results have been tabulated in the column marked E of Table I. At temperatures above 1200° K., Foote's formula for the total emissivity of metals gives results which are much too low. Below about 500° or 600° K. Foote's formula is probably quite accurate.

5. The intrinsic brilliancy of a black body as a function of the temperature has been calculated in terms of the mechanical equivalent of light from Nutting's visibility data, by means of the Planck equation. By comparing these results with the experimentally determined brilliancies of tungsten and correcting for the known emissivity of tungsten the mechanical equivalent of light is found to be .00121 watt per lumen. This is close to the value (.00125) calculated by Ives from Nernst's data on the brilliancy of a black body, but differs considerably from Ives and Kingsbury's direct determinations of the mechanical equivalent (.00159) The discrepancy is hard to account for.

6. The intrinsic brilliancy of a black body is calculated from the experimental data for tungsten and is compared with similar data obtained by Nernst and Pirani. These data are given in Table VI.

7. The color of the light from tungsten filaments is distinctly bluer than that from a black body at the same temperature and corresponds to that from a black body at a temperature 40 to 80° higher.

8. The linear thermal expansion of tungsten at temperatures from 1200 to 2500° K. is given approximately by the equation 30.

The writer wishes to express his indebtedness to Mr. G. M. J. Mackay and Mr. C. V. Ferguson for the active part they have taken in obtaining the experimental data upon which this paper is based.

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ON EWELL'S METHOD OF MEASURING SINGLE POTENTIAL DIFFERENCES.

By S. J. Barnett.

IN the October number of this journal Professor Ewell[1] describes an interesting method of determining single differences of potential which he believes free from the fallacies involved in methods hitherto proposed.

The theory of the method, which the paper does not give in systematic form, can best be discussed with the aid of a diagram, such as Fig. 1. The black parts of this diagram represent homogeneous substances without electric fields in the steady state, while the white parts represent those electric fields which it is necessary to consider. The (constant) double layers in the electrometer are not shown; and for simplicity all parts of the metal case A, surrounding the apparatus, the metal parts of the electrometer and its connections, and the metal coat G of the vessel containing the liquid under experiment are assumed to be made of the same substance.

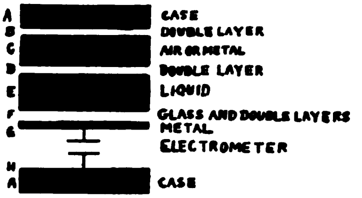

Fig. 1.

In making the experiments the liquid E is first momentarily connected to the surrounding conductor A, to which G, the metal coat of the glass vessel F containing the liquid, is also connected.

After waiting several hours, when the residual conductivity of the insulators is assumed, on the basis of certain experiments with mercury (see below) in place of the liquid E, to have reduced to zero the voltage from the liquid E to the conductor A,[2] the zero reading of the electrometer (unchanged since the preceding operation if there is no drift) is taken.

[1] A. W. Ewell, Phys. Rev., 6, 271, 1915.

[2] By using an ionizing agent such as X-rays it would seem that the regularity of the results could be greatly increased and the time consumed in an experiment greatly reduced.

Then G is insulated; and one end of the metal C' under test is connected to A, and the other then dipped in E. This in effect substitutes the metal C' for the air C. The electrometer is then read and the single potential difference between the metal and the liquid is calculated. The process involved in this calculation will now be examined.

Before the insulation of G and the electrometer the voltage V from the liquid E to A is the sum of the voltage V_D across the double layer D and the voltage V_B across the double layer B. That is,

$$V = V_D + V_B. \tag{1}$$

This is also equal to the voltage from E to G across F. If the capacity of F is C_1, the charge at the lower surface of E is

$$C_1V = C_1(V_D + V_B). \tag{2}$$

When now G is insulated and the metal C' substituted for the air C, the voltage V' from the liquid to A is

$$V' = V_D' + V_B' = V_1' + V_2', \tag{3}$$

where V_D' and V_B' are the voltages through the new double layers and V_1' and V_2' are the new voltages through F and the electrometer. The charge of F is now C_1V_1' and the free charge of the electrometer with capacity C_2 is

$$C_2V_2' = C_1(V_1' - V). \tag{4}$$

From (4), (3), and (1) we obtain the relation

$$V_D' + (V_B' - V_D - V_B) = \left(1 + \frac{C_2}{C_1}\right) V_2' = \text{Constant} \times \text{Deflection}, \tag{5}$$

the constant being determined by separate calibration experiments.[1]

The method thus determines only $V_D' + (V_B' - V_D - V_B)$, while V_D' alone is the quantity sought. V_B' is the contact voltage from the metal to the surrounding conductor A, V_D the voltage from the electrolyte to air, and V_B the voltage from the air to the metal A. It is thus not apparent that the quantity within the parentheses is zero, as it must be if the experiment determines V_D'.

Ewell concludes that the quantity within the parentheses is zero from a series of experiments on a number of metals with mercury substituted for the electrolyte E. In each of these experiments he found, at least approximately, with the nomenclature used here,

$$V_d' + (V_b' - V_d - V_b) = 0, \tag{6}$$

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[1] These calibration experiments involve errors of the same kind as those involved in the principal experiments, but for the sake of simplicity the calibration will be assumed correct. If the electrometer is a long range instrument, and if its deflections are proportional to voltages throughout its range, the calibration by Ewell's second method may be made with negligible error by using a calibration voltage much greater than the contact voltages involved.

the small letters corresponding to the capitals used above when the electrolyte was in the place of the mercury.

If we neglect metal-metal contact voltages, the first member of (5) becomes

$$V_D' - (V_D + V_B), \tag{7}$$

and equation (6) gives

$$(V_d + V_b) = 0. \tag{8}$$

If now, neglecting metal-metal contact voltages, we admit that (8) is correct, or approximately correct, as it may be since V_d is a voltage from a *metal to air* and V_b a voltage from *air to a metal*, we still cannot conclude, as Ewell concludes, that *therefore also* $(V_D + V_B)$, whose first term is a voltage from an *electrolyte to air* and whose second term is a voltage from *air to a metal*, is zero. The assumption, moreover, that metal-metal contact voltages are negligible is one which the evidence does not warrant at present.[1]

We thus appear to be forced to the conclusion that the method under discussion cannot give with any certainty the single potential differences which other methods have always shown so elusive.

THE OHIO STATE UNIVERSITY,
November, 1915.

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[1] See especially the work of Millikan and his collaborators at the Ryerson Laboratory, published in recent volumes of this journal.

A MICA X-RAY SPECTROMETER.

BY W. S. GORTON.

IN the literature of X-ray spectroscopy and crystal analysis mica has frequently been mentioned as a substance capable of " reflecting " X-rays strongly. Many investigators have reported this fact but have given no detailed information as has been done in the case of other crystals such as rock salt, calcite, etc. De Broglie and Lindemann,[1] and Rohmann[2] have described an X-ray spectrometer employing mica as analyzer which offers some advantages over the usual form employing rock salt, calcite, etc., but no details as to the performance of the instrument were given. This spectrometer consists essentially of a sheet of mica, bent to a cylindrical shape, on which a beam of X-rays falls. The various components of the beam are reflected at different angles and are received on a photographic plate as shown in Fig. 2. The advantage of this form of spectrometer consists in the curved shape of the analyzing crystal which enables a beam of X-rays comprised within a very narrow angle to strike the analyzer in such a way as to allow a large range in the possible angles of reflection. In the particular instrument used in the present work a beam having an angle of 5½° allows a range of 45½° in the angles of reflection. Thus one is enabled to do in one exposure what requires many exposures in the ordinary form of spectrometer. Rock salt, as well as mica, is capable of being used as analyzer as it can be bent under hot water but mica, of course, is more easily manipulated. In view of the advantage offered by this type of spectrometer, it seemed well worth while to make a thorough investigation of the properties of the instrument. Such an investigation would also throw light on the structure of mica.

The instrument used in the present work is shown in section in Fig. 1; the drawing is to scale. A sheet of mica 7.5 cm. long and 2.5 cm. wide is fastened on the surface of a wooden cylinder 8.35 cm. in diameter. Extending partly under the cylinder is a holder for the photographic plate. This holder has a top of black paper which offers a very small resistance to the passage of the X-rays. The open end of the holder is closed by a piece of wood not shown in the drawing. The instrument is protected by a covering of sheet lead $\frac{1}{16}$ inch (1.6 mm.) thick. The slit is in this lead cover and is $\frac{1}{4}$ mm. wide. Above and to one side of the slit is placed a lead screen in order to prevent any rays but those reflected from the mica from reaching the plate. In order to find the angle of reflection corresponding to any line on the plate, an accurate drawing of

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[1] De Broglie and Lindemann, Comptes Rendus, 158, 944, 1914.

[2] Rohmann, Physikalische Zeitschrift, 15, 510, 1914.

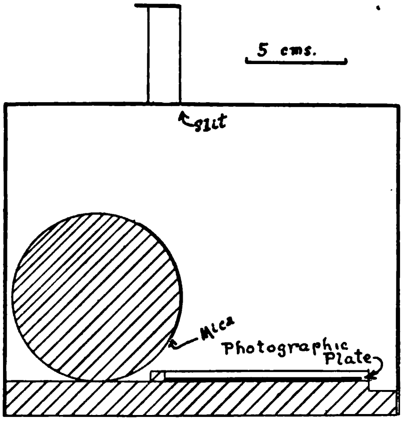

Fig. 1.

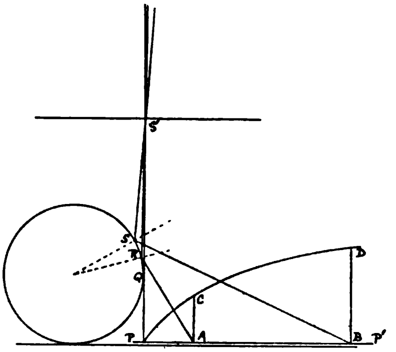

Fig. 2.

the apparatus was made as shown in Fig. 2. Lines $S'S$, $S'R$, etc., were drawn, making various angles θ with the surface of the cylinder. The reflections of these lines cut the surface of the photographic plate PP' at B, A, etc. CA, BD, etc., were drawn with lengths proportional to $\sin \theta$, and the points C, D, etc., were connected by a curve. Sin θ for any line on the plate is readily obtained by laying the plate on the drawing and reading the length of the ordinate of the curve. The wave-length λ can be calculated from the formula

$$n\lambda = 2d \sin \theta, \tag{1}$$

as soon as d, the distance apart of the atomic planes, and n, the order of the spectrum, are known. In order to find d it is necessary to use first radiations of known wave-length and measure θ and n. In the present work it was most convenient to use for this purpose the characteristic radiations of tungsten as produced in an X-ray tube having a tungsten anticathode. Ordinary X-ray tubes were used. The time of exposure was generally five minutes, using currents of the order of 15 milliamperes and spark gaps of anywhere from 1 to 6 inches.

Examples of the results obtained are shown in Plate I. Fig. 1 of this plate represents the usual result of an exposure and shows some strongly marked lines which are present on practically every plate taken. Fig. 2

TABLE I.

Line.	Series.	Order.	Exposure No.				
			22	23 (Plate I., Fig. 2).	24 (Plate I., Fig. 1).	27 (Plate I., Fig. 3).	29
			Sin θ Intensity.	Sin θ Intensity.	Sin θ Intensity.	Sin θ Intensity.	Sin θ Intensity.
a	2	2	.457 v.f.	......	......	......	......
..	..	..	......	......	.393 v.f.	......	.391 v.f.
a_1	1	1	......	......	.380 v.f.	......	.379 v.f.
a	1	1	.379 s.	......	.377 s.	.377 f.	.377 s.
b_1	1	1	......	......	.333 v.f.	.333 v.f.	.333 v.f.
b	1	1	.332 f.	......	.328 m.	.329 s.	.327 m.
c	1	1	......	......	.324 v.f.	.324 v.f.	.323 v.f.
d	1	1	.322 v.f.	......	.319 f.	.320 f.	.319 f.
..	..	..	......	......	.305 v.f.	.304 v.f.	.304 v.f.
g	1	1	......	......	.284 v.f.	.285 f.	.284 v.f.
h	1	1	......	......	......	.274 v.f.	......
k	1	1	......	......	......	.266 f.	......
a	3	1	......	......	......	.258 v.f.	......
a	2	1	......	.233 m.	.230 v.f.	.230 s.	.229 v.f.
b	3	1	......	.228 e.f.	......	.226 v.f.	......
c	3	1	......	.223 f.	......	.223 f.	......
d	3	1	......	......	......	.220 v.f.	......
b_1	2	1	......	......	......	.202 v.f.	......
b	2	1	......	.204 s.	......	.199 s.	......
c	2	1	......	.200 f.	......	.197 v.f.	......
d	2	1	......	.196 m.	......	.194 m.	......
g	3	1	......	......	......	.191 m.	......
h	3	1	......	......	......	.186 f...	......
g	2	1	......	.173 v.f.	......	.171 f.	......
h	2	1	......	......	......	.165 v.f.	......
k	2	1	......	......	......	.160 v.f.	......
..	..	..	......	.152 v.f.	......	.152 f.	......
a	4	1	......	.140 m.	......	.138 v.f.	......
..	..	..	......	......	......	.130 v.f.	......
b	4	1	......	.120 m.	......	......	......
c	4	1	......	.118 v.f.	......	.118 v.f.	......
d	4	1	......	.117 f.	......	.117 v.f.	......
g	4	1	......	.104 v.f.	......	.100 v.f.	......
h	4	1	......	......	......	.098 v.f.	......
..	..	..	......	.077 v.f.	......	.075 v.f.	Exposure 58
..	..	..	......	.066 v.f.	......	.064 v.f.	......
g	1	2	......	......	......	......	.574 v.f.
k	1	2	......	......	......	......	.538 v.f.

shows the sole exception to the rule set forth in the previous sentence; a large number of very fine lines, some of which extend only part of the way across the plate, are seen. These lines are nearer the central image than those of Fig. 1. Fig. 3 represents a condition of very infrequent

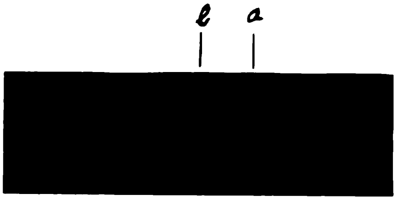

Fig. 1.

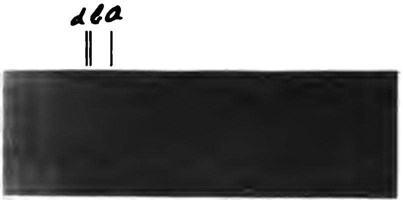

Fig. 2.

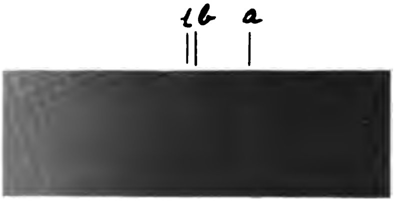

Fig. 3.

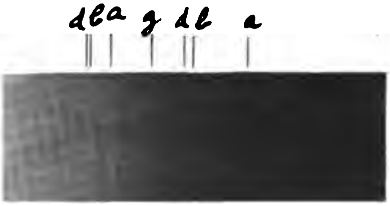

Fig. 4.

A Mica X-ray Spectrometer

W. S. GORTON.

occurrence; it contains all the lines found in both Figs. 1 and 2. Fig. 4 was taken with a piece of mica much thinner than was used for the rest of the work; the thickness was 0.0005 inch (0.0013 cm.) as against 0.0033 inch (0.0083 cm.) for the other figures. The lines, however, occupy the same positions as those shown in Fig. 1. They are not so strong and exhibit a curious splotchiness but they are very evidently the same lines. In all the numerous exposures made the lines were invariable in position. Sometimes lines were absent from a plate but when they were present their positions were always the same. Consequently it will suffice to give here the measurements of the quantity sin θ for only a few exposures (Table I.). The intensity of the lines is characterized as strong, medium, faint, or very faint.

A glance at Table I. shows that as many as 32 lines are sometimes found on a plate. The question at once is whether these lines are due to as many wave-lengths. This question must be answered in the negative. In a previous paper[1] the author reported the analysis of the X-ray spectrum of tungsten by means of calcite and rock salt. Ten lines in all were found. The wave-lengths are given in Table II. In view of the undoubted greater simplicity of the structure of calcite and rock salt as compared with that of mica it must be considered that the characteristic radiation of tungsten has but ten components. This conclusion is confirmed by the results reported below. The problem is, then, to pick out in the spectrum the lines due to any one wave-length. For the stronger lines this is easily done. In the tungsten spectrum the most striking groups of lines are those consisting of a_1 and a, and b_1, b, c_1 and d. A glance at almost any one of the exposures, say No. 27, enables one to pick out the lines for which sin θ = .377, .333, .329, etc., as being due to a, b_1, b, etc., respectively. The text for identity is to calculate the value of sin θ/λ for each of the lines. This was done and the results are given in Table III., series 1. The lines g, h, and k were always faint and could

TABLE II.

Line.	$\lambda \times 10^8$	Intensity.
a_1	1.476	f.
a	1.466	s.
b_1	1.292	f.
b_2	1.283	v.f.
b	1.275	s.
c	1.256	f.
d	1.237	m.
g	1.094	s.
h	1.057	f.
k	1.025	f.

TABLE III.

Line.	Series.			
	1.	2.	3.	4.
	$\frac{\sin\theta}{\lambda}\cdot 10^{-8}$	$\frac{\sin\theta}{\lambda}\cdot 10^{-8}$	$\frac{\sin\theta}{\lambda}\cdot 10^{-8}$	$\frac{\sin\theta}{\lambda}\cdot 10^{-8}$
a_1	.2576	——	——	——
a	.2572	.1570	.1760	.0942
b_1	.2578	.1563	——	——
b	.2580	.1560	.1773	.0941
c	.2580	.1570	.1775	.0940
d	.2588	.1570	.1779	.0946
g	.2606	.1562	.1745	.0915
h	.2595	.1562	.1760	.0929
k	.2595	.1561	——	——
Average value	.2586	.1565	.1765	.0936

only be picked out by this test. The measurements of sin θ are estimated to be accurate to ½ per cent. for values around 0.350, the accuracy being inversely proportional to sin θ. The ratio (sin θ)/λ for the above mentioned series of lines is constant within this limit and shows conclusively the identity of the lines. In addition to these lines it was easy to pick out the lines for which sin θ = .230, .202, .199, etc. (exposure 27), as also being due to a, b_1, b, etc. The values of (sin θ)/λ are given in Table III., series 2; they are constant to within the limits of accuracy of the measurements. The designations of the lines as shown by Table III. are also given in Table I. With the exception of the two above-mentioned series it was at first thought possible to identify hardly any of the lines. Sin θ = .457 (exposure 22) is very probably the second order of sin θ = .230, an a line. Possibly sin θ = .304 is a second-order line corresponding to a first order sin θ = .152, but beyond that little can be said from a mere inspection of the plates. In an effort to identify the remaining lines a table was constructed on the following principle: For each value of sin θ, values of the ratio (sin θ)/λ were calculated using in turn the values of λ corresponding to a, b, c, d, g, and h. If any of the lines form a series they can be picked out easily by looking for a constant value of (sin θ)/λ. This attempt at identification met with considerable success. A third series of lines was found beginning with the a line, sin θ = .258, and a fourth series beginning with the a line, sin θ = .138. The remaining lines reported in Table I., six in all, have not been identified. All lines except one (sin θ = .457) were, as far as could be ascertained, in the first order.

[1] Gorton, PHYS. REV., Feb., 1916.

The above data show that mica is not the equal of such crystals as calcite and rock salt in the analysis of X-rays. In any one order of the spectra due to mica are found not one but several series of lines. This fact complicates matters considerably, but on the other hand the lines produced by mica are generally narrower and sharper than those produced by the other crystals mentioned, some of the lines, in fact, being not more than one tenth the width of the lines due to calcite or rock salt. The presence of the numerous, very fine, lines, a few of which have not been identified, might be considered an objection to the use of mica but almost all of these lines lie below sin $\theta = .195$ and, moreover, do not appear on many of the plates; consequently their existence need never be troublesome. As for the lines of the four series, the lines of series 1 and 2 can, for the most part, be identified at a glance. Most of the lines of series 3 and 4 are very faint and could probably be neglected for most work. The conclusion to be drawn from these data is that mica is a reliable and convenient substance for use as an X-ray analyzer. Its property of being bent easily is, as explained in the early part of this paper, of great service in expediting the work. The form of spectrometer here described has the disadvantage that the relative intensity of the lines on the photographic plate is, to some extent, a function of the distribution of intensity of radiation on the anti-cathode of the X-ray tube and so is apt to vary from exposure to exposure, the variation sometimes being so great as to cause certain lines to be absent from the plate; but this disadvantage it shares with every other form of spectrometer using a stationary crystal. The present spectrometer should be of considerable service in all work on the effects produced by X-rays where it is desired to ascertain accurately the effect of variation in wave-length on those effects.

Some conclusions can be drawn from the present data as to the structure of mica. For the purpose of the present article the crystal axes of mica may be considered to be mutually orthogonal. The unit distances on the three axes are in the ratios, $a : b : c = 0.577 : 1 : 3.313$. The cleavage planes are parallel to the *ab* plane. Consequently a sheet of mica may be considered to be built up of small rectangular parallelopipeds, the greatest dimension of each parallelopiped being perpendicular to the surface of the sheet; the atoms probably lie at the corners of the parallelopipeds. It is at once evident that the planes rich in atoms are, in the order of richness, the cleavage planes, planes parallel to the *ac* plane, planes parallel to the *bc* plane, and lastly other planes. It is evident further that there are no planes rich in atoms which make a small angle with the cleavage plane; this would be so even if atoms were also situated at the centers, or at the centers of the faces, of the parallelopipeds. All of such planes make angles with the cleavage plane which are nearer to 90° than to 0°. Now the presence of four series of lines in the spectrum can be explained in two ways. First, they may be due to four distinct sets of planes all parallel to the cleavage plane and having different values of d. Second, they may be due to sets of planes making small angles with the cleavage plane; as a consequence the grazing angle of the incident rays would not be the angle θ, and equation (1) would not apply. In view of the theory of structure outlined above the first of these hypotheses is to be preferred. By application of equation (1), using the values of λ given in Table II., the values of d for the several sets of planes are as follows:

Series	d
1.	1.93×10^{-8} cm.
2.	3.19×10^{-8} "
3.	2.83×10^{-8} "
4.	5.35×10^{-8} "

The sets of planes responsible for the lines of series 1 and 2 are certainly essential constituents of the mica as these lines appear in exposures made with the thinnest lamina it was possible to split off (thickness 0.0013 cm.; *see above*). As to the sets of planes to which series 3 and 4 are due, these are probably not essential constituents of the mica. The lines of these series do not appear on plates made with the thinnest lamina, although this may be due to their faintness, and, more significant still, most of them extend only part of the way across the plate. This indicates that the lines of series 3 and 4 are due to a substance which is mixed in with the mica and scattered non-uniformly through it.

It will be noticed that the values of d for series 1 and 2 are very nearly in the ratio 3 : 5. This suggests that there may be a set of planes in the mica for which d has one third the value of d for series 1. In order to investigate this point a special plate-holder was made which would hold the photographic plate vertically before the mica surface in such a way as to enable it to register values of $\sin \theta$ up to 0.95. The only lines observed, however, were two for which $\sin \theta$ equaled 0.574 and 0.538 respectively. These lines are very probably lines g and k of the first series in the second order. Consequently the unit in the structure of mica would seem to be a parallelopiped containing atoms spaced in such a way as to form sets of planes having the inter-planar distances 1.93×10^{-8} and 3.19×10^{-8} cm.

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A PHYSICAL STUDY OF THE THERMAL CONDUCTIVITY OF SOLIDS.

By Arthur H. Compton.

SINCE the physical interpretation of the variation of the specific heat of solids with temperature has been found to be of considerable theoretical importance, it is to be expected that a study of the physical nature of the thermal conductivity of solids may also bring interesting results. The present paper may be considered as a start at such a study.

In order not to be disturbed by thermal conduction due to electrons, we shall consider only non-metallic substances. For the sake of further simplicity we shall consider in the first place the conduction of heat through a crystal, in which the atoms are closely packed. Consider two infinite, parallel planes in such a medium, at a distance x apart, and maintained at a constant difference of temperature, $T_1 - T_0$. Let δx be the average distance in the x direction through which the energy of an atomic collision is transmitted. The temperature drop along this distance δx will evidently be

$$\delta T = \frac{\delta x}{x}(T_1 - T_0).$$

This quantity represents the average difference in temperature between the atom which gives out energy at each collision, and the atom or atoms which absorb it. According to ordinary kinetic theory, if the collisions of the atoms are elastic, the average amount of energy transmitted at each collision may be shown to be

$$\delta\epsilon = \frac{4m_1m_2}{(m_1 + m_2)^2}R\delta T,$$

where m_1 and m_2 are the masses of the atoms in collision, and R is the gas constant for a single molecule. This may be written

$$\delta\epsilon = bR\delta T,$$

where b is in most cases approximately equal to 1.

In the kind of a medium we are considering, the number of atomic collisions in unit time on unit area of the surface of a plane separating adjacent layers of atoms will be equal to the number of atoms per unit area multiplied by the frequency of oscillation of each atom. That is,

$$\eta = n^2\nu,$$

where ν is probably equal to the natural frequency of the atoms as determined by optical methods, and n^2 is the number of atoms per unit area of the plane. n^2 will be equal to the 2/3 power of the number of atoms in unit volume in the case of cubic crystals, but not necessarily for other kinds. The rate of transfer of energy per unit area across a plane perpendicular to the x axis will therefore be

$$\frac{dE}{dT} = \eta \cdot \delta\epsilon = n^2\nu \cdot bR \cdot \frac{\delta x}{x}(T_1 - T_0);$$

and since there can be no piling up of energy at any point of the substance this is also the rate of transfer of energy from one boundary plane to the other. But by definition,

$$\frac{dE}{dT} = k\frac{T_1 - T_0}{x},$$

where k is the coefficient of thermal conductivity, so that

$$k = bn^2R\nu\delta x,$$

or

$$\delta x = k/bn^2R\nu.$$

According to the experiments of Eucken,[1] the thermal conductivity of crystals is approximately inversely proportional to the temperature. If we call the factor of proportionality, c,

$$k = c/T,$$

and the distance through which the energy of an atomic collision is transmitted is

$$\delta x = c/T \cdot \frac{1}{bn^2R\nu}. \qquad (1)$$

That is, as the temperature falls, and the atoms become more and more closely packed, the distance through which the effect of an atomic collision is transmitted increases, as appears plausible.

Thus we have the distance δx for a crystal expressed in terms of known quantities. For rock-salt at 0° C. these quantities are:

$c/T = 1.36 \times 10^6$ erg cm.$^{-1}$ sec.$^{-1}$ deg.$^{-1}$,
$b = 0.955$,
$n = 3.58 \times 10^7$ cm.$^{-1}$,
$\nu = 5.8 \times 10^{12}$ sec.$^{-1}$,
$R = 1.35 \times 10^{-16}$ erg deg.$^{-1}$;

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[1] A. Eucken, Ann. d. Physik, 34, 185 (1911).

and thus

$$\delta x = 1.42 \times 10^{-6} \text{ cm.}$$

But the distance between two neighboring atoms in rock-salt is $d = 2.80 \times 10^{-8}$ cm., or $\delta x = 51d$. The value of δx for other crystals is of the same order of magnitude. We find, therefore, that the energy of a collision between two atoms in a crystal is transmitted through a distance which is approximately inversely proportional to the temperature, and many times the distance between two neighboring atoms.

If it should happen that the crystals of which a substance is composed are of smaller dimensions than the average distance through which the effect of a thermal disturbance is transmitted, a decrease in temperature will no longer mean a decrease in the δx of equation (1), since this distance will evidently be limited by the size of the component crystals. On the other hand, it is probable that the frequency of the collisions between the atoms of neighboring crystals will diminish as the amplitude of the vibrations becomes less, since the atoms will not be closely packed at the juncture of the crystals. This would result in a decrease in the conductivity with temperature, such as is actually observed in the case of all amorphous substances.

That something of this kind actually occurs in amorphous substances becomes evident when we consider a definite example. In the case of fused quartz at $-190°$ C., for instance,

$$k = 6.61 \times 10^{4} \text{ erg cm.}^{-1} \text{ sec.}^{-1} \text{ deg.}^{-1},$$

$$b = .95$$

$$\nu^{6} = 1.82 \times 10^{13} \text{ sec.}^{-1} \text{ (mean value)},$$

$$n = 4.1 \times 10^{7} \text{ cm.}^{-1},$$

and if we calculate δx according to equation (1) we obtain

$$\delta x = 1.7 \times 10^{-8} \text{ cm.} = 0.7d.$$

Thus if a collision occurred at each oscillation of an atom, we should have to conclude that the effect of the collision was transmitted through less than the distance between two atoms, which would be meaningless. It is evident, therefore, that some of the atoms in an amorphous substance must oscillate without affecting appreciably the atoms next to them, as suggested above.

The physical interpretation of this result, that the effect of each atomic collision in a crystal is felt at a considerable distance, is an interesting problem. According to the agglomeration theory of the variation of the specific heat of solids with temperature,[1] at low temperatures the atoms of solids lose degrees of freedom in a manner very similar to that in which a degree of freedom is lost when two atoms combine chemically. It is evident that if large groups of atoms should thus be formed which would vibrate as units, the thermal conductivity would be greatly increased. Benedicks[1] has accordingly considered the sharp increase in the heat conductivity of crystals at low temperatures to be an indication of the formation of such groups, and hence as a support of the agglomeration hypothesis. If these atomic aggregations vibrate independently, the distance through which the effect of a collision will be transmitted will be equal to their average diameter. As we have seen above, for rock-salt at 0° C. this distance is some 50 times the distance between two neighboring atoms; but the probability that a degree of freedom shall be agglomerated is in this case only 1/3. Thus it is seen that the size of the groups formed by a possible agglomeration of the atoms is not of the proper order of magnitude to account for the thermal conductivity of crystals.

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[1] C. Benedicks, Ann. d. Physik, 42, 133 (1913); A. H. Compton, Phys. Rev., 6, 377 (1915).

A more probable interpretation would seem to be that each atom transmits the greater part of the energy of every thermal disturbance which reaches it, retaining or scattering the remaining part, the atoms thus acting as if imperfectly elastic. If we assume that the energy of each collision is transferred from atom to atom, being diminished by a definite factor as it traverses each atom, it can be shown by reasoning similar to that above that the thermal conductivity of a crystal in which adjacent atoms are arranged in lines parallel to the axis of conduction is

$$(2) \qquad k = bn^2dR\nu \cdot \frac{2-\alpha}{\alpha^2};$$

where d is the distance between successive atoms in the direction of conduction, and α is the fraction of the energy absorbed by each atom as the thermal disturbance traverses it. This absorbed energy is given out again at the next collision of the atom. For other kinds of crystals the function of α is somewhat different, but is in any case approximately proportional to $1/\alpha^2$ as long as α is small. For rock-salt at 0° C.

$$\alpha = 0.202.$$

According to this explanation the energy of an atomic collision is not transmitted through a definite distance, but dies out gradually as it passes through a substance. The *mean* range, however, must be the same as the δx which we have used above.

If we assume that the fraction of the energy of each thermal disturbance which an atom retains is proportional to its amplitude of vibration, and hence, according to ordinary kinetic theory, to the square root of the temperature, *i. e.*,

$$\alpha = c\sqrt{T},$$

we have,

$$k = bn^2dR\nu \cdot \frac{2 - cT^{\frac{1}{2}}}{c^2T}, \tag{3}$$

where c is determined by the conductivity at a given temperature. The following tables show how this expression compares with Eucken's experimental data. The first calculated values are found according to the expression

$$k = c/T,$$

and the second according to equation (3). The value of α at 273° K. is calculated according to equation (2), using the value of d determined by X-ray measurements,[1] and ν for quartz in the mean value, 1.82×10^{13}, determined from specific heat data.[2] k is expressed in calories cm.$^{-1}$ sec.$^{-1}$ deg.$^{-1}$.

TABLE I.

T.	NaCl; $\alpha_{273} = 0.202$.			KCl; $\alpha_{273} = 0.175$.		
	k (Calc.$_1$).	k (Obs.).	k (Calc.$_2$).	k (Calc.$_1$).	k (Obs.).	k (Calc.$_2$)
373°	0.0122	0.0116	0.0119	0.0121	0.0118	0.0118
273	0.0167	0.0167	0.0167	0.0166	0.0166	0.0166
195	0.0233	0.0249	0.0238	0.0232	0.0248	0.0237
83	0.0548	0.0636	0.0577	0.0545	0.0502	0.0567
22	——	——	——	0.206	0.125	0.215
	SiO$_2$ ‖; $\alpha_{273} = 0.200$(3).			SiO$_2$ ⊥; $\alpha_{273} = 0.309$(3).		
373	0.0236	0.0215?	0.0230	0.0127	0.0133	0.0123
273	0.0325	0.0325	0.0325	0.0173	0.0173	0.0173
195	0.0455	0.0467	0.0465	0.0242	0.0241	0.0250
83	0.1067	0.1170	0.1124	0.0569	0.0586	0.0620
22	——	——	——	0.214	0.58	0.234

The data are not sufficiently consistent to fit any formula accurately, but it is apparent that equation (3) fits the data in general somewhat more closely than does the strictly linear law except for quartz ⊥, in which case equation (3) does not strictly apply.[3]

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[1] Cf. W. H. Bragg and W. L. Bragg, "X-rays and Crystal Structure," pp. 88 and 160.

[2] The data used are those of Nernst (Ann. d. Physik, 36, 395) and F. Koref (Ann. d. Physik, 36, 64). ν is calculated according to the formula $\nu = \tau/\beta$, where β is a universal constant and τ is determined by the specific heat at a given temperature. Cf. A. H. Compton, loc. cit.

[3] According to the structure of quartz as found by Professor Bragg, the arrangement of the atoms parallel to the axis is probably not greatly different from a linear distribution; but perpendicular to the axis the arrangement is more irregular. Thus while the function of β for quartz ‖ is probably nearly that given in equation (2), for quartz ⊥ this function may be different. The value of n^2 which should be used in this case is also doubtful, which could account for the large value of α for quartz ⊥.

We have so far been considering the average energy of a degree of freedom in a solid to be RT, instead of the somewhat different value which is found by a study of the specific heat. If we use instead the energy assigned by the quantum hypothesis,

$$h\nu\left\{\frac{1}{e^{\frac{h\nu}{RT}}-1}+\frac{1}{2}\right\},$$

two changes must be made. In the first place, the average amount of energy given out by an atom at each collision is no longer

$$\delta\epsilon = bR\delta T,$$

but is

$$\delta\epsilon = b\frac{\partial}{\partial T}\left\{h\nu\left(\frac{1}{e^{\frac{h\nu}{RT}}-1}+\frac{1}{2}\right)\right\}\delta T$$

$$= \frac{bh^2\nu^2}{RT^2}\frac{e^{\frac{h\nu}{RT}}\delta T}{\left(e^{\frac{h\nu}{RT}}-1\right)^2}.$$

In the second place, if α is proportional to the amplitude of vibration, or the square root of the energy, *i. e.*,

$$\alpha = c\left\{\frac{1}{e^{\frac{h\nu}{RT}}-1}+\frac{1}{2}\right\}^{\frac{1}{2}},$$

the expression for the thermal conductivity corresponding to equation (3) is

$$(4)\qquad k = bn^2d\nu\cdot\frac{h^2\nu^2}{RT^2}\frac{e^{\frac{h\nu}{RT}}}{\left(e^{\frac{h\nu}{RT}}-1\right)^2}\cdot\frac{2-c\left\{\frac{1}{e^{\frac{h\nu}{RT}}-1}+\frac{1}{2}\right\}^{\frac{1}{2}}}{c^2\left\{\frac{1}{e^{\frac{h\nu}{RT}}+1}+\frac{1}{2}\right\}}.$$

This expression makes the thermal conductivity approach zero at very low temperatures, giving values much smaller than are actually observed. This is shown clearly in the following table, for the cases of rock-salt and sylvine (calc.$_3$).

TABLE II.

T.	NaCl; $\alpha m = 0.202$.			KCl $\alpha : m = 0.175$.		
	k (Calc.$_3$).	k (Obs.).	k (Calc.$_4$).	k (Calc.$_3$).	k (Obs.).	k (Calc.$_4$).
373°	0.0124	0.0116	0.0118	0.0127	0.0118	0.0117
273	0.0167	0.0167	0.0167	0.0166	0.0166	0.0166
195	0.0214	0.0249	0.0240	0.0223	0.0248	0.0240
83	0.0263	0.0636	0.0590	0.0323	0.0502	0.0577
22	——	——	——	0.0015	0.125	0.217

In order to obtain a satisfactory expression for the thermal conductivity, if the energy of a degree of freedom is that assigned by the quantum hypothesis, it is necessary to assume that α is at least approximately proportional to the expression

$$\left\{\frac{h^2\nu^2}{RT}\cdot\frac{e^{\frac{h\nu}{RT}}}{\left(e^{\frac{h\nu}{RT}}-1\right)^2}\right\}^{\frac{1}{2}},$$

which is the square root of the product of the heat capacity of a degree of freedom according to Einstein's formula by the absolute temperature. It is hard to conceive of any possible direct connection between this quantity and the amount of "inelasticity" α of an atom. It seems difficult, therefore, to obtain on the basis of the quantum hypothesis a justifiable formula for the heat conductivity.

The case is different, however, if use is made of the agglomeration hypothesis. According to this hypothesis, the number of existing degrees of freedom is less than the greatest possible number by the factor $e^{-\frac{\beta\nu}{T}}$, where β is a universal constant; but the degrees of freedom which actually exist have the amount of energy assigned by equipartition, that is RT. This being the case, the energy given out at each collision is, as originally,

$$\delta\epsilon = bR\delta T;$$

but the number of collisions is diminished, since the number of existing degrees of freedom is reduced, in such a manner that

$$\eta = n^2\nu e^{-\frac{\beta\nu}{T}}.$$

The average energy of each *atom* is, however, $3RTe^{-\frac{\beta\nu}{T}}$, and if as before we consider the fraction of the passing energy which each atom absorbs to be proportional to its amplitude of vibration, we have

$$\alpha = cT^{\frac{1}{2}}e^{-\frac{\beta\nu}{2T}}.$$

The expression for the thermal conductivity then becomes:

$$k = bn^2dR\nu\cdot e^{-\frac{\beta\nu}{T}}\cdot\frac{2-cT^{\frac{1}{2}}e^{-\frac{\beta\nu}{2T}}}{c^2Te^{-\frac{\beta\nu}{T}}},$$

$$(4)\qquad = bn^2dR\nu\cdot\frac{2-cT^{\frac{1}{2}}e^{-\frac{\beta\nu}{2T}}}{c^2T}.$$

That this expression is practically equivalent to equation (2) will be seen on comparing the values of $k(\text{calc.}_4)$ in Table II, with the corresponding values of $k(\text{calc.}_2)$ of Table I. Thus the agglomeration hypothesis leads directly to a satisfactory formula for the heat conductivity while the quantum hypothesis apparently does not.

The essential difference between these two hypotheses is that while according to the quantum hypothesis all of the possible degrees of freedom possess energy, but in an amount which differs from that assigned by equipartition, according to the agglomeration hypothesis only a part of the possible degrees of freedom possess energy, but those which have it carry the amount assigned by equipartition. The fact that the latter of these hypotheses leads naturally to a satisfactory formula for the thermal conductivity while the former seems to give such an expression only with the use of a highly improbable assumption may be taken as evidence against the quantum hypothesis, and as a verification of the law of equipartition of energy as applied to the degrees of freedom of solids.

SUMMARY.

In the first place it has been shown that the average distance through which the energy of an atomic collision in a crystal is transmitted is approximately inversely proportional to the temperature, and is many times the distance between the atoms.

This result may be explained on the assumption that each atom in a crystal absorbs or scatters a small fraction of the energy of every thermal disturbance which traverses it and transmits the rest.

In order to obtain a satisfactory formula for the thermal conductivity on the basis of the quantum hypothesis it seems necessary to make certain improbable assumptions; the agglomeration hypothesis, on the other hand, leads to such a formula with much more plausible assumptions.

This study of the thermal conductivity therefore brings evidence against the quantum hypothesis, and supports the law of the equipartition of energy as applied to the degrees of freedom in a solid.

I am indebted to my brother Professor Karl T. Compton for his suggestions and to Professor E. P. Adams for his helpful criticism in connection with this paper.

PALMER PHYSICAL LABORATORY,
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ON THE LOCATION OF THE THERMAL ENERGY OF SOLIDS.

By Arthur H. Compton.

FROM a consideration of the variation of the specific heat of solids with temperature, Debye has suggested[1] that the thermal energy of a solid lies not in the independent motion of the individual atoms, as assumed by Einstein,[2] but in the elastic vibrations of the body as a whole. These include the fundamental tonic vibrations as determined by the dimensions of the body, and all the harmonics up to and including the natural vibrations of the atoms themselves. On this assumption he has been able to obtain an expression for the specific heat of solids, based on the quantum hypothesis, which fits the experimental data acceptably throughout a considerable range of temperature. A formula for the specific heat of solids based on an agglomeration hypothesis has also been recently obtained,[3] but this hypothesis is found to lead to an accurate result only on the assumption that the thermal energy of solids lies not in their elastic vibrations, but in the motions of the individual atoms. In the present paper an attempt will be made to find out whether the vibrations which have a greater period than that of the individual atoms affect in any important manner the specific heat of solids.

It has been pointed out in a previous paper[4] that Debye's assumption that the thermal energy of a solid is distributed throughout all its possible elastic vibrations leads to a value of the thermal conductivity which is of a much greater order of magnitude than that observed experimentally. This makes his assumption in its original form untenable. In order to account for the magnitude of the thermal conductivity, it has been found[5] that the energy of each atomic collision is transmitted through an average distance of some 50 times the atomic distances in the case of a crystal, and probably not many times the atomic distances in the case of an amorphous substance. It is evident, therefore, that there must be a maximum value of the wave-length of the elastic vibrations which can possess heat energy, which is determined by the distance a thermal disturbance can be transmitted through the substance. The existence of such a maximum wave-length changes somewhat the form of Debye's specific heat formula, and the amount of this change will be shown to be in general greater for amorphous than for crystalline substances. If, therefore, the vibrations of frequency less than the natural frequency of the atoms possess any considerable amount of thermal energy, we should expect to find a certain difference between the specific heats of amorphous and of crystalline forms of the same substance. If, on the other hand, only the independent motions of the individual atoms possess a considerable amount of energy, the distance through which the effect of an atomic collision is transmitted cannot affect the specific heat, and no difference is to be expected on this account between the specific heat of amorphous and that of crystalline substances.

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[1] P. Debye, Ann. d. Phys., 39, 789 (1912).

[2] A. Einstein, Ann. d. Phys., 22, 180 (1907).

[3] A. H. Compton, Phys. Rev., 6, 377 (1915).

[4] A. H. Compton, loc. cit., p. 387.

[5] A. H. Compton (Preceding paper on Thermal Conductivity).

We shall first consider the effect on Debye's formula for the specific heat of introducing a lower limit to the frequency of vibration which is comparable in magnitude with the upper limit. Debye has shown[1] that for a perfectly elastic sphere the total number of degrees of freedom for elastic vibrations of frequency less than ν, where ν is large compared with the fundamental frequency, may be taken as

$$Z\nu = FV\nu^3,$$

if V is the volume of the sphere, and F is a quantity determined by the elastic constants and the density of the substance. If ν_m is the upper and ν_l the lower limit of the frequencies to be considered, the total number of degrees of freedom is therefore

$$Z = Z\nu_m - Z\nu_l = FV(\nu_m^3 - \nu_l^3).$$

But by Rayleigh's formula, the total number of degrees of freedom in a solid is

$$Z = 3N,$$

and thus

$$FV = 3N/(\nu_m^3 - \nu_l^3).$$

The number of degrees of freedom between the frequencies ν and $\nu + d\nu$ is

$$dn = \frac{\partial Z\nu}{\partial \nu}d\nu = 3FV\nu^2 d\nu$$

$$= \frac{9N\nu^2 d\nu}{\nu_m^3 - \nu_l^3}.$$

If with Debye we assume that each degree of freedom has the amount of energy assigned by the quantum hypothesis,

$$h\nu/(e^{\frac{h\nu}{RT}} - 1),$$

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[1] P. Debye, loc. cit.

the total energy content of the sphere is

$$U = \int_{\nu_l}^{\nu_m} \frac{h\nu}{e^{\frac{h\nu}{RT}} + 1} \cdot \frac{9N}{\nu_m^3 - \nu_l^3}\, \nu^2 d\nu,$$

and we find for the specific heat at constant volume,

$$C\nu = \frac{dU}{dT} = \frac{9NR}{x^3 - y^3}\left\{4\int_y^x \frac{\xi^2 d\xi}{e^\xi - 1} - \frac{x^4}{e^x - 1} + \frac{y^4}{e^y - 1}\right\};$$

where

$$x = \frac{h\nu_m}{RT}; \quad y = \frac{h\nu_l}{RT}; \quad \xi = \frac{h\nu}{RT}.$$

The ratio of the specific heat to its value at very high temperatures is

$$\frac{C\nu}{C\infty} = \frac{C\nu}{3NR} = \frac{3}{x^3 - y^3}\left\{4\int_y^x \frac{\xi^3 d\xi}{e^\xi - 1} - \frac{x^4}{e^x - 1} + \frac{y^4}{e^y - 1}\right\}.$$

On integration this becomes:

$$\frac{C\nu}{C\infty} = \frac{3}{x^3 - y^3}\left\{\frac{y^4}{e^y - 1} - \frac{x^4}{e^x - 1} - 4\sum_{n=1}^{\infty}\left(\frac{e^{-nx}}{n}\left[x^3 + \frac{3x^2}{n} + \frac{6x}{n^2} + \frac{6}{n^3}\right]\right.\right.$$
$$\left.\left. - \frac{e^{-ny}}{n}\left[y^3 + \frac{3y^2}{n} + \frac{6y}{n^2} + \frac{6}{n^3}\right]\right)\right\}.$$

which is identical with the expression obtained by Debye if y is put equal to zero. The difference between the value of $C\nu/C\infty$ according to this equation when y has a finite value and when y is zero may be shown to be

$$\left(\frac{C\nu}{C\infty}\right)_{y=y} - \left(\frac{C\nu}{C\infty}\right)_{y=0} = \frac{3}{x^3 - y^3}\left\{\frac{y^4}{e^y - 1}\right.$$
$$\left. - 24\sum_{n=1}^{\infty}\frac{1}{e^{ny}}\left(\frac{y^4}{4!} + \frac{ny^5}{5!} + \frac{n^2y^6}{6!} + \cdots\right)\right\} + \frac{y^3}{x^3}\left(\frac{C\nu}{C\infty}\right)_{y=0}.$$

The value of this correction for small values of y at ordinary temperatures is approximately $-3(y/x)^3$, and does not vary rapidly with the temperature.

If the vibrations which are longer than the natural period of the atoms possess heat energy, as Debye assumes, his formula for the specific heat must be modified by some expression such as this. If, however, only the vibrations of the individual atoms possess thermal energy, as assumed in the writer's agglomeration formula, no such correction is to be applied. If, therefore, there is found to be any effect on the specific heat due to the fact that the energy of an atomic collision is transmitted a relatively small distance through a solid, Debye's assumption in a somewhat modified form will be justified.

For rock-salt at 0° C. it is found[1] that the mean distance through which the energy of an atomic collision is transmitted is $\delta x = 1.42 \times 10^{-6}$ cm. The maximum wave-length to be considered will therefore be at least approximately $4\delta x$, or 5.6×10^{-6} cm. The velocity of transmission of a vibrational disturbance in rock-salt is about 3.5×10^{5} cm. sec.$^{-1}$, so the lower limit of the frequency is of the order of magnitude of

$$\nu_l = 3.5 \times 10^{5}/5.6 \times 10^{-6} = 6 \times 10^{10} \text{ sec.}^{-1}$$

The maximum frequency, ν_m, is approximately the same as the natural frequency of the atoms, and this is

$$\nu_m = 5.8 \times 10^{12} \text{ sec.}^{-1}$$

The ratio y/x is thus,

$$y/x = \frac{h\nu_l}{RT} \bigg/ \frac{h\nu_m}{RT} = \nu_l/\nu_m = 10^{-2}$$

approximately. Thus for rock-salt at 0° C. the correction to be applied to Debye's formula is of the order of magnitude of 10^{-6}, and is entirely negligible.

It is found,[2] however, that the quantity δx in amorphous substances may be of the order of magnitude of the distance between the atoms, which would make the frequency ν_l approach the frequency of the vibration of the individual atoms. The value of δx is too uncertain for amorphous substances to calculate ν_l with sufficient accuracy to make a definite estimate of the magnitude of the correction to be applied in such a case, but this correction might be expected to be rather large. Such a correction would not necessarily mean that the specific heat of a substance should be less in the amorphous than in the crystalline state, since the frequency of the vibrations of the atoms in the two forms is generally different. It is possible, also, that similar atoms in an amorphous substance may be so arranged as to have widely different natural frequencies. Such a distribution of the frequencies of the atoms would not affect greatly the form of the specific heat curve at higher temperatures, but would tend to *increase* the relative value of the specific heat at low temperatures. The effect of a correction term such as we have been discussing would be, on the other hand, to change the specific heat curve in such a manner as to *decrease* the relative magnitude of the specific heat at low temperatures. Thus if experimental evidence is found to indicate a relatively smaller specific heat at low temperatures for amorphous than for crystalline substances, such a correction term may be inferred to exist; while if experiment indicates the opposite, no definite conclusion can be drawn as to the existence of such a correction term.

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[1] A. H. Compton (Preceding paper on Thermal Conductivity).

[2] Ibid.

The following table will show how the specific heats of certain crystalline and amorphous substances compare for rather high relative temperatures. T_m is the arithmetic mean of the absolute temperatures between which the mean specific heat per gram molecule C_m is determined. T and C represent corresponding quantities determined at definite temperatures.

AgCl (Crystalline).[1]		AgCl (Amorphous).[1]		Silicon (Crystalline).[2]		Silicon (Amorphous).[2]	
T_m.	C_m.	T_m.	C_m.	T_m.	C_m.	T_m.	C_m.
301°	12.53	301°	12.29	297°	4.84	300°	5.08
235	12.31	234	12.89	234	4.10	234	4.26
137	10.83	137	10.52	138	2.44	138	2.58

Although for both of these substances the actual magnitude of the specific heat of the two forms is appreciably different, it can be shown that the specific heat curve of the amorphous and crystalline modifications are of practically the same form. The result is different, however, at low relative temperatures, as is shown in the case of quartz.

SiO_2 (Rock Crystal)		SiO_2 (Amorphous Quartz).		SiO_2 (Rock Crystal).		SiO_2 (Amorphous Quartz).	
T	C[3]	T	C[3]	T_m	C_m[4]	T_m	C_m[4]
25.8°	0.416	26.25°	0.637	137	5.30	137	5.32
28.75	0.520	29.4	0.644	234	8.81	235	8.87
31.2	0.527	35.2	0.844				
36.1	0.794	42.6	1.33				
84.3	3.03	84.0	3.14				
89.0	3.25						
92.6	3.39						

It will be seen at a glance that at low temperatures the specific heat of the amorphous form is decidedly greater than that of the crystalline form, while at higher temperatures the specific heats are nearly the same. This difference in the rate of the variation of the specific heat of the two modifications is, as we have seen above, of such a nature as to indicate that similar atoms in the amorphous form have different natural frequencies. If the fact that the energy of an atomic collision is transmitted through a shorter distance in an amorphous than in a crystalline substance has any effect on the relative specific heat of the two forms, this effect is entirely masked, at least in the case of quartz, by the effect of the differing frequencies of the atoms in the amorphous state. This test, therefore, gives no evidence of the kind of a difference between the specific heat of amorphous and of crystalline substances that would be expected if the elastic vibrations of the solid possess thermal energy.

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[1] R. Ewald, Ann. d. Phys., 44, 1213 (1914).

[2] A. S. Russel, Phys. Zeitschr., 13, 61 (1912).

[3] W. Nernst, Ann. d. Phys., 36, 395 (1911).

[4] F. Koref, Ann. d. Phys., 36, 64 (1911).

We cannot conclude from this that all the heat energy lies in the vibrations of the individual atoms, but the above result would indicate that the vibrations of longer period probably play no important part in the specific heat. Further measurements of the specific heat of amorphous substances at very low temperatures may very possibly lead to important information on this point.

SUMMARY.

An explanation of the specific heat according to the quantum hypothesis implies that the heat energy of a solid lies in its elastic vibrations, while according to the writer's agglomeration hypothesis this energy lies in the vibrations of the individual atoms.

A previous study of the thermal conductivity has shown that the distance through which the energy of an atomic collision is transmitted is in a crystal many times the distance between neighboring atoms, but is in amorphous substances probably not many times the atomic distances. This implies that the maximum wave-length of the elastic vibrations which can possess thermal energy is greater for crystalline than for amorphous substances, which is shown to mean that if it is the elastic vibrations which possess the heat energy the specific heat of amorphous substances should be comparatively less at low relative temperatures than that of crystalline substances.

The fact that experiment shows no such difference in the specific heat of substances in the amorphous and in the crystalline form indicates that the vibrations of period greater than that of the individual atoms probably play no important part in the specific heat; though the evidence to this effect cannot be taken to be conclusive.

PALMER PHYSICAL LABORATORY,
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December 10, 1915.

A DIRECT PHOTOELECTRIC DETERMINATION OF PLANCK'S "*h*."[1]

By R. A. Millikan.

I. Introductory.

QUANTUM theory was not originally developed for the sake of interpreting photoelectric phenomena. It was solely a theory as to the mechanism of absorption and emission of electromagnetic waves by resonators of atomic or subatomic dimensions. It had nothing whatever to say about the energy of an escaping electron or about the conditions under which such an electron could make its escape, and up to this day the form of the theory developed by its author has not been able to account satisfactorily for the photoelectric facts presented herewith. We are confronted, however, by the astonishing situation that these facts were correctly and exactly predicted nine years ago by a form of quantum theory which has now been pretty generally abandoned.

It was in 1905 that Einstein[2] made the first coupling of photo effects and with any form of quantum theory by bringing forward the bold, not to say the reckless, hypothesis of an electro-magnetic light corpuscle of energy $h\nu$, which energy was transferred upon absorption to an electron. This hypothesis may well be called reckless first because an electromagnetic disturbance which remains localized in space seems a violation of the very conception of an electromagnetic disturbance, and second because it flies in the face of the thoroughly established facts of interference. The hypothesis was apparently made solely because it furnished a ready explanation of one of the most remarkable facts brought to light by recent investigations, viz., that the energy with which an electron is thrown out of a metal by ultra-violet light or X-rays is independent of the intensity of the light while it depends on its frequency. This fact alone seems to demand some modification of classical theory or, at any rate, it has not yet been interpreted satisfactorily in terms of classical theory.

While this was the main if not the only basis of Einstein's assumption, this assumption enabled him at once to predict that the maximum energy of emission of corpuscles under the influence of light would be governed by the equation

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[1] An abstract of this paper was presented before the Am. Phys. Soc. in April, 1914. (Phys. Rev., IV., 73, '14.) The data on lithium were however first reported at the meeting of the Am. Phys. Soc. in April, 1915. (Phys. Rev., VI., 55, '15.)

[2] Ann. d. Phys. (4), 17, 132, 1905, and (4), 20, 199, 1906.

$$\tfrac{1}{2}mv^2 = V.\,e = h\nu - p, \qquad (1)$$

in which $h\nu$ is the energy absorbed by the electron from the light wave, which according to Planck contained just the energy $h\nu$, p is the work necessary to get the electron out of the metal and $\frac{1}{2}mv^2$ is the energy with which it leaves the surface, an energy evidently measured by the product of its charge e by the P.D. against which it is just able to drive itself before being brought to rest.

At the time at which it was made this prediction was as bold as the hypothesis which suggested it, for at that time there were available no experiments whatever for determining anything about how P.D. varies with ν, or whether the hypothetical h of equation (1) was anything more than a number of the same general magnitude as Planck's h. Nevertheless, the following results seem to show that at least five of the experimentally verifiable relationships which are actually contained in equation (1) are rigorously correct. These relationships are embodied in the following assertions:

1. *That there exists for each exciting frequency ν, above a certain critical value, a definitely determinable maximum velocity of emission of corpuscles.*
2. *That there is a linear relation between V and ν.*
3. *That $\frac{dV}{d\nu}$ or the slope of the $V\,\nu$ line is numerically equal to h/e.*
4. *That at the critical frequency ν_0 at which $v = 0$, $p = h\nu_0$, i. e., that the intercept of the $V\,\nu$ line on the ν axis is the lowest frequency at which the metal in question can be photoelectrically active.*
5. *That the contact E.M.F. between any two conductors is given by the equation*

$$\text{Contact E.M.F.} = h/e(\nu_0 - \nu_0') - (V_0 - V_0').$$

No one of these points except the first[1] had been tested even roughly when Einstein made his prediction and the correctness of this one has recently been vigorously denied by Ramsauer.[2] As regards the fourth Elster and Geitel[3] had indeed concluded as early as 1891, from a study of the alkali metals, that the more electro-positive the metal the smaller is the value of ν at which it becomes photo-sensitive, a conclusion however which later researches on the non-alkaline metals seemed for years to contradict.

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[1] Lenard (Ann. d. Phys. (4), 8, 149, 1902) discovered a maximum velocity though he did not find the effect of frequency upon it.

[2] Ann. d. Phys., 45, 1120, 1914 and 45, p. 961.

[3] Wied. Ann., 43, 225, 1891.

During the ten years which have elapsed since Einstein set up his equation the fifth of the above assertions has never been tested at all, while the third and fourth have never been subjected to careful experimental test under conditions which were even claimed to permit of an exact and definite answer, nor indeed can they be so subjected without simultaneous measurements *in vacuo* of both contact potentials and photo-potentials *in the case of metals which are sensitive throughout a long range of observable frequencies.* In making this statement I am not underrating at all the exceptionally fine work of Richardson and Compton,[1] who in common with most other observers interpreted their results in terms of Einstein's equation, but who saw the significance of that equation much more clearly than most of their predecessors had done. I am merely calling attention to the fact that the slope mentioned in (3) and the intercept mentioned in (4) cannot possibly be determined with any approach to certainty unless the region of wave-lengths open to study is larger than it is in the case of any save the alkali metals, and also, in the case of (4) unless simultaneous measurements are made in vacuo upon photo-potentials and contact E.M.F.'s. The fact that Richardson and Compton's slopes vary by more than 60 per cent. is sufficient evidence of the correctness of this statement. Also on the one alkali metal, sodium, on which they made measurements, they made no attempt at contact E.M.F. determinations.[2] Furthermore, since their work appeared two or three theories have been developed to explain their observed departures from the relationships predicted by Einstein's equation[3]—departures which the work at the Ryerson Laboratory appears to prove to be altogether non-existent. In a recent review Pohl and Pringsheim[4] whose own large experience with photoelectric work possibly predisposes them to see the difficulties even too clearly, go so far as to assert not only that any conclusions regarding the validity of (3) and (4) are premature and quite unsupported by any experiments with light waves, but that (2) also, to the testing of which practically all of the work on the energies of photo-emission has thus far been directed, is not deducible from existing data. Furthermore Sir J. J. Thomson[5] writing in December, 1914, says: "the experimental results hitherto obtained seem on the whole to confirm this result [that stated in (2)] though it must I think be confessed that the direct experimental evidence in favor of the strict proportionality is too meagre to enable us to maintain that it is a direct result of experiment."

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[1] Phil. Mag. (6), 24, p. 575, 1912.

[2] Professor Richardson in his note in the Phys. Rev., IV., p. 523, 1914, rightly states that they developed a "novel" method of estimating contact E.M.F.'s *in vacuo*, and for this they deserve much credit. But they could not use this method in the case of sodium, for they definitely state that on account of fatigue effects, they were unable to obtain any complete photo current-potential curve for sodium. Further my own work shows that this novel method is not *always* a valid one for determining contact E.M.F.'s.

[3] See Hughes, Phil. Mag. (6), 25, 683, '14 and Sutton, Phil. Mag., 29, 734, '15.

[4] Phil. Mag., 26, 1017, 1913.

[5] Proc. Phys. Soc., London, XXVII., 105, December 15, 1914.

The difficulties which have stood in the way of reaching a reliable conclusion even as to (2) will be seen from the following review. The first measurements on the relation between the frequency and the maximum energy of emission (which has always been measured by the retarding P.D. necessary to just stop the discharge of electrons under the influence of ultra-violet light) were made in 1907 by E. Ladenburg[1] who concluded that $\sqrt{V}$ was proportional to ν, *i. e.*, that the maximum *velocity* rather than the maximum *energy* of emission was proportional to frequency. Joffe[2] worked over Ladenburg's experimental data and showed that it fitted just as well the Einstein relation $V \propto \nu$. The reason for this wide uncertainty lay simply in the fact that the range within which the relation between V and ν could be followed in Ladenburg's experiments was between 2,000 and 2,600 Ångstroms, and with so small a strip of wave-length to investigate, and relatively large experimental errors in making the settings, practically any kind of a theoretical curve might be fitted by the experimental data.

Unquestionably the most reliable work in this field is that done by Richardson and Compton[3] and by Hughes.[4] But Hughes like Ladenburg made his observations at but three frequencies, viz., those corresponding to $\lambda = 1{,}849$, $\lambda = 2{,}257$, and $\lambda = 2{,}537$ and the first of these he finds very hard to get on account of the extreme feebleness of the light. He uses the observations at two of these points, viz., 1,849 and 2,537 to fix his line, and then finds that the third point is slightly closer to this line when ν is assumed proportional to V than when it is assumed proportional to $\sqrt{V}$. This is evidence favorable to the first law as opposed to the second, but it obviously does not show a linear relation between V and ν as opposed to a relation of the type $V \propto \nu^{\frac{3}{2}}$ as implied by Lindemann's theory,[5] or as opposed to any one of several other relations which might be suggested. Hughes indeed adds to his case the argument that his straight line cuts the ν axis at about the frequency at which the metal is found to first become photo-sensitive, but since he fails to take any account of the contact E.M.F.'s between his opposing plates these crossing points should be incorrect as Pohl and Pringsheim have pointed out. According to these latter authors, barring the very uncertain points at $\lambda = 1{,}800$ and $\lambda = 2{,}000$ Å., even a $V \propto \nu^2$ relationship is just as well fitted by both the Richardson and Compton and the Hughes work as a $V \propto \nu$ relationship. The whole argument, according to these critics, rests, in Richardson and Compton's case, on the value of the potential at $\lambda = 2{,}000$ which is so far in the ultra-violet as to yield a V which they think unreliable and which in fact fits neither curve well, while in Hughes's case the argument rests on the value of the potential at $\lambda = 1{,}894$ which they think even more unreliable.

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[1] Verh. d. D. Phys. Ges., 9, 504, 1907.

[2] Ann. d. Phys. (4), 24, 939, 1907.

[3] L. c.

[4] Phil. Trans. Roy. Soc. London, *A*, CCXII., 205, 1912.

[5] Verh. d. D. Phys. Ges., 15, 637, 1913.

The work at the Ryerson Laboratory on energies of emission began in 1905. How the present investigation has grown out of it will be clear from the following brief summary of its progress and its chief results.

1. It was found first that these energies are independent of temperature,[1] a result unexpected at the time but simultaneously discovered by Lienhop[2] and thoroughly confirmed by others later. This result showed *that photoelectrons do not share in the energies of thermal agitation as they had commonly been supposed to do*, and this result still stands.

2. The apparent energies of emission, that is, the volts which had to be applied to just stop the emission were determined for eleven different metals and found to differ among themselves by more than one volt.[1] This point has recently been tested again by Richardson and Compton[3] and by Page,[4] both of whom find *no* differences. The present work shows that differences do in general exist though possibly not under the conditions used by the other experimenters.

3. The energy of emission was found to vary considerably with time and illumination, a result which I interpreted as due to the disturbing influence of a surface film which exerted under different conditions different retarding influences on the escape of electrons.[5]

4. The results in 3 revealed the necessity of questioning the validity of all results on photopotentials unless the effects of surface films were eliminated either by removal of the films or by simultaneous measurement in vacuo of photopotentials and contact E.M.F.'s, or by both procedures at once. Accordingly I initiated in 1910 on a somewhat elaborate scale simultaneous measurements on photoeffects and contact E.M.F.'s in vacuo on film free surfaces. Some of the results of these measurements on the common metals have been recently reported by Mr. Hennings.[6] Others are contained in this paper, and still others will be reported in a later paper.

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[1] Phil. Mag. (6), 14, 188, 1907.

[2] Ann. d. Phys. (4), 21, 284.

[3] Phil. Mag. (6), 24, 582, 1912.

[4] Am. Jr. Sci., 36, 501, 1913.

[5] Phys. Rev., 30, 287, 1910, and 34, 68, 1912.

[6] Phys. Rev., 4, 228, 1914.

5. The marked difference between the apparent effects on the energy of emission of different types of sources such as the spark and the arc, even when the same wave-length was employed, were traced to the extreme difficulty of eliminating disturbances when spark sources are employed—a difficulty of course appreciated from the first, but thought to have been disposed of because screening of the direct light from the arc removed the differences.[1] After these disturbing influences were eliminated powerful spark sources of given wave-length were found to produce exactly the same energies of emission as arc sources of the same wave-length and of about the same mean intensity, but of only one thousandth the instantaneous intensity.[2] *This furnished very exact proof of the independence first discovered by Lenard of the energy of emission upon intensity, even when the intensity of illumination in one wave-length, viz.,* $\lambda = 3650$, *was as high as* $10000 \frac{\text{erg}}{\text{cm}^2\,\text{sec}}$.

6. The relation between V and ν was tested with spark sources without bringing to light at first anything approaching a linear relationship. These results were reported by Dr. Wright.[3] A question as to their validity was, however, raised by my subsequent proof of the insufficiency of such screening devices as had been used in the case of spark sources. Accordingly Dr. Kadesch took up again the relation between V and ν with powerful spark sources, using film-free sodium and potassium surfaces, and obtained results which spoke definitely and strongly in favor of a linear relation between the maximum P.D. and ν. The range of wave lengths studied was from 3,900 Å. to 2,200 Å. These results have been published[4] since Pohl and Pringsheim wrote their critique, else I think they would not have felt that the common assumption since 1905 of a linear relation between V and ν rested upon so insecure an experimental basis.

7. At the same time I undertook to investigate with as much exactness as possible, using as a source the monochromatic radiations of the quartz-mercury arc, the third, fourth and fifth of the above assertions of Einstein's equation, and in the vice-presidential address before the American Association for the Advancement of Science in December, 1912, expressed the hope that we should soon be able to assert whether or not Planck's *h* actually appeared in photoelectric phenomena as it has been usually assumed for ten years to do.[5] At that time the papers of Hughes and of

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[1] Verh. d. D. Phys. Ges., XIV., 720, 1912.

[2] PHYS. REV. (2), 1, 73, 1913.

[3] PHYS. REV., 33, 41, 1911.

[4] PHYS. REV., 3, 63, May, 1914.

[5] Science, 37, 119, 1913. The following is the quotation from that address: "But not only

PHYSICAL REVIEW, VOL. VII., SECOND SERIES.
March, 1916.

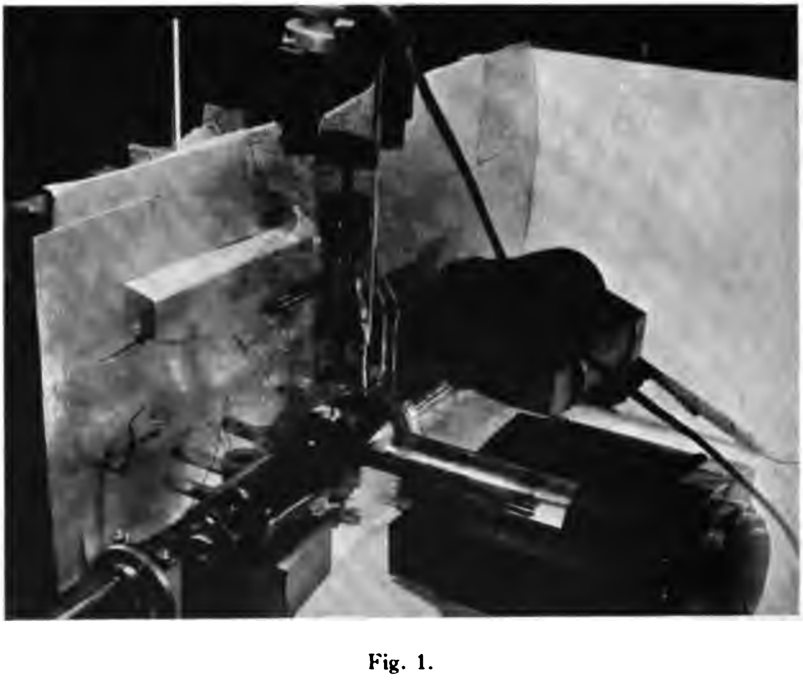

Fig. 1.

R. A. MILLIKAN.

Richardson and Compton had just appeared, though the latter paper I had unfortunately not seen at the time of writing and hence made no reference to it. These authors found the value of h in the Einstein photoelectric equation varying in the eight metals studied from 3.55×10^{-27} to 5.85×10^{-27}. Planck's h was 6.55×10^{-27}, a difference which Hughes tried to explain by assuming either that only a fraction of the energy $h\nu$ was absorbed or that the energy of emission against the direction of the incident light was less than that in the direction of the incident light.

2. Experimental Arrangements.

In all of this photoelectric work at the Ryerson Laboratory the same general method has been employed, namely, the substances to be studied or manipulated have been placed in the best obtainable vacuum on an electromagnetically controlled wheel and all the needed operations have been performed by movable electromagnets placed outside. As new operations have been called for the tubes have by degrees become more and more complicated until it has become not inappropriate to describe the present experimental arrangement as a machine shop in vacuo. The operations which are now needed in all the tubes which are being used are

1. The removal in vacuo of all surface films from all surfaces.
2. The measurement of the photocurrents and photopotentials due to these film-free surfaces.
3. The simultaneous measurement of the contact E.M.F.'s of the surfaces.

To construct tubes in which these operations can all be performed in rapid succession, particularly when the substances to be studied are, as in this case, the inflammable alkali metals, sodium, potassium and lithium, is not at all easy, and such success as is being obtained in these experiments is due in no small degree to the skill and experience of the mechanician, Mr. Julius Pearson, who has from the start made all the tubes and contributed not a little toward their design. Dr. A. E. Hennings and Dr. W. H. Kadisch, whose papers have already been referred to, have also assisted materially in the evolution of these tubes. That shown in Fig. 1 is the ninth tube of this general plan which has been constructed. Since the detail in the photograph is not very clear it will be explained in connection with the diagram shown in Fig. 2. This diagram, however, relates to a tube used before that shown in the photograph and one in which the motions of the plate S in the measurement of the contact

is the absorption of energy by an electron from a light wave proportional to ν; its numerical magnitude is approximated a least by multiplying the frequency of the light by Planck's, h. It is true there is here no accurate agreement as yet for part of the energy absorbed by the electron is lost in getting out of the metal and the exact amount of this loss has not been measured with as much accuracy as we hope soon to be able to obtain," etc.

E.M.F.'s were made mechanically instead of electromagnetically, otherwise the tubes are identical.

The three metals on the periphery of the wheel *w* (Fig. 2) are cast cylinders of sodium, potassium and lithium. The measurements, herewith reported relate only to the sodium and the lithium, an accident having prevented the inclusion of data on potassium at the present time. The wheel *w* is rotated by the electromagnet (not shown in Fig. 2) which is nearest the observer in Fig. 1, until it is opposite the rotary knife *K*. The electromagnet *F* (Fig. 2) is then energized and slipped slightly forward. It thus advances the armature *M′* which is rigidly attached to the knife *K* by a rod which slips freely through the second armature *M* until the stop *N* strikes against *M*. In this motion the small centering rod which projects in front of the knife *K* enters a small cylindrical hole in the middle of the sodium cylinder. The magnet *F* is then demagnetized and slipped still farther forward until it is opposite the armature, *M*, re-energized and rotated on an axis coinciding with the axis of the tube. This motion advances the knife *K*, which is rigidly attached to *M*, by pushing the screw between *K* and *M* forward in the brass nut carried by a frame which is rigidly attached to the walls of the tube (Fig. 2).

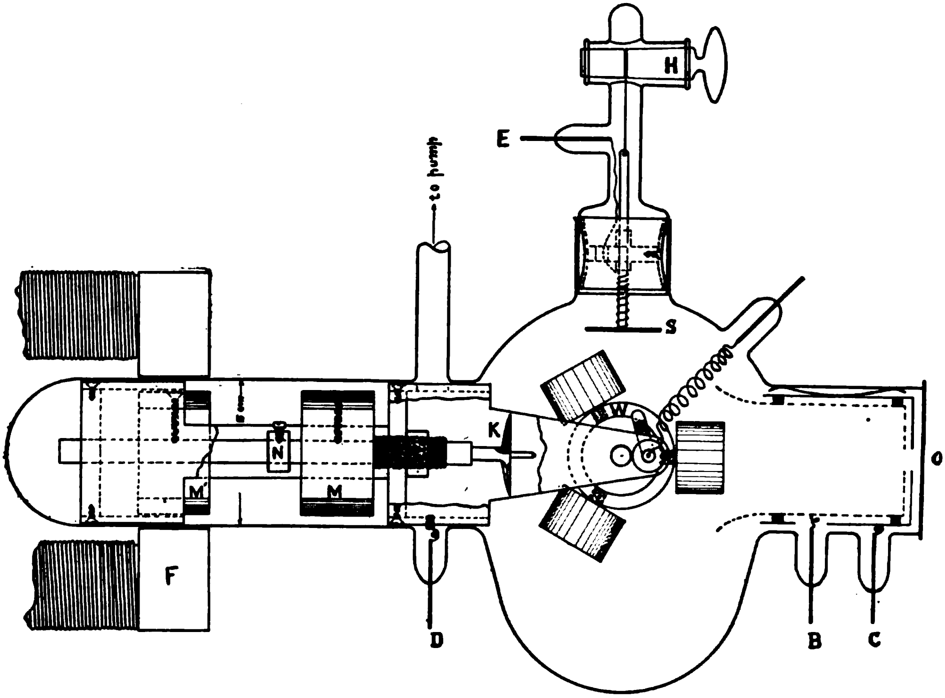

Fig. 2.

K is thus advanced far enough so that upon rotation it will cut a thin shaving from the face of the sodium cylinder. *F* is then demagnetized, slipped back to its position opposite *M′*, energized and rotated and the shaving cut off. It will be clear that by suitably setting *M* the thickness of the shaving can be regulated at will. The shaving falls down into the enlargement on the bulb beneath the wheel where it is of use in freeing the tube from residual oxygen. The knife *K* is next withdrawn from the sodium by slipping back the electromagnet *F*. The wheel *W* is then rotated until the freshly cut sodium surface is normal to the beam of monochromatic light entering at *Q*, and a number of photocurrents (see below) are taken corresponding to various P.D.'s applied between the sodium and the Faraday cylinder into which it projects. This Faraday cylinder consists of an inner cylinder of copper gauze and an outer one of solid copper. The gauze cylinder, of mesh 1/20 cm., is strongly oxidized by dipping it into a solution of copper nitrate and then holding it in a bunsen flame. The inner gauze cylinder and the outer solid one are insulated from each other by ebonite rings 3 mm. thick, though in this experiment they were always at the same potential because of the joining of the electrodes *B* and *C*. This type of Faraday cylinder, originally designed by Ladenburg and Markow, was here used for the sake of giving the corpuscles which have just enough energy to reach the gauze a chance to go through the mesh and be caught between the gauze and cylinder, the idea being that the photocurrent-potential curve would plunge into the potential axis more suddenly and thus locate the maximum positive potential more accurately than if a single cylinder were used. Furthermore, the gauze projected out beyond the sodium so as to catch all electrons which left the latter (see Figs. 1 and 2).

At any desired time the freshly cut sodium or lithium surface was rotated until it was opposite the test plate *S*, which was of copper treated as to oxidizing, washing, drying, etc., exactly as was the copper cylinder, so that the contact E.M.F. between the sodium and *S* should be the same as that between the sodium and the gauze cylinder. This contact E.M.F. was measured by the Kelvin method, *i. e.*, by applying, with a potentiometer system, just such a P.D. between the sodium and the plate *S* that lifting *S* from a position say two tenths millimeter above the face of the sodium caused no motion in an electrometer needle connected to *W*. The accuracy of this determination was better than ½ per cent. All potentials measured in this work were in terms of the indications of a Weston Standard Laboratory voltmeter certified correct to 0.1 per cent. by the Bureau of Standards at Washington.

Since the aim was to test with the utmost possible accuracy the slope of the line connecting frequency with the maximum P.D. it was necessary first to know ν with great precision and second to see that no trace of light of frequency greater than that plotted got through the slit of the spectroscope. To this end a Heracus 200 volt quartz-mercury lamp was used as a source, the lines of a powerful spark source such as that used by Kadesch and also by Lenard and Ramsauer having been found much too impure for the purpose in hand. A grating photograph of the spectrum of the mercury lamp was taken and only such lines were chosen for use as had no companions anywhere on the short wave-length side. This photograph is shown in Fig. 3. The central lines belong to the iron comparison spectrum, the mercury lines being above and below it. The lines chosen for use were $\lambda = 5{,}461$ (not shown in the photograph) $\lambda = 4339.4$, $\lambda = 4046.8$, $\lambda = 3650.2$, $\lambda = 3125.5$, $\lambda = 2534.7$ and $\lambda = 2399$. The wave-lengths were kindly determined by Mr. Walter T. Whitney. A glance at the photograph will show how well adapted these lines are for use as accurately determined monochromatic sources so far at least as the short wave-length side is concerned. Light filters to cut out stray short wave-length light were also used as described below. Since the measurement was to be made on the maximum P.D. and since this increases with decreasing wave-length it was not of great importance that the source be of great purity on the long wave-length side. The spectroscope used was a Hilgar monochromator, the end of which is shown in Fig. 1. The slit width was uniformly .01 inch save in the case of lines 2,535 and 2,650 with which slit-widths as large as .04 inch were sometimes used.

The pressure in the apparatus, which was exhausted by a Gaede molecular pump, assisted by the chemical action of the sodium shavings, was carried down to a ten-millionth of a millimeter or less as measured by a McLeod gauge sensitive enough to permit estimates as low as the value mentioned. Such a gauge gives, of course, no indication as to the amount of mercury vapor present, but there appears to be no reason for thinking that this is not carried out by the molecular pump (which ran continuously throughout the observations unless otherwise stated) nearly as completely as are other gases. I have, however, made no tests upon this point and hence can make no positive assertions.

3. SOURCES OF ERROR AND THEIR ELIMINATION.

(*a*) *Magnitude of Photocurrents.*—Since photopotentials are independent of light intensity, it is evident that the accuracy with which the point of intersection of the photocurrent-potential curve with the potential axis can be determined is directly proportional to the magnitude of the

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March, 1916.

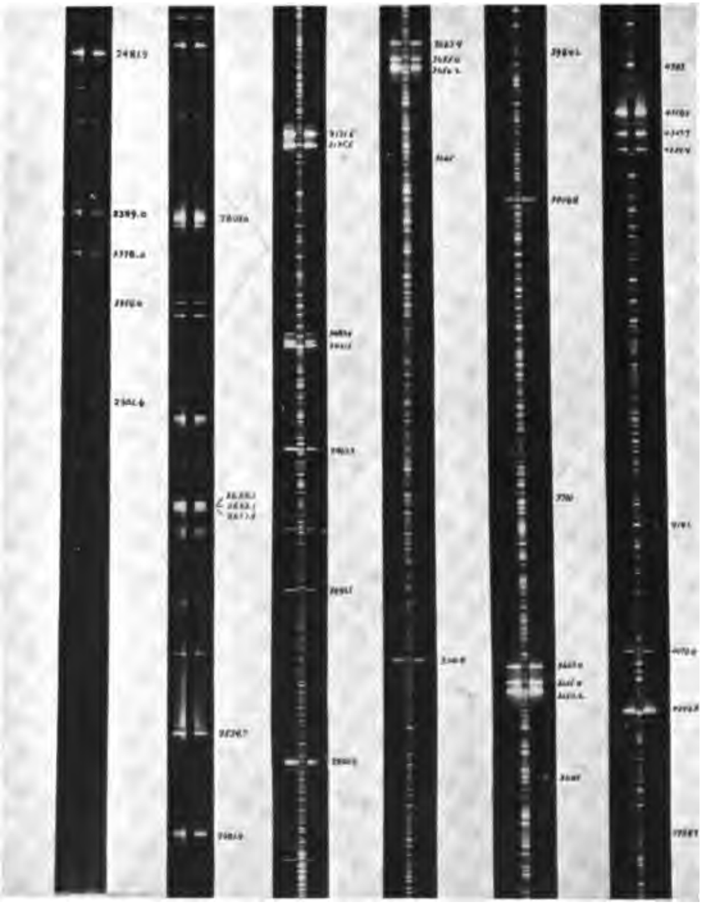

Fig. 3.

R. A. MILLIKAN.

photocurrents at any particular potential, for example at saturation. The photocurrents from a freshly shaved sodium surface are, in the best attainable vacuum, hundred of times larger than those from surfaces a few hours old. Results of especial interest in this connection will be found in a following paper.

The method employed for obtaining the maximum energy of emission in volts was not to attempt to locate experimentally just that retarding potential which would prevent the escape of electrons from the sodium or lithium to the gauze—a very uncertain procedure at best—but rather to plot the photocurrent-potential curve in the neighborhood of the balance point and then to locate this point graphically. These currents were measured with a Dolezalek electrometer connected to the joined electrodes B and C (Fig. 2). This electrometer had a sensibility of 2,500 mm. per volt, and the scale could be read to 0.2 mm.

The currents were measured by the deflections obtained in an illumination of 30 seconds, and the deflections used in the location of the balance potentials in the case of the sodium ranged from 3 mm. to 80 mm. *But with all the lines studied the saturation currents were from 75 to 500 times these maximum currents used in locating the balance points.* Thus the saturation current for line 5,461 was 3,000 mm., that for line 4,339 was 6,000 mm., that for line 4,047 was 6,000 mm., that for line 3,650 was 31,000 mm., that for line 3,125 was 17,000 mm., that for line 2,652 was 3,000 mm. and that for line 2,535 was 7,800 mm. This means that the complete curves the beginnings of which are shown in Fig. 5 would be from three to twenty meters high. These enormous currents, when measured in scale divisions, make the location of the intercepts on the potential axis a matter of much precision. Smaller currents were used in part of the measurements to be reported both because light filters were introduced (see below) and because a considerable portion of the energy passing through the slit was absorbed in a thermopile introduced for the sake of making simultaneous photoelectric and energy measurements.

(*b*) *Electrons Liberated by Reflected Light.*—In general the balance potential does not correspond at all to the potential necessary to stop the electrons having the highest energy of emission, but rather to the potential at which the stream of electrons escaping from the illuminated surface is equal to the stream liberated from the walls opposite that surface by light reflected to these walls. With small photocurrents the difference between the two potentials may be very large. Of course, the attempt is usually made to measure and allow for this reverse electronic current by applying a large positive potential to the illuminated electrode and noting the apparent positive leak, but a correction made in this way is very uncertain. It was because of this error that Richardson and Compton placed less reliance on their determination of maximum energy of emission that on that of mean energy. But the validity of Einstein's equation can only be tested by accurate determinations of the maximum energy. This "back leakage" was here eliminated completely by using only *light for which the illuminated electrode was photoelectrically sensitive, but for which the surrounding walls were not photo-sensitive.* In the following experiments with sodium the long wave-length limit of the oxidized copper walls was $\lambda = 2688$ Å,[1] but the sodium was sensitive up to $\lambda = 6800$ Å. *For any intermediate wave-lengths, though the saturation currents were measured by a deflection of from 3 to 31 meters, there was not a quarter of a mm. of deflection in 30 seconds when the sodium was given* a positive potential of 6 volts. Hence the balance potentials corresponding to the lines from 3,125 to 5,461, inclusive, were obtained with very great precision, so far as this effect alone was concerned. Line 2,535 was below the long wave-length limit of the walls of the Faraday cylinder which was used in the measurements on sodium, so that a positive potential of 6 volts applied to the sodium produced a back leak of 20 mm. in 30 seconds, because of electrons liberated by the reflected light. This was corrected for in plotting line 2,535, but I consider the balance potential for this line as less reliable than those for the other lines. When it is remembered that most photopotential-frequency curves have been obtained from observations on lines between 2,500 and 1,800 where this positive leak is always very large it is not to be wondered at that most erratic and uncertain results have been obtained.

(*c*) *Contact Electromotive Forces.*—The measurement of the contact E.M.F. between the alkali metals and the test plate showed that it varied from 0.4 volt up to 3.0 volts—depending upon the condition as to age, absorbed gas, etc., of the surfaces tested. This shows the futility of attempting to obtain accurate information about photopotential-frequency relations in the case of the alkali metals without simultaneous measurement in vacuo of photopotentials and contact E.M.F.'s. Furthermore, these changes in contact E.M.F. were found to take place quite rapidly immediately after a fresh surface had been formed by shaving. These results will be more fully reported in a following paper. For the purpose of this study it was more convenient and more accurate to find conditions under which the contact E.M.F. would remain constant over a time sufficiently long to make possible a full series of photopotential-frequency measurements, than to correct each photopotential measurement by a simultaneous contact E.M.F. determination. Such conditions were found after considerable experimenting and oddly enough they were obtained not in the best obtainable vaccum, but with a pressure of as much as .01 mm. of gas in the bulb. Under such conditions there was obtained the very desirable combination of a constant contact E.M.F. and very large and fairly constant photocurrents so that the most reliable photopotential frequency curve yet taken on sodium (that shown in Figs. 5 and 7) was taken at a pressure as high as .01 mm. of mercury. These results were in perfect agreement, however, so far as concerns "*h*," with others obtained in the best obtainable vaccum by another method (see below).

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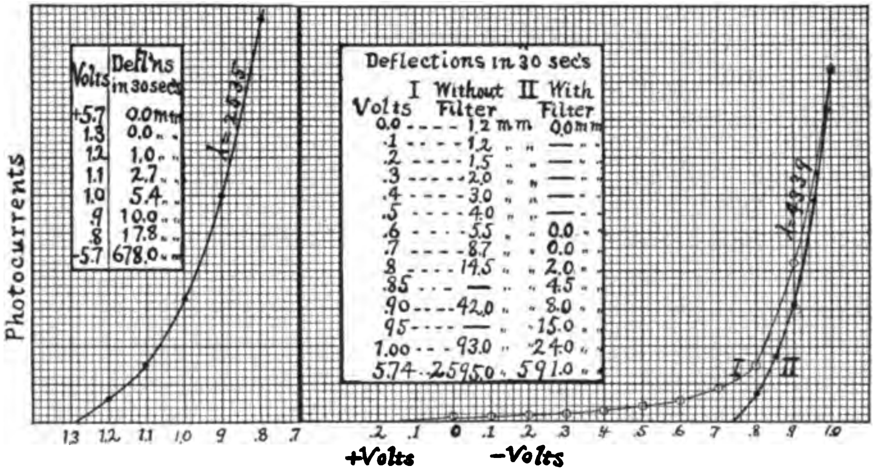
[1] In the experiments with lithium the receiving cylinder has been changed and its long wave-length limit was distinctly and definitely lower, namely $\lambda = 2535$ A°.

(*d*) *The Length of the Photopotential-Frequency Curve.*—The smallness of the range of frequencies throughout which reliable photopotentials can be measured has already been mentioned as the gravest source of uncertainty in attempting to test any one of assertions 2, 3, 4 or 5 of page 356. Ladenburg's whole wave-length range, viz., from 260 $\mu\mu$ to 201 $\mu\mu$ gave him a curve of length but 59 $\mu\mu$ and that in a region in which, in view of the sensitiveness to the reflected light of all substances of which the collecting Faraday cylinder can be made, accurate measurements of photopotentials cannot be obtained. Hughes's range of 69 $\mu\mu$ was in a less favorable region 185 $\mu\mu$ to 253 $\mu\mu$ even than Ladenburg's. All of Richardson and Compton's published photocurrent-potential curves save those on Al and Na correspond to wave-lengths between $\lambda = 210\ \mu\mu$ and $\lambda = 275\ \mu\mu$, *i. e.*, the range is 65 $\mu\mu$ or about the same as in Ladenburg's experiments, and the spectral region is the same. With aluminum the range was from 200 $\mu\mu$ to 310 $\mu\mu$, or a total of 110 $\mu\mu$, and it was against the validity of the conclusions from the work on Al that Pohl and Pringsheim critique was chiefly directed. On sodium, Richardson and Compton obtained no complete photocurrent-potential curve, but made measurements on maximum positive potentials with wave-lengths 436, 366, 313, 254, and 210 $\mu\mu$. The settings on 254 and 210 are subject to the same difficulty due to reflected light as are my own readings on these lines, so that although they made no measurements of contact E.M.F.'s of sodium in vacuo, their range of wave-lengths at which reliable settings on maximum energy of emission might have been made if the contact E.M.F.'s had been known was from 430 $\mu\mu$ to 313 $\mu\mu$ or 117 $\mu\mu$.

Kadesch's observations on sodium and potassium covered the range of 390 $\mu\mu$ to 220 $\mu\mu$ or 170 $\mu\mu$.

The present observations on sodium cover the range from 680 $\mu\mu$, to 240 $\mu\mu$, or a total of 440 $\mu\mu$ and the potentials determined with a very high precision (about .01 volt) cover the range 577.0 to 265.0 or 312 $\mu\mu$, *i. e.*, the range is four times that of Ladenburg and Hughes and the region in which thoroughly trustworthy settings can be made is nearly three times that of Richardson and Compton or of Kadesch.

(*e*) *Scattered Short Wave-Length Light.*—With the very large sensibility of from 2,000 to 31,000 mm. of deflection for saturation currents, settings on the potential necessary to entirely stop a negative discharge would be incorrect if there were the slightest trace of scattered light of wave-length considerably shorter than that of the line being studied. There were in these experiments slight indications of such traces of short wave-length light, deflections of several millimeters in 30 seconds persisting in some cases considerably beyond the potential at which the photocurrent-potential curve was approaching the potential axis. In other words with increasing positive potential this curve would fall very rapidly toward the potential axis until it almost touched it and then shoot out nearly horizontally and approach it asymptotically. That these flat "feet" to the photocurrent-potential curves were due to stray short wave-length light diffusely reflected from the prism faces, lenses, or the plane reflector of the Hilgar monochromator was rendered probable by the fact that they were more pronounced with the longer wave-lengths than with the shorter, and also by the fact that they changed in amount when the plane reflector furnished with the instrument was replaced by one of speculum metal made in the laboratory. It was conclusively proved when it was found that the interposition of filters which absorbed all wave-lengths shorter than the one under investigation caused these "feet" to disappear entirely. Figure 4 shows a rather extreme case of the falsification of the true photocurrent-potential curve by such stray short-wave-length light. The metal studied was here lithium. The slit width was .02 inch and a considerable portion of the light which came through it was intercepted by a thermopile so that the saturation currents were smaller than those mentioned above, as the data on the figure shows. It will be seen from this data that line 2,535, which is the last strong line at the short-wave-length end of the mercury arc spectrum, did not show any trace of this stray light effect, for at + 1.3 volts, only .01 volt beyond the intercept there was not a trace of a deflection although .2 mm. could have been easily detected. It will be seen too that at +5.7 volts there was also no trace of a deflection, thus showing that in this case line 2,535 was above the long-wave-length limit of the receiving Faraday cylinder. These readings together with the shape of the curve constitute the best of evidence that neither stray short-wave-length light nor reverse electronic currents (Richardson and Compton's "back leakage") vitiate in any way this curve corresponding to line 2,535. But the curve corresponding to line 4,339 has a very pronounced "foot" (see Curve I.) which however disappeared completely upon interposing a filter of aesculin in a glass trough—a combination which was found to cut out entirely all lines below 4,339 including the strong adjacent line 4,047. Note that there was now no trace of a deflection either at −.6 or at −.7 volt (see Curve II.). *These curves and a great many similar ones which I have taken seem to me to establish beyond question the contention that there is a definite, exactly determinable maximum velocity of emission of corpuscles from a metal under the influence of light of a given frequency.* Ramsauer's[1] recent conclusion to the contrary is due, I think, to the fact that the errors caused by stray short-wave-length light (falsches licht) were so large as to falsify entirely the lower parts of his velocity distribution curves. His correction for "falsches licht" actually amounted to as much as 10 per cent. of his saturation currents.[2] It will be seen that the effect of this stray short-wave-length light is to push the maximum potentials corresponding to the longer wave-lengths too far in the direction of positive potentials, while the effect of reverse electronic currents is to push the intercepts corresponding to the shorter wave-lengths, which alone they influence, in the opposite direction. These two errors are probably chiefly responsible for the fact that the slope h/e has heretofore come out so erratically and so low.



Volts	Defl'ns in 30 secs
+5.7	0.0 mm
1.3	0.0 " "
1.2	1.0 " "
1.1	2.7 " "
1.0	5.4 " "
.9	10.0 " "
.8	17.8 " "
−5.7	678.0 " "

Deflections in 30 sec's

Volts	I Without Filter	II With Filter
0.0	1.2 mm	0.0 mm
.1	1.2 " "	— " "
.2	1.5 " "	— " "
.3	2.0 " "	— " "
.4	3.0 " "	— " "
.5	4.0 " "	— " "
.6	5.5 " "	0.0 " "
.7	8.7 " "	0.0 " "
.8	14.5 " "	2.0 " "
.85	— " "	4.5 " "
.90	42.0 " "	8.0 " "
.95	— " "	15.0 " "
1.00	93.0 " "	24.0 " "
5.74	2595.0 " "	591.0 " "

Fig. 4.

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[1] Ann. der Phys., 45, p. 1120, 1914.

[2] For more complete discussion of Ramsauer's experiments see R. A. Millikan, Phys. Rev., January, 1916.

In the present work there were selected, for each line studied, filters which absorbed all, or nearly all, of the energy of wave-length shorter than that of the line under investigation. Since, however, these filters diminish largely the total saturation currents their use tends to diminish the accuracy of locating the intercepts. Hence after it has been established that the flat " feet " of the curves are due to stray light, it is perhaps just as well to reduce these feet as much as possible, without the use of filters, that is, by having all transmitting and reflecting surfaces as clean and as perfect as possible and all absorbing surfaces as black as possible, and then simply to cut the feet off. Thus even in the extreme case shown in Fig. 4, Curve I., very little error in the intercept would have been made by drawing the curve altogether without reference to the last eight small readings. In the work on the sodium, after a study of all the lines had been made with filters and the stray light effects reduced to the order of a millimeter in 30 seconds, the filters were dispensed with entirely and curves plotted corresponding to observed deflections between 3 mm. and 80 mm. The result is shown in Fig. 5.

(*f*) *Maximum Energy of Emission vs. Most Frequent Energy.*—Richardson and Compton place least reliance upon their determinations of maximum energies of emission and draw most of their conclusions from a consideration of the *most frequent* energy of emission, making the assumption that with all wave-lengths the ratio between the most frequent energy and the maximum energy is a constant. In the present experiments with sodium this was not found to be the case, and in general it seems probable that the assumption cannot be strictly true. For even if the surface of the sodium were a perfectly homogeneous substance (evidence against this view will be brought forward in a second paper), the differences in the penetrating power of light of different wave-lengths into the sodium, as well as the differences in the absorptions of electrons of different speeds in the layers of sodium through which they emerge, would seem to require differences in the velocity distribution curves obtained with different spectral lines. Hence, there would seem to be no warrant for expecting to obtain a reliable test of any of the assertions of Einstein's equation by comparing the most frequent velocities of emission produced by different lines. *If Planck's " h " is actually involved in photoelectric phenomena its presence can be expected to be brought to light only by accurate measurements on the maximum energies of emission under the influence of lights of different frequencies.* Attention was, therefore, here directed entirely to this end.

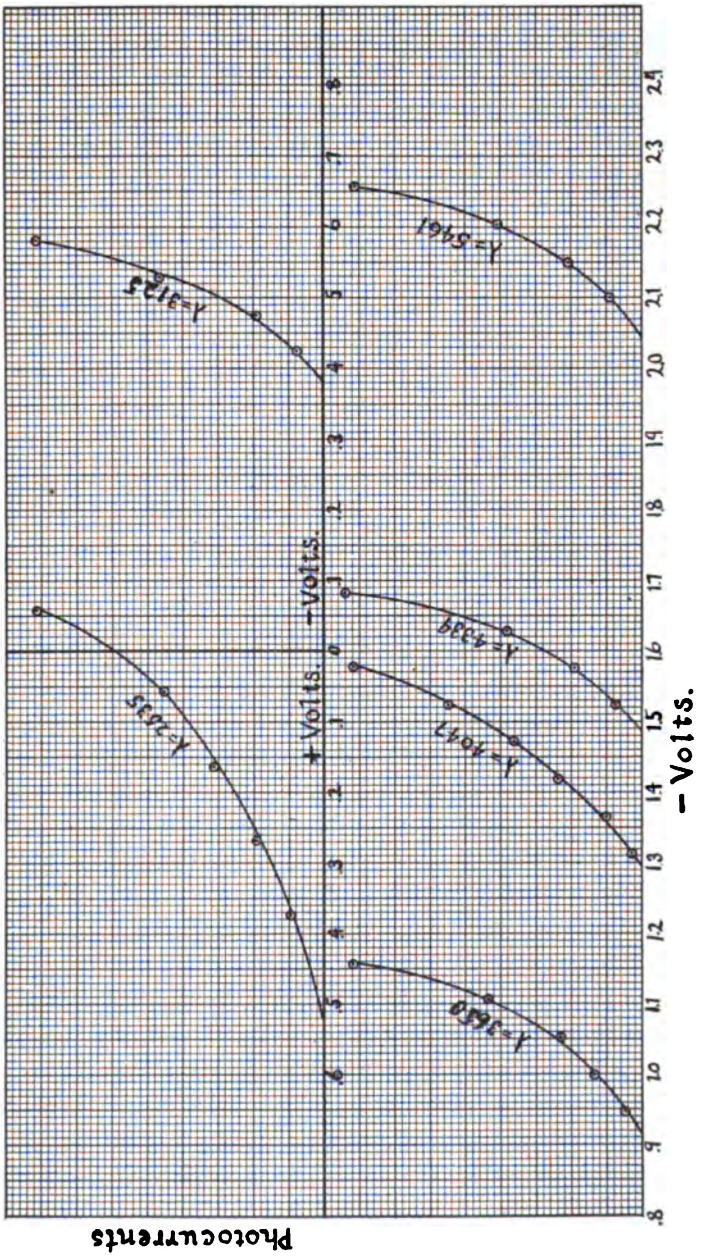
λ=5461
λ=4339
λ=4047
λ=3650
λ=3125
λ=2535
+Volts.
-Volts.
- Volts.
Photocurrents
.8
.7
.6
.5
.4
.3
.2
.1
0
.1
.2
.3
.4
.5
.6
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
2.1
2.2
2.3
2.4

Fig. 5.

4. THE DETERMINATION OF *h*.

Fig. 5 shows the most reliable single set of photocurrent potential curves yet taken for the different lines. For compactness the whole right half of the figure, namely the half containing lines 3,650 to 5,461, is placed below the half containing lines 2,535 and 3,125. It will be seen that all the curves, save that corresponding to $\lambda = 2{,}535$, strike the potential axis on the side of negative volts. This means simply that *e* times the contact P.D., which, since sodium is positive with respect to copper, acts as a retarding potential, exceeds the energy of emission save in the case of line 2,535. It will be seen also that the maximum possible error in locating any of the intercepts is say two hundredths of a volt and that the total range of volts covered by the intercepts is more than 2.5. Each point, therefore, of a potential-frequency curve should be located with not more than a per cent. of uncertainty. The frequencies are, of course, known with great precision. In the case of a group of lines such as that about 3,650 or 4,339 (see Fig. 3) the shortest wave-length of the group is taken since it alone determines the position of the intercept. The actual potentiometer readings and the corresponding deflections in mm. for all these lines are given in Table I. In the plotting the deflections in mm. were multiplied by such factors as were necessary to give all the curves practically the same maximum ordinate. No corrections of any kind were applied to the readings save in the case of line 2,535 (see section 3, *b* above).

TABLE I.

5,461.		4,339.		4,047.		3,650.		3,126.		2,535.	
Volts.	Def'n., mm.	Volts.	Def'n., mm.	Volts.	Def'n., mm.	Volts.	Def'n., mm.	Volts.	Def'n., mm.	Volts	Def'n., mm.
2.257	28	1.581	44	1.576	82	1.157	67½	.5812	52	−.0576	68
2.205	14	1.629	20	1.524	55	1.105	36	.5288	29	+.0576	38
2.152	7	1.576	10	1.471	36	1.0525	19	.4765	12	+.1620	26
2.100	3	1.524	4	1.419	24	1.0002	11	.4242	5	+.2670	16½
				1.367	3	.9478	4	.3718	2½	+.3720	8

The result of plotting the intercepts on the potential axis against the frequencies is given in Fig. 6. It will be seen that the first result is to strikingly confirm the conclusion reached successively by Joffé, Hull, Kunz, Hughes, Richardson and Compton and Kadesch as to the correctness of the predicted linear relationship between maximum P.D. and *ν*, no point missing the line by more than about a hundredth of a volt.

The slope of this line was fixed primarily by a consideration of the five

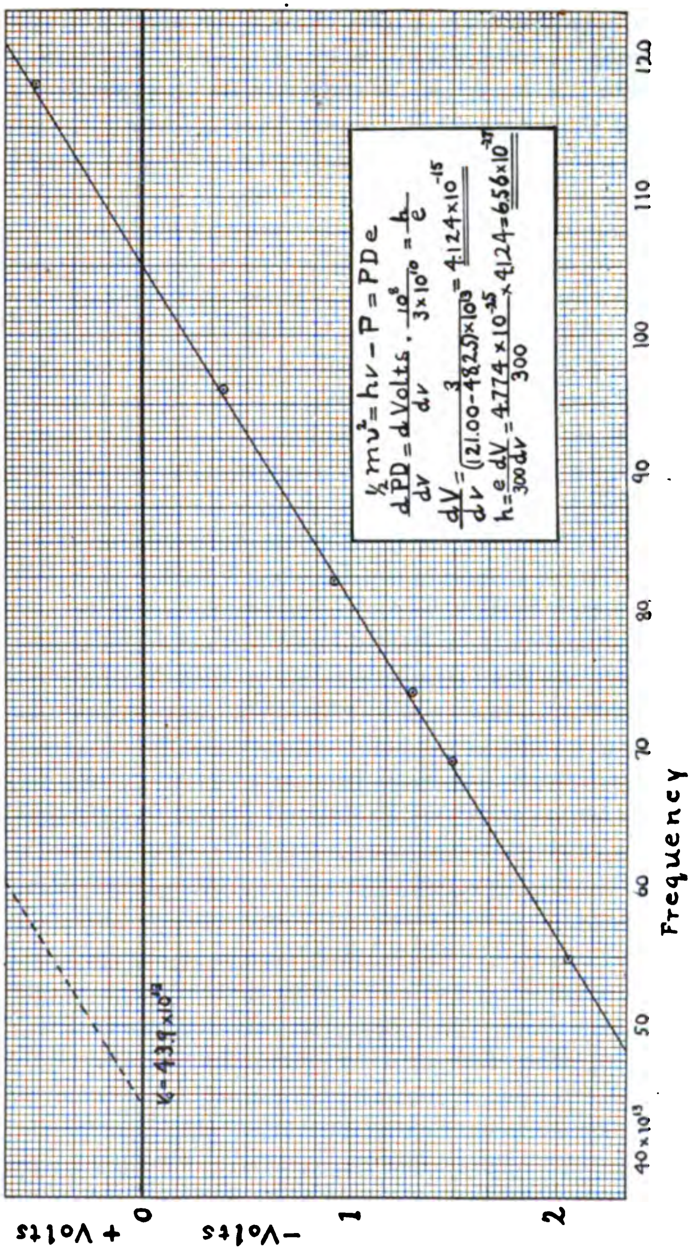
½ mv² = hν − P = PDe
d PD / dν = d Volts / dν · 10^8 / 3×10^10 = h / e
dV / dν = 3 / (121.00 − 48.25)×10^13 = 4.124×10^-15
h = e dV / 300 dν = 4.774×10^-10 / 300 × 4.124 = 6.56×10^-27
K = 4.39×10^14
40×10^13
50
60
70
80
90
100
110
120
Frequency
+ Volts
0
− Volts
1
2

Fig 6.

points corresponding to lines 5,461, 4,339, 4,047, 3,651 and 3,125, of which the first, the fourth and the fifth are particularly reliable.

Since no potential departs from the line by more than .01 volt and since the range of volts is about 2, it is a conservative estimate to place the maximum uncertainty in the slope at about 1 in 200 or 0.5 per cent. The value of this slope in volt-frequencies is $4.124 \times {}^{-15}$. Setting this, when reduced to absolute units, equal to h/e and inserting my value of e, namely 4.774×10^{-10}, there results $h = 6.56 \times 10^{-27}$.

The measurements thus far reported were all made upon a particular surface several hours after it had been cut, the conditions being exactly the same with all the lines, the only change made throughout the whole series of observations consisting in shifting the incident light from one wave-length to another by the adjustments provided on the Hilger monochromator. Such a shift required but a couple of minutes at the most. The gas pressure within the bulb amounted to as much as a hundredth of a millimeter, for the reason that after considerable experimenting it was found that under these conditions, though the contact E.M.F. varied rapidly and largely immediately after shaving, in an hour or so it reached a constancy which it maintained for several hours. Furthermore, this constant contact E.M.F. was accompanied by very large and fairly constant photocurrents—a condition very essential to accuracy in the determination of photo-potentials. In a high vacuum, though the contact E.M.F. does not change rapidly after shaving, the photocurrents often decay to a third or a fourth of the initial value in half an hour, so that it is impossible to get a complete set of curves like the above.

Having placed beyond a doubt, however, the fact of the linear relation between V and ν, it was possible to determine the slope of the line in the highest attainable vacuum by locating two distant points on it in rapid succession. Wave-length 5,461 was always chosen for one of these points because its great intensity and large wave-length *i. e.*, steep slope, adapt it admirably to an accurate determination of the intercept on the potential axis. This intercept was determined a few minutes after shaving, while the molecular pump was kept running and the McLeod gauge showed a pressure of perhaps a ten millionth of a millimeter, then the intercept of the other chosen line was found as quickly as possible by the method used above, after which the intercept of 5,461 was again taken. If the contact E.M.F. was changing uniformly this change should be eliminated by taking the mean intercept of line 5,461. The results of nine different determinations of slope made in this way on different days and with different wave-lengths to compare with 5,461 are given in Table II.

TABLE II.

Wave-Length Compared with 5,461.	Slope in Volt-Frequencies.
3,126	4.11×10^{-15}
3,650	4.14 "
3,126	4.10 "
3,650	4.12 "
3,126	4.24 "
4,047	3.98 "
2,535	4.04 "
3,126	4.24 "
4,047	4.21 "
	Mean........4.131×10^{-15}

The estimated uncertainties in these slopes, as set down at the time of taking, range from 1 per cent. to 4 per cent., those corresponding to lines 3,650 and 3,120 being most reliable. I have not, however, attempted to weight the individual observations in taking the mean. This is seen, however, to be in very close agreement with the value found from Fig. 5. We may conclude then that the slope of the volt-frequency line for sodium is the mean of 4.124 and 4.131, namely 4.128×10^{-15} which, with my value of e, yields

$$h = 6.569 \times 10^{-27} \text{ erg. sec.}$$

The measurements on lithium were made in a new tube and the new receiving Faraday cylinder was found to have a long wave-length limit 150 Å. lower than its predecessor so that the maximum positive potential due to line 2,535 could now be determined with precision. Lithium was found to differ from sodium in that the surface formed by shaving in vacuo remained strongly photo-sensitive for an indefinite period. It was studied without shaving through a period of six months during which time it showed no marked decay. The set of observations in Table III was taken a few days after shaving without filters but with

TABLE III.

4339.		4047.		3650.		3125.		2535.	
Volts.	Def'n., mm.	Volts.	Def'n., mm.	Volts.	Def'n., mm.	Volts.	Def'n., mm.	Volts.	Def'n., mm.
−6.00	1,300.0	−6.000	1,950.0	−6.000	6,000.0	−6.000	6,000.0	−6.000	3,500.0
−1.200	10.0	−1.087	21.5	−0.680	19.0	− .077	16.0	+0.736	18.0
−1.153	5.7	−1.027	10.5	−0.628	9.5	.000	6.3	+0.777	10.5
−1.103	2.7	−0.984	6.7	−0.575	5.0	+ .058	2.3	+0.814	6.7
−1.058	1.7	−0.941	3.5	−0.539	2.7	+ .100	1.1	+0.873	4.0
−1.000	1.1	−0.900	2.5	0.495	1.5	+ .151	.5	+3.928	2.0
− .950	0.6	−0.845	1.0	0.437	.6	+ .200	.2	+0.969	1.0
− .870	0.5	−0.793	0.5	0.390	.2			+1.019	0.5
		−0.737	0.25					+1.069	0.0
								+6.000	0.0

all the precautions for avoiding stray short-wave-length light which had been used with the sodium. Line 5,461 was entirely above the long-wave-length limit of the lithium, but the other lines gave readings as follows:

It will be seen that the observations were purposely extended down to very minute deflections where the effects of stray short-wave-length light

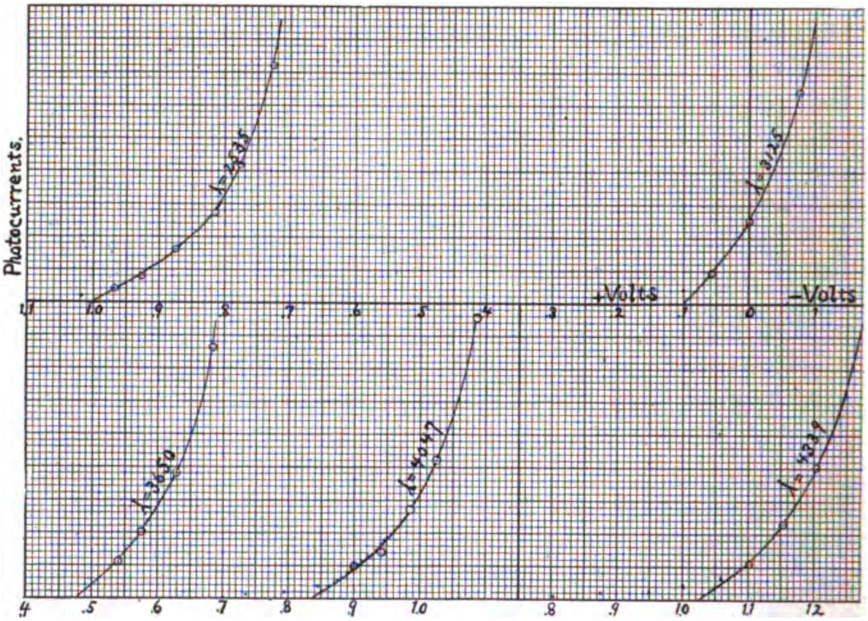

Fig 7.

are observable, but in drawing the curves it was planned beforehand to ignore all deflections under 3 mm., so as to have the results comparable in every way with those on sodium. In this way the intercepts in Fig. 7 were all determined precisely as they now stand before plotting as in Fig. 8 was tried at all. It will be seen from Fig. 8 that the points so found are in perfect alignment, and that the slope of the line is extraordinarily close to that found with sodium. Several months later, on the then old lithium surface, another set of observations was taken using a new spectrometer. The observations are those shown in Fig. 4. Fig. 9 shows the same observations graphed so as to determine *h*. This quantity is seen to come out exceedingly close to the value found several months earlier on the new surface. From the two determinations on lithium we obtain the mean

$$h = 6.584^{-27} \text{ erg. sec.}$$

the uncertainty here being perhaps as much as 1 per cent. In the matter of precision I place the greater reliance on the value 6.57^{-27} obtained from

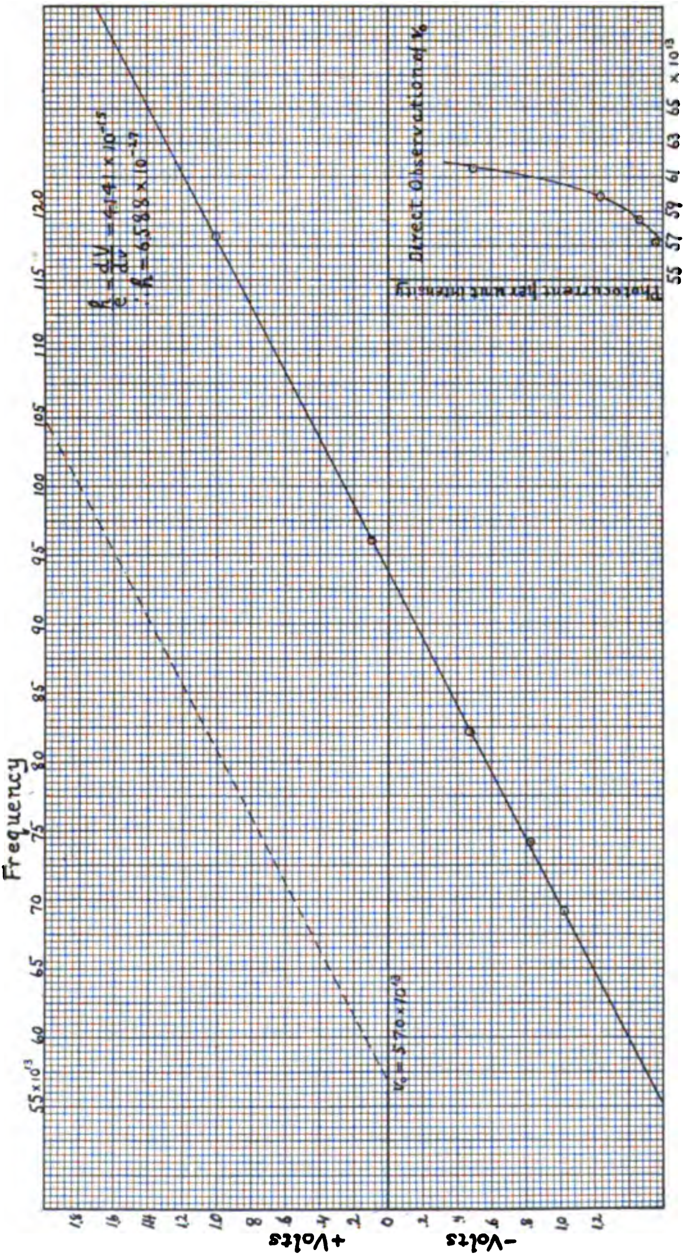
Frequency
55×10^13
60
65
70
75
80
85
90
95
100
105
110
115
120
h/e = dV/dν = 4.124 × 10^-15
∴ h = 6.588 × 10^-27
V₀ = 5.70 × 10^13
Direct Observation of V₀
Photocurrent per unit intensity
55
57
59
61
63
65 × 10^13
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
0
.2
.4
.6
.8
1.0
1.2
+Volts
-Volts

Fig. 9.

the experiments on sodium. If, as estimated above, this is in error by no more than .5 per cent. it is by far the most reliable value of h which is thus far available. Planck's original value was very close to this, namely 6.55×10^{-27}, but it was obtained from a product ab^4 in which Planck used $a = 7.061 \times 10^{-15}$ (erg./cm.^{3} degree^4) and $b = .294$ cm. degrees. In the latest edition of the "Wärmestrahlung," published in 1913, the value taken for a is 7.39×10^{-15}, while that for b is 0.289. The original value of h was obtained, then, from the product of two factors, the first of which Planck has since changed by 4.7 per cent. (the actual uncertainty in it was twice this amount), the second by 1.7 per cent. $\times\ 4 = 6.8$ per cent. so that the original value of h contained at the lowest possible estimate, an uncertainty of $\sqrt{(4.7)^2 + (6.8)^2} = 8$ per cent. The value of h at which Planck arrives in the last edition of the Wärmestrahlung is 6.415×10^{-27}, but Westphal whose value of a Planck used, has now raised his value by 2.35 per cent. while the uncertainty in b (*i. e.*, in c_2) is still at least 0.6 per cent., since this is the difference between the Reichsanstalt value $c_2 = 1.436$ and Coblenz's value $c_2 = 1.4456$. There is still, then, in each of the factors ab^4 an uncertainty of at least 2.5 per cent. and hence an uncertainty in h of $\sqrt{(2.5)^2 + (2.5)^2} = 3.5$ per cent.

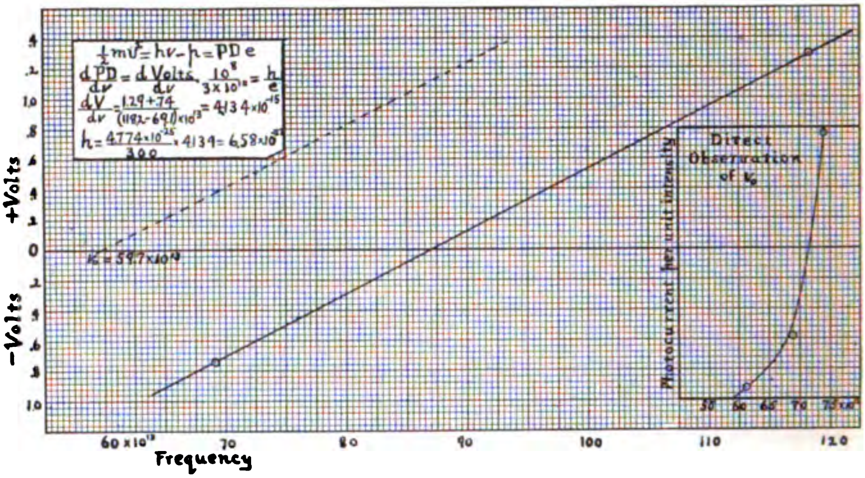

Fig. 9.

Planck's equations combined with radiation data are not then as yet able to yield a value of h which is comparable in accuracy with that obtained from these photoelectric measurements.

If, however, Planck's equation is correct we can determine h with much precision from my value of e and recent data on the total radiation constant which is related to the a of Planck's equation through $a = 4\sigma/c$, c denoting the velocity of light.

Thus Planck's equation is

$$h = \frac{k^{\frac{4}{3}}}{c}\left(\frac{48\pi\alpha}{a}\right)^{\frac{1}{3}}$$

and if we insert my value of k,[1] which has the same precision as e and the value of a corresponding to Westphal's latest conclusions as to the value of σ, namely 5.67×10^{-5}, we obtain

$$h = \frac{(1.372 \times 10^{-16})^{\frac{4}{3}}}{2.999 \times 10^{10}}\left(\frac{48 \times 1.0823}{7.56 \times 10^{-15}}\right)^{\frac{1}{3}} = 6.57 \times 10^{-27}$$

The above value of σ Westphal has kindly sent me by letter as the result of new work at the Physical Institute in Berlin. It agrees exactly with Shakespear's value,[2] and Coblenz's recent work on σ seems to indicate that it is not likely to be in error by more than a per cent. If its error is no greater than this, then the above value of h, which involves $a^{\frac{1}{3}}$ should be in error by no more than ⅓ per cent. This value will be seen to be in exact agreement with the present photoelectric determination which I have estimated as involving an uncertainty of ½ per cent.

We are now in position to compute with the aid of Planck's equation both of the constants σ and c_2 of black body radiation from my direct measurements on e and h. σ is of course seen from the preceding calculation to come out 5.67×10^{-5}, while the value of c_2 is found to be 1.436 cm. degrees.[3] This is exactly the Reichsanstalt value found in 1913.[4]

Planck's "h" appears then to stand out in connection with photo-electric measurements more sharply, more exactly and more certainly than in connection with any other type of measurements thus far made. The reason it has not appeared with greater certainty before despite the fact that the experiments of Ladenburg, Kunz, Hull, Hughes, Richardson and Compton and Kadesch have all been interpreted in terms of Einstein's theory is found, I think, chiefly in the uncertainty in determining volt-frequency relations when special precautions are not taken to avoid stray short-wave-length light, and when *the wave-lengths used are below the long-wave-length limit of the receiving surface, i. e., below about* $\lambda = 2{,}700$. Richardson and Compton's work was all done below this limit save in the case of sodium and aluminum and although the values of h obtained with these substances should have been the most reliable, the sodium gave $h = 5.2$ or 20 per cent. too low, while the Al gave $h = 4.3$, or 35 per cent. low.

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[1] Phys. Rev., Vol. II., p. 143, 1913.

[2] Proc. Roy. Soc., 86, 180, 1911.

[3] See Phys. Rev., II., 142, 1913.

[4] Ann. der Phys., 40, 609, 1913.

5. The Long-Wave-Length Limit of Photosensitiveness.

The observations corresponding to Figs. 5 and 6 were all taken between one and seven hours after shaving when the gas pressure within the bulb amounted to one or two hundredths of a millimeter. A contact E.M.F.-time curve was then taken under identical conditions, and with the following results. The contact E.M.F. before shaving was 2.819 volts, it fell upon shaving to 2.30 volts. Then rose at a fairly uniform rate for half an hour to 2.520 volts, where it remained so nearly constant for nine hours that an observation made 6 hours after shaving gave a value 2.495 volts, while one made nine hours after shaving gave 2.493 volts. After nine hours the molecular pump was started and the gas all pumped out, whereupon the contact E.M.F. rose at once to 2.820 volts, its initial value. The contact E.M.F., then, during the period of taking the photopotential curve remained constant at a value which was within a hundredth of a volt of 2.51 volts. Accordingly, in order to find the value of the frequency at which, if there were no contact field, the light could just cause an electron to escape from the metal, *i. e.*, in order to find the frequency which causes an electron to leave the surface of the metal with zero velocity it is only necessary to push up the volt frequency line of Fig. 6 by the amount of the contact E.M.F., viz., 2.51 volts, and then to find its intercept on the frequency axis. The dotted line of Fig. 6 shows this line thus displaced. The intercept is seen to be at $\nu_0 = 43.9 \times 10^{13}$ which corresponds to $\lambda_0 = 6{,}800$ Å. This is far above the long-wave-length limit of sodium as computed by Richardson and Compton who published the two estimates 5,770 and 5,830. Yet it was found by direct measurement that the long-wave-length limit of this sodium was at least in the neighborhood of 6,800. For the yellow lines 5,790 and 5,770 produced together a saturation current which was $\frac{56}{118}$ of that produced by line 5,461. Indeed the balance potential for these lines was determined and found to lie on the straight line of Fig. 6 within the limits of error of its determination, but the point corresponding to 5,461 was so near to it and could be determined with so much greater accuracy, that no attempt to use the former point in the determination of *h* was made and it was not placed on the diagram of Fig. 6. Also the red line 6,235 produced a marked saturation current of value $\frac{1}{20}$ of that of the yellow group. With the aid of Mr. Wilmer Souder who is making in this laboratory photoelectric-efficiency measurements on these same surfaces, the energies in the lines 5,461, 5,780 and 6,235 were found with a thermopile to be in the ratios 71, 100, 10, respectively. From these data the saturation photocurrents per unit intensity for these three lines are found to have the ratios 140, 50, 25. Plotting these numbers against wave-lengths and drawing a smooth curve, we can locate the intercept on the wave-length axis that is, the long-wave-length limit, within perhaps 100 Å. The result is shown in Fig. 10. This procedure is probably the simplest way of avoiding the disturbing effects of stray short-wave-length light in determining directly the long-wave-length limit. Evidence will be presented in a later paper to show that this limit is by no means a constant—a fact also emphasized by Pohl and Pringsheim,[1] but the two sorts of measurements upon it herewith reported are made at practically the same time so that their agreement is a real test of the validity of Einstein's equation.

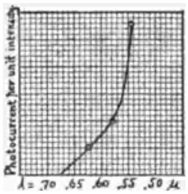

Fig. 10.

Lithium is better suited to this sort of a test than is sodium for the reason that its long-wave-length limit falls in the middle of the visible spectrum where all the elements necessary to the comparison are most easily and reliably observable. Both Fig. 8 and Fig. 9 show such comparisons, the first on a new lithium surface, and the second on the same surface several months later. It is to be particularly noted that while the measured contact E.M.F. between the lithium and the oxidized copper plate was 1.51 volts in the observations shown in Fig. 8, it was only 1.11 volts at the time the observations shown in Fig. 9 were taken, yet the displacing of the V ν line by the amount of this measured contact E.M.F. yields in each case excellent agreement between the ν_0 determined from the intercept on the ν axis and that determined by direct observation (see lower right hand corners of Figs. 8 and 9). I should be unwilling to claim that the direct observations fix the long-wave-length limit with a precision greater than 100 Å. Nevertheless these observations seem to leave no room whatever for doubt that the agreement demanded by Einstein's equation between the two methods of determining ν_0 actually exists.

6. Contact E.M.F. and Long-Wave-Length Limits.

I have shown elsewhere[2] that Einstein's equation demands that the contact E.M.F. between any two conductors be given in terms of photoelectrically measured quantities thus

$$\text{Contact E.M.F.} = \frac{h}{e}(\nu_0 - \nu_0') - (V_0 - V_0^1), \qquad (2)$$

in which ν_0 and ν_0' are the long-wave-length limits of the two conductors and V_0 and V_0^1 are the maximum potentials which must be externally applied to just prevent corpuscular discharge by light of any particular wave-length from these two conductors to any third conductor. If the long-wave-length limit of the copper oxide receiving cylinder can be determined this equation can be quite accurately tested by means of the experiments recorded above. The desired measurement on the Faraday cylinder was made with no little precision by utilizing the observation mentioned in section 3*b* that when the sodium for example was charged to a positive potential of say 6 volts, if the wave-length used was longer than the long-wave-length limit of the cylinder, there was no trace of an apparent positive leak to the cylinder upon illuminating the sodium, but as soon as, with decreasing wave-length, the long-wave-length limit of the cylinder is reached, such a leak will appear because of the liberation of electrons from the walls of the cylinder by the light reflected to them. I have already mentioned that with line 3,125 although it produced with -6 volts on the sodium a current of 17,000 mm. in 30 seconds, when the sodium was given a potential of $+6$ volts and illuminated there was not a quarter of a millimeter of deflection. Since the greater part of the light incident on the sodium is reflected back to the Faraday cylinder, this shows that line 3,125 is completely and definitely above the long-wave-length limit of the copper oxide. Lines 3,022 and 2,967 and 2,804 are weaker and their indications are accordingly less definite, but no positive leak was obtained with them. But with wave-length 2,652, which with -6 volts give a deflection of 5,300 mm., $+6$ volts produced a deflection in 30 seconds of 3 mm. and with $\lambda = 2{,}535$, which with -6 volts gave 7,800 mm., $+6$ volts produced a deflection of 20 mm. The energies of the two lines were found in the ratio 5.3 to 9.1. Plotting then deflections per unit intensity, viz., $20/9.1 = 22$ and $3/5.3 = 5.66$ against the corresponding wave-lengths and extending the straight-line connecting these two points to the wave-length axis we find the intercept at $\lambda = 2{,}688$ Å. which may be taken as the long-wave-length the limit of the copper oxide, and in view of the steepness of the curve it is not likely that this point is in error by more than say 30 Å. The corresponding ν_0' is 111.6×10^{13}. From the relation d volts$/d\nu = 4.128 \times 10^{-15}$ we can compute the maximum energy of emission in volts of corpuscles under the influence of line 2,535 (which corresponds to $\nu = 118.2 \times 10^{13}$) from a surface for which this energy is zero at $\lambda = 2{,}688$ Å. It is

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[1] Phil. Mag., 26, 1017, 1913.

[2] Phys. Rev., January, 1916.

$$4.128 \times 10^{-15}(118.2 - 111.6) \times 10^{13} = .27 \text{ volt.}$$

This is V_0^1. With line 2,535 Fig. 5 shows that the observed V_0 for sodium was .52 volt, so that $V_0 - V_0^1 = .52 - .27 = .25$ volt. But $h/e(\nu_0 - \nu_0') = 4.128 \times 10^{-15}(118.2 - 43.9) \times 10^{13} = 2.79$ volts. The measured contact E.M.F. at this time was 2.51 volts (see p. 382 and Fig. 6). Hence equation 2 becomes

$$2.51 = 2.79 - .25 = 2.54,$$

which is but about 1 per cent. in error.

Similar computations made with lithium showed equally good agreement. As stated above the Faraday cylinder used with the lithium was a new one and with line 2,535, + 6 volts on the lithium failed to reveal any trace of a reverse electronic current. But with line 2,400 Å this reverse current was distinctly and definitely present, although line 2,400 is weak in energy compared with 2,535. Line 2,535 was then in this case very close to the long-wave-length limit. Considering the limit at this line we have $V_0' = 0$. But Fig. 8 shows that at that time the contact E.M.F. was 1.52 volts, while V_0 for line 2535 was 1.00 volt and $\nu_0 = 57 \times 10^{13}$. Hence $h/e(\nu_0 - \nu_0') = 4.128 \times 10^{-15}$ $(118.2 - 57.0) \times 10^{13} = 2.526$ volts, and equation (2) becomes

$$1.52 = 2.526 - 1.00 = 1.526.$$

In the measurements taken six months later, although the contact E.M.F. had dropped to 1.11 volts, equation 2 still predicts the observed value. For now (see Fig. 9) $h/e(\nu_0 - \nu_0') = 4.128 \times 10^{-15}(118.2 - 59.7) \times 10^{13} = 2.415$ volts and $V_0 = 1.29$ volts, so that equation 2 becomes

$$1.11 = 2.415 - 1.29 = 1.125.$$

Einstein's equation appears then to stand up accurately to all of the five quantitative tests demanded of it on page 356.

9. Theories of Photo Emission.

Perhaps it is still too early to assert with absolute confidence the general and exact validity of the Einstein equation. Nevertheless, it must be admitted that the present experiments constitute very much better justification for such an assertion than has heretofore been found, and if that equation be of general validity, then it must certainly be regarded as one of the most fundamental and far reaching of the equations of physics; for it must govern the transformation of all short-wave-length electromagnetic energy into heat energy. Yet the semi-corpuscular theory by which Einstein arrived at his equation seems at present to be wholly untenable. I have pointed out elsewhere[1] that this theory was but a very particular form of the ether-string theory advanced by J. J. Thomson[2] two years earlier, for it simply superposed upon that theory the additional hypotheses (1) that the bunches of energy which are assumed to travel along the ether strings are proportional to the impressed frequency and (2) that they are transferred upon absorption as wholes to an electron. This being the case, the objections to an ether-string theory, that is, to any theory in which the energy remains localized in space instead of spreading over the entire wave-front, must hold for the Einstein theory. Lorenz[1] and Planck[2] have pointed out some of these. Despite these objections, however, Sir J. J. Thomson[3] and Norman Campbell[4] still adhere to it. I wish to call attention to one more difficulty which in itself seems to me to be very serious.

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[1] Science, Vol. XXXVII., p. 130, 1913.

[2] Electricity and Matter, pp. 62–70.

If a static electrical field has a fibrous structure, as postulated by any form of ether-string theory "each unit of positive electricity being the origin and each unit of negative electricity the termination of a Faraday tube,"[5] then the force acting on one single electron between the plates of an air condenser cannot possibly vary *continuously* with the potential difference between the plates. Now in the oil-drop experiments[6] we actually study the behavior in such an electric field of one single, isolated electron and we find, over the widest limits, exact proportionality between the field strength and the force acting on the electron as measured by the velocity with which the oil drop to which it is attached is dragged through the air.

When we maintain the field constant and vary the charge on the drop, the granular structure of electricity is proved by the discontinuous changes in the velocity, but when we maintain the charge constant and vary the field the lack of discontinuous change in the velocity disproves the contention of a fibrous structure in the field unless the assumption be made that there are an enormous number of ether strings ending in one electron. Such an assumption takes all the virtue out of an ether string theory.

Despite then the apparently complete success of the Einstein equation, the physical theory of which it was designed to be the symbolic expression is found so untenable that Einstein himself, I believe, no longer holds to it. But how else can the equation be obtained?

Before attempting to answer this question, let us consider the energy relations which it imposes. It requires the absorption at some time or other by the escaping electron of at least the energy $h\nu$ from incident waves of frequency ν. The total luminous energy falling per second from a standard candle on a square centimeter at a distance of three meters is one erg.[1] Hence the amount falling per second on a body of the size of an atom, *i. e.*, of cross section 10^{-15} cm. is 10^{-15} ergs. But the energy $h\nu$ for light of wave-length equal to 5,000 Å is 4×10^{-12} ergs or 4,000 times as much. Since not a third of this energy is in wave-lengths shorter than 5,000 Å, a photoelectric cell which is sensitive up to 5,000 Å. should require at least 12,000 seconds or 4 hours of illumination by a standard candle 3 meters away before any of its atoms could absorb enough energy to discharge a corpuscle, yet the corpuscle is observed to shoot out the instant the light is turned on. If then we must abandon the Thomson-Einstein hypothesis of localized energy, which is, of course, competent to satisfy these energy relations, *there is no alternative but to assume that the corpuscles which are ejected are already possessed of an energy almost equal to* $h\nu$. Since this energy must have come from the incident frequency, for otherwise it could not be proportional to this frequency, it is necessary to assume, if the absorption is due to resonance (and we know of no other way in which to conceive it after eliminating as above the possibility that a single ether pulse drives out a free electron) that there are oscillators of all frequencies within the absorbing body and that these oscillators are at all times in all stages of energy loading up to the value $h\nu$. But this is impossible if the oscillators, when not exposed to radiation, emit any energy at all; for if they did so, they would in time lose all their store and we should be able, by keeping bodies in the dark, to put them into a condition in which they should show no photoelectric effect until after at least 4 hours of illumination with a standard candle. Since this is contrary to experiment, we are forced, even when we discard Einstein's theory of localized energy and discontinuous absorption, to postulate electronic absorbers which do not radiate at all while they are absorbing until the absorbed energy has reached a certain critical value when explosive emission occurs. The photoelectric effect then, however it is interpreted, if only it is correctly described by Einstein's equation, furnishes a proof which is quite independent of the facts of black-body radiation of the correctness of the fundamental assumption of the quantum theory, namely, the assumption of a discontinuous or explosive emission of the energy absorbed by the electronic constituents of atoms from ether waves. It materializes, so to speak, the quantity "*h*" discovered by Planck through the study of black body radiation and gives us a confidence inspired by no other type of phenomenon that the primary physical conception underlying Planck's work corresponds to reality.

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[1] Phys. Zeit., 11349, 1910.

[2] Ann. der. Phys., 39, 1912. Berliner Ber., 723, 1911.

[3] Proc. Phys. Soc. London, XXVII., 105, December 15, 1914.

[4] Modern Electrical Theory, Cambridge Press, 1913, p. 248.

[5] J. J. Thomson's Electricity and Matter, p. 9.

[6] PHYS. REV., 2, 109, 1913.

But to return to a substitute for Einstein's theory. Planck[2] and Sommerfeld[1] have both made alternative suggestions, neither of which, however, seems competent, in general, to furnish a physical basis for the Einstein equation. Indeed Sommerfeld and Debye[2] definitely admit the inadequacy of their theory to account for the normal photoelectric effect, which is precisely the effect herewith investigated, while Planck's theory as thus far formulated does not yield the Einstein equation at all. Originally it had nothing whatever to say about the escape of corpuscles from atoms under the influence of ether waves, and even in the 1912 formulation[3] the energy of an oscillator was assumed to be given up only in the form of electromagnetic waves, but in 1913[4] the theory was extended so as to permit part of the energy to be emitted in the form of an ejected corpuscle. The oscillator of frequency ν was assumed to absorb energy continuously without radiating any of it until its energy content became $n_1 h\nu$, in which n_1 is an integer, the value of which on the average depends on the intensity of the incident waves. This energy was then assumed to shoot out explosively, the amount $\sigma h\nu$, σ being a fraction less than one, going off in the form of an ether wave, and the amount $(n - \sigma)h\nu$ appearing in the form of the kinetic energy of the escaping corpuscle. This energy $(n - \sigma)h\nu$ was then assumed to be transformed entirely into electromagnetic waves of frequency ν_2, when the corpuscle fell into a new oscillator of natural frequency ν_2.

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[1] Drude, Lehrbuch der Optik, 1906, p. 472.

[2] Ann. d. Phys., 37, 644, 1912.

Now when we compare this theory with the above experimental results we observe that there is not the slightest indication in the latter that the energy absorbed by an escaping corpuscle from an incident light wave of frequency ν is ever more than $h\nu$, or that σ has any value at all and yet the setting of $n_1 = 1$ and $\sigma = 0$ in the above expression of Planck is not consistent with his way of deducing his black body radiation law.[5]

Further the assumption that all bodies which show the normal photoelectric effect, and there is some evidence that even gases when stimulated by waves of frequency ν shoot off corpuscles with energy $h\nu$, contain sharply tuned oscillators possessing every conceivable natural frequency is a somewhat troublesome one. Certainly both the phenomena of absorption and of emission show that in most substances the great bulk of the oscillators possess natural frequencies of one or two particular values and the characteristic waves which they emit are of these frequencies, the condition for such emission being in general merely that the stimulating frequency be greater than the characteristic frequency.

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[1] Phys. Zeit., 12, 1057, 1911, Solvey Congress, 1911.

[2] Ann. d. Phys., 41, 873, 1913.

[3] Sitz. Ber. d. k. Preuss. Akad., 18, 350, 1913.

[4] Ann. d. Phys., 37, 644, 1912.

[5] My experimental results might perhaps be reconciled with Planck's theory if, under the conditions under which I work, his expression $\frac{1 - \eta}{\eta} = pI$ (Wärmestrahlung, p. 159) were a quantity very much smaller than unity. As a matter of fact, however, the intensity of illumination K which I used in these experiments was measured and found, with line 5461, to be about $10 \frac{\text{erg}}{\text{cm}^2 \text{sec}}$. Since $K = I \frac{32\pi^2}{3c}$ (see "Wärmstrahlung," p. 162) and $p = \frac{3c^3}{32\pi^2 h\nu^3}$ we see that $pI = \frac{c^2 K}{h\nu^3}$. Inserting $c = 3 \times 10^{10}$, $h = 6 \times 10^{-27}$, $\nu = 6 \times 10^{14}$ we find $pI = 7000$—a very large quantity instead of a very small one. This seems to make my results irreconcilable with Planck's theory.

In spite of these difficulties, however, a modification of Planck's formulation seems to me able to account for all the relations thus far known between corpuscular and ethereal radiations. Most of the oscillators of a given substance may be assumed to have a particular frequency or frequencies characteristic of the substance, but, just as the line spectrum of a gas is always superposed upon a faint continuous spectrum, so a few oscillators of every conceivable frequency may with no little plausibility be assumed to be mixed with the enormously larger number of oscillators which have a frequency or frequencies characteristic of the given substance. This is only making for all substances precisely the assumption which Planck makes with such conspicuous success for black substances. In other words it is assuming that all substances are to a certain degree black. If any particular frequency is incident upon such a substance the oscillators in it which are in tune with the impressed waves may be assumed to absorb the incident waves until the energy content has reached a critical value when an explosion occurs and a corpuscle is shot out with an energy $h\nu$. This free corpuscle may then be assumed to fritter away some, and in many cases all, of its energy by impacts with atoms, but if it strikes an oscillator of any frequency ν_2 while its energy exceeds $h\nu_2$, it either stimulates directly new ether waves of frequency ν_2, or else it modifies the condition of the atom in such a way that in a subsequent readjustment it emits waves of this frequency. If the substance possesses these particular oscillators ν_2 in great abundance then these emitted waves are its characteristic X-rays. Whether they are stimulated by the return of a displaced corpuscle or by the act of displacement by a corpuscle of energy greater than $h\nu_2$ may be left unsettled, though the former view is at present the more attractive.

In the photoelectric effect we study not the stimulation of these characteristic ether waves but the antecedent emission of corpuscles from the atom, this emission appears to take place to some extent at all frequencies of the incident light, though the emitted corpuscle never leaves the metal unless its energy of emission from the atom is greater than $h\nu_0$, but it takes place especially copiously when the impressed frequency coincides with a " natural frequency." According to this point of view the selective photoelectric is simply the normal effect taking place in the neighborhood of an absorption band where oscillators of one particular frequency predominate. This is little more than Planck's theory with the possibility of a corpuscle being emitted from an atom with an energy greater than $h\nu$ eliminated for the sake of reconciling it with the experimental facts above presented. It is to be hoped that such a theory will soon be shown to be also reconcilable with the facts of black body radiation.

I have to thank Mr. Wilmer H. Souder for able assistance during the latter portion of this investigation.

10. SUMMARY.

1. Einstein's photoelectric equation has been subjected to very searching tests and it appears in every case to predict exactly the observed results.

2. Planck's h has been photoelectrically determined with a precision of about .5 per cent. and is found to have the value

$$h = 6.57 \times 10^{-27}.$$

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THE CONSTANTS OF RADIOACTIVITY.

By Gerald L. Wendt.

It is now two years since the publication of Kolowrat's table of radioactive constants.[1] The numerous determinations since published require a retabulation of the fundamental values, which is presented herewith, the publication of Le Radium having been suspended because of the war.

In the subjoined table the present system of nomenclature is retained; it is probably still too early to devise a wholly satisfactory one. Since the simplification introduced by Rutherford and Geiger[2] the discovery of branching at the three *C* members of the disintegration series has again led to confusion. It seems that the term thorium C_2 should be replaced by thorium C', as was done by Soddy,[3] to correspond with radium C' and actinium C', and to indicate the striking analogy between these short-lived beta ray products of the *C* members. Radium C_2 then remains for the present as the analogue of thorium *D* and actinium *D*.

The known radioelements are now thirty-six in number. Varder and Marsden[4] have confirmed the existence of actinium C', previously observed by Marsden and Wilson,[5] and Marsden and Perkins,[6] and indicated by the work of Miss Blanquies.[7] The disintegration series of the three emanations are therefore precisely similar until the *D* members are reached. Antonoff's discovery[8] of uranium *Y* has been repeatedly confirmed.[9] It is, however, difficult to assign it to a position in the disintegration series. It is isotopic[10] with uranium X_1 and ionium, and is derived either from uranium 1 or uranium 2 by a subordinate alpha ray change. Both of these origins are anomalous since they demand that one element undergo two different alpha ray disintegrations. Still more a puzzle is the origin of actinium; its solution has not resulted even from the generalization of Fajans,[11] Soddy,[12] and Russell.[13]

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[1] Le Radium, *11*, 1 (1914).

[2] Phil. Mag., *22*, 621 (1911).

[3] Chemistry of the Radioelements, Pt. II. Longmans, 1914.

[4] Phil. Mag., *28*, 818 (1914).

[5] Nature, *92*, 29 (1913).

[6] Phil. Mag., *27*, 690 (1914).

[7] Le Radium, *7*, 159 (1910); Comptes Rendus, *151*, 57 (1910).

[8] Phil. Mag., *22*, 419 (1911); *26*, 332, 1058 (1913).

[9] Soddy, Phil. Mag., *27*, 215 (1914). Hahn and Miss Meitner, Physik. Zeitschr., *15* 236 (1914).

The latter is now well established. No exception to it is known. It is used in the chemical classification of the short-lived elements: U X_2, Ra C_2, Act D, Th D, and the three C' members. The chemical properties of the others are taken from the researches of v. Hevesy,[14] v. Hevesy and Paneth,[15] Fleck,[16] McCoy and Viol,[17] Metzener,[18] and Klemensiewicz.[19] The elements of all the three series which are in the same chemical group and on the same side of the emanations in the series are isotopic. That in spite of real differences in atomic weight isotopes are both chemically and spectroscopically indistinguishable is further proved by numerous determinations of the atomic weight of lead of radioactive origin.[20] Hönigschmid and Horowitz[21] have prepared what is probably pure radium G, with an atomic weight of 206.04, as compared with 207.18 for ordinary lead.[22] This points to radium G as the end product of the uranium series and justifies the assumption that the emission of an alpha ray results in the loss of four units in atomic weight. The difference between Hönigschmid's values for radium (225.97) and for radium G is just five times the weight of an atom of helium. Yet the genetic relation between the radioelements and ordinary lead is obscure. Rutherford and

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[10] From ἴσος, equal, and τόπος, place, as suggested by Soddy, in reference to their identical position in the periodic table of the elements. The growing use of the term *isotropic* in this sense is quite indefensible.

[11] Physik. Zeitschr., *14*, 136 (1913); Le Radium, *10*, 61 (1913).

[12] Chem. News, *107*, 97 (1913). Nature, *91*, 571 (1913).

[13] Chem. News, *107*, 49 (1913).

[14] Physik. Zeitschr., *13*, 672 (1912). Phil. Mag., *25*, 390 (1913), *27*, 586 (1914).

[15] Le Radium, *10*, 65 (1913); Monatshefte d. Chemie, *34*, 1393, 1593 (1913), *36*, 795 (1915), Physik. Zeitschr., *15*, 797 (1914), *16*, 45 (1915).

[16] Jour. Chem. Soc., *103*, 381, 1052 (1913).

[17] Phil. Mag., *25*, 333 (1913).

[18] Berichte Chem. Ges., *46*, 979 (1913).

[19] Comptes Rendus, *158*, 1889 (1914).

[20] Richards and Lembert, Jour. Amer. Chem. Soc., *36*, 1329 (1914); Comptes Rendus, *159*, 248 (1914). Maurice Curie, Comptes Rendus, *158*, 1676 (1914). Hönigschmid and Horowitz, ibid., 1796.

[21] Monatshefte d. Chemie, *36*, 355 (1915).

[22] Baxter and Grover, Jour. Amer. Chem. Soc., *37*, 1027 (1915).

Andrade[23] found that the atomic numbers of radium *B* and lead are the same, while recent work by Richards and Wadsworth[24] shows that the atomic volumes of radium *G* and lead are the same. Searching for the end product of thorium, Soddy and Hyman[25] found that lead from thorium minerals showed an abnormally high atomic weight, as if thorium *E* were stable and an isotope of lead. Hönigschmid and Horowitz[21] found no such effect, and Holmes and Lawson[26] could determine no relation between the lead and thorium contents of ancient minerals. Miss Meitner[27] has shown that pure bismuth is not radioactive and is therefore not a beta ray product of lead, as suggested by Fajans and Towara.[28]

New determinations of the ranges of the actinium elements by Meyer, Hess, and Paneth[29] have effected an excellent alignment with Geiger and Nuttall's rule.[30] The graph for the actinium series cuts that for uranium at uranium 2. Radioactinium was found to give two sets of alpha rays, of which the shorter gives exact correspondence with the above rule. McCoy and Leman,[31] however, found a single range of 4.4 cm. The former investigators confirm Hahn and Rothenbach's discovery[32] of alpha rays emitted by actinium itself. The elements which on the present data do not conform with the Geiger-Nuttall rule are actinium *A*, thorium, and ionium. The rule is used for the estimation of the periods of uranium 2, actinium, and the *C′* members.

The values of the transformation constant, λ, the half-life period, *P*, and the range, *R*, which have been changed from Kolowrat's table are based on the work of Soddy and Miss Hitchins[33] for ionium; Meyer, Hess, and Paneth[28] for ionium, polonium and the actinium series; Hahn and Miss Meitner[9] for uranium *Y*; Thaller[34] for radium *D* and *E*; Miss Heimann[35] for thorium; and McCoy and Leman[31] for radioactinium.

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[23] Phil. Mag., 27, 854 (1914).

[24] Jour. Amer. Chem. Soc., *38*, 221 (1916).

[25] Transact. Chem. Soc., *105*, 1402 (1914).

[26] Phil. Mag., *28*, 823 (1914), *29*, 673 (1915).

[27] Physik. Zeitschr., *16*, 4 (1915).

[28] Naturwissenschaften, *2*, 685 (1914).

[29] Sitzber. kais. Akad. Wiss., Wien, Abt. II*a*, *123*, 1459 (1914).

[30] Phil. Mag., *22*, 613 (1911).

[31] Phys. Rev., *4*, 409 (1914).

[32] Physik. Zeitschr. *14*, 409 (1913).

[33] Phil. Mag., *30*, 209 (1915).

[34] Sitzber. kais. Akad. Wiss., Wien, Abt. II*a*, *121*, 1611 (1912); *123*, 157 (1914).

[35] Ibid., 1369.

The absorption coefficients of the gamma rays in aluminium, μ_γ, have required amendment only in the case of radioactinium[36]; those for the beta rays, μ_β, are unchanged.

Substances.	λ (Sec.$^{-1}$)	P	Chem. Group.	Rays.	$R_{15°}$ (Cms.)	μ_β(Al) (Cm^{-1})	μ_γ(Al) (Cm.$^{-1}$)
Uranium 1	$4.3\cdot10^{-18}$	$5\cdot10^{9}$ yr.	6	α	2.50		
Uranium X_1	$3.3\cdot10^{-7}$	24.6 d.	4	β		510	24; 0.70; 0.140
Uranium X_2	0.01	1.15 min.	5	β		14.4	
Uranium 2	$1.1\cdot10^{-14}$	$2\cdot10^{6}$ yr.?	6	α	2.90		
Uranium Y	$7.5\cdot10^{-6}$	25.5 hrs.	4	β		300	
Ionium	$2.2\cdot10^{-13}$	10^{5} yr.	4	α	3.11		
Radium	$1.26\cdot10^{-11}$	1730 yr.	2	αβ	3.30	200	354; 16; 0.27
Ra Emanation	$2.085\cdot10^{-6}$	3.85 d.	0	α	4.16		
Radium A	$3.85\cdot10^{-3}$	3.0 min.	6	α	4.75		
Radium B	$4.33\cdot10^{-4}$	26.7 min.	4	β		75	230; 40; 0.51
Radium C_1	$5.93\cdot10^{-4}$	19.5 min.	5	αβ		13.5	0.115
Radium C_2	$8.3\cdot10^{-3}$	1.4 min.	3	β			
Radium C′	$7\cdot10^{+5}$	10^{-6} sec.?	6	α	6.94		
Radium D	$1.39\cdot10^{-9}$	15.83 yr.	4	β		130	45; 0.99
Radium E	$1.66\cdot10^{-6}$	4.85 d.	5	β		43.3	like Ra D
Polonium (RaF)	$5.90\cdot10^{-8}$	136 d.	6	αβ?	3.84		585
Thorium	$1.2\cdot10^{-18}$	$1.5\cdot10^{10}$ yr.	4	α	2.72		
Mesothorium 1	$4.0\cdot10^{-9}$	5.5 yr.	2				
Mesothorium 2	$3.1\cdot10^{-5}$	6.2 hr.	3	β		30	26; 0.116
Radiothorium	$1.09\cdot10^{-8}$	2.02 yr.	4	α	3.87		
Thorium X	$2.20\cdot10^{-6}$	3.64 d.	2	α	4.30		
Th Emanation	0.0128	54 sec.	0	α	5.00		
Thorium A	5.0	0.14 sec.	6	α	5.70		
Thorium B	$1.8\cdot10^{-5}$	10.6 hr.	4	β		110	160; 32; 0.36
Thorium C_1	$1.9\cdot10^{-4}$	60 min.	5	αβ	4.80	16.3	
Thorium D	$3.7\cdot10^{-3}$	3.1 min.	3	β			0.096
Thorium C′	$7\cdot10^{+10}$	10^{-11} sec.?	6	α	8.60		
Actinium	$1\cdot10^{-10}$	200 yrs.?	3	α?	3.56		
Radioactinium	$4.25\cdot10^{-7}$	18.88 d.	4	αβ	4.2	170	25; 0.190
[Radioactinium′]?	$3.2\cdot10^{-6}$?	60 hr.?	?	α?	4.61		
Actinium X	$7.6\cdot10^{-7}$	11.4 d.	2	α	4.26		
Act. Emanation	0.18	3.9 sec.	0	α	5.57		
Actinium A	350	0.002 sec.	6	α	6.27		
Actinium B	$3.2\cdot10^{-4}$	36.1 min.	4	β		soft	120; 31; 0.45
Actinium C_1	$5.37\cdot10^{-3}$	2.15 min.	5	αβ?	5.15		
Actinium D	$2.26\cdot10^{-3}$	4.71 min.	3	β		28.5	0.198
Actinium C′	700	0.001 sec.	6	α	6.45		

The following are general relations involving these constants:

$\lambda P = ln2 = 0.69315.$

$\log^P = A + B \log R$, where A and B are constants for each series.

$R_{T_1P_1} = R_{T_2P_2}((T_1P_2)/(T_2P_1))$, where T is the absolute temperature and P is the pressure.

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[36] Phil. Mag., *26*, 937 (1913), *27*, 112 (1914).

$V^3 = aR$, where V is the velocity of the alpha ray and a is a constant. For Ra C', $V = 1.922 \times 10^9$ cm. per sec.[37]

$Q = kR^{2/3}$, where Q is the total ionization due to an alpha ray,[31] and k is a constant.

$\nu_\gamma = c\mu_\gamma{}^{2/5}$, where ν_γ is the frequency of a gamma ray, and c is a constant. For the penetrating gamma rays from Ra B the wave-length is 1.64×10^{-9} cm.[38]

The Wolcott Gibbs Memorial Laboratory,
Harvard University,
Cambridge, Massachusetts.

[37] Phil. Mag., *28*, 552 (1914).

[38] Phil. Mag., *28*, 263 (1914).

THE FLUORESCING SODIUM URANYL PHOSPHATE.

By H. L. Howes and D. T. Wilber.

IN an early article[1] by G. G. Stokes on the ultra-violet spark spectra of the metals mention is made of a powerfully fluorescent screen made from the phosphate of uranium. Since the ordinary phosphate is only slightly fluorescent it seemed of interest to prepare the phosphate employed by Stokes and to determine the nature of its fluorescence. The Stokes method of preparation is to treat the ordinary phosphate with a solution of phosphoric acid and sodium or ammonium phosphate. The non-fluorescent salt, which is a yellow amorphous insoluble powder, is changed into the fluorescent double salt as the mass dries out. The addition of sufficient of the reagents to produce strong fluorescence does not produce any visible change in the uranyl phosphate except to make the color lighter and more greenish due to the fluorescence.

The chemical problem is to find the composition of the fluorescent double salt. Although Stokes used the acid double phosphate as indicated above, the following specimens were prepared to determine whether the "neutral" sodium uranyl double salt was fluorescent. For specimen No. 1 a mixture was made consisting of uranyl phosphate and sodium phosphate in the ratio of four molecular weights of $HUO_2PO_4 \cdot 3\frac{1}{2}H_2O$ to one molecular weight of HNa_2PO_4. The proportions for specimen No. 2 were two to one and for No. 3 were one to one. It will be seen by referring to Fig. 1 that these three specimens, when cooled to − 180° C., yielded precisely similar spectra on excitation with the carbon arc.

To ascertain the rôle played by phosphoric acid, several specimens were prepared by mixing increasing amounts of phosphoric acid with sodium uranyl phosphate. Specimen No. 4 was made by adding one molecule of phosphoric acid to two molecules of uranyl phosphate and one molecule of sodium phosphate, giving the composition $H_3NaUO_2(PO_4)_2$. This was a powder resembling specimen No. 2. The spectrum of this specimen consists partly of lines occupying approximately the same positions as in the spectrum of specimen No. 1, 2 and 3; and partly of a broad band series. Specimen No. 5 was made by adding one molecule of phosphoric acid to one molecule of uranyl phosphate and two molecules of sodium phosphate. When dried this contained much free sodium phosphate. Specimen No. 6 was made by adding two molecules of phosphoric acid to one molecule of uranyl phosphate and one molecule of sodium phosphate. This specimen did not dry but remained syrupy at room temperature and appeared to be vitreous at − 180° C. The spectrum consisted entirely of broad bands. Specimen No. 7 was made by dissolving some uranyl phosphate in syrupy phosphoric acid. This gave a great excess of acid and no sodium. The specimen presented a clear glassy appearance even at 20° C. The spectrum, like that of No. 6, consisted of one series of broad bands.

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[1] Phil. Trans. Roy. Soc. Lond., 152, 599–617 (1862); Chem. News, 7, 147–149, (1863); Abstract Proc. Roy. Soc., 12, 166 (1862); Jahrsbericht Fortsch. Chem., 106 (1863).

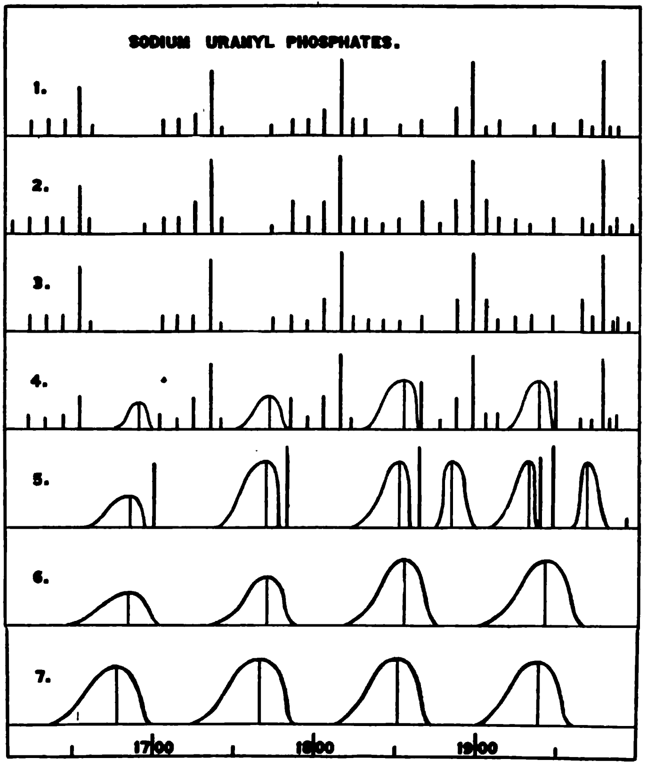

Fig. 1.

In examining the spectra the specimens were mounted in glass, immersed in liquid air, and excited by the carbon arc. A wave-length spectrometer was used in locating the bands. In Fig. 1 the bands are plotted to a frequency scale, *i. e.*, $(1/\lambda) \times 10^3$ is one frequency unit when μ is the unit of wave-length. Thus the abscissa 1,800 corresponds to $\lambda = 5{,}555$ A. U.

It will be seen that the spectra of specimens 1, 2 and 3 consist of recurring groups of bands. Careful measurements show that the homologous members of these groups form constant interval series. The interval of any one of the line series has a value lying between 80 and 81 units, which is the shortest interval yet observed in the study of the fluorescence spectra of the uranyl salts. The broad bands of Specimens 4 and 5 form series with a constant interval of 82.5 units. Evidently the increase in the proportion of phosphoric acid tends to suppress the strongest line series and merge the dimmer series into broad bands. With the increasing predominance of the broad bands,—caused by the increasingly larger proportion of acid present,—there is a simultaneous increase in interval from 82.5 units to 85.1 units for Specimen No. 6, and 87.0 units for specimen No. 7.

A comparison of the spectra of uranyl nitrate in nitric acid solutions[1] with the spectra of sodium uranyl phosphate in phosphoric acid solutions shows a striking resemblance in the matter of resolution when the proportion of acid is increased. In each of these solutions the acid acts as the solvent even at low temperatures, and this fact may well be the cause of the vitreous appearance. If the acid were crystalline at low temperatures presumably the broad bands would not appear. Evidence gained from the study of several other uranyl salts, having different crystalline forms or no crystalline form, leads us to believe that a crystalline form is a necessity where a resolved spectrum is to be obtained.

Since Stokes used an acid powder to make his screen, he could not have used specimens Nos. 1, 2 or 3, since they are neutral. Again, since he used a powder he could not have used specimens No. 6 or No. 7 because they are semi-solid solutions; hence he must have employed No. 4 or No. 5. Either of these specimens is extremely brilliant; the fluorescence being more intense in fact than that of the average uranyl salt; it appears therefore that Stokes employed, for the purpose of making his fluorescence screen, one of the best substances he could have obtained.

PHYSICAL LABORATORY,
CORNELL UNIVERSITY,
December 2, 1915.

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[1] H. L. Howes, PHYS. REV., Vol. VI., No. 3, pp. 202–203, Sept., 1915.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

"AN ATTEMPT TO DETECT A CHANGE IN THE SPECIFIC HEAT OF SELENIUM WITH A CHANGE IN THE ILLUMINATION, AND ALSO WITH THE APPLICATION OF AN ELECTRIC FIELD."[1]

By L. P. Sieg.

THE ordinary theory of free electrons is unsatisfactory in dealing with specific heat. While solid conductors should have, in accordance with this theory, higher atomic heats than do dielectrics, experiment fails to substantiate this. It would be interesting to be able to measure the specific heat of a single substance that can be thrown at will from one of the above classes to the other. Selenium, while not serving this purpose fully, still can be made to increase its conductance many times by light action, or by the application of a strong electric field, or by both.

Experiments were performed on a thin plate of crystalline selenium to determine the specific heat, first in the light and in the dark. A Jolly steam calorimeter was employed. This apparatus seemed to offer the most sensitive arrangement for the purpose, considering the kind of substance that was employed. In spite of the fact that the illumination on the plate was changed from zero to that from the light of an arc at a distance of 50 cm., no change in the specific heat was detected, although a change of one fifth of one per cent. could have been detected.

Separate experiments were then made in which the apparatus was so arranged that a potential difference of 220 volts could be applied to the plate of selenium. This gave a drop of about 50 volts per cm. This potential could be applied both in the light and in the dark. It was thought that perhaps the light alone might not actually free the electrons, but that the establishment of the electric field was the thing necessary for their liberation. Negative results, however, were still obtained.

As a result of these experiments the difficulties connected with the ordinary theory of free electrons become even more formidable. The experiments, however, are not entirely conclusive, for there is no way of making certain just how many electrons are liberated by the light. The change in conductance seems no sure measure, for changes in the mean free paths of the electrons could well account for the changes in conductance observed, without assuming any change in their number at all. The experiments do specifically indicate that, if one accepts the theory of free electrons, then under the conditions of light and field intensity that prevailed, the number of electrons freed must have been less than one fifth of one per cent. of the number of atoms.

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[1] Abstract of a paper presented at the Columbus meeting of the Physical Society, December 28–30, 1915.

If one accepts either Einstein's,[1] or Thompson's[2] theory of specific heat one would not be led to expect a change in the specific heat of selenium under the action of light, of an electric field, or of both acting together.

PHYSICS LABORATORY,
STATE UNIVERSITY OF IOWA.

MEASUREMENTS OF ELECTRIC CURRENTS BY THEIR HEATING EFFECT.[3]

BY S. LEROY BROWN.

THE object of this paper is to give a description of a substitute for the hot-wire ammeter and describe its use. The essential feature of this instrument is a small coil of fine wire which surrounds the element of a sensitive resistance thermometer. This element may be a thin layer of lead oxide between two wires. These two wires are joined together in a molten globule of the oxide and upon cooling the oxide forms a junction between the wires which has a moderately high resistance. Such a resistance element is easily sensitive to a change of temperature of .001° C. and hence is very sensitive to the heat produced by a current in the fine wire heating coil. Any method of resistance measurement may be employed to register the change in resistance of this element due to the heating effect of a current in the coil.

An instrument can easily be built on this plan which is sensitive to a few milliamperes. For larger currents, the coil would be made of larger and lower resistance wire and for very small currents, a heating coil of very fine high resistance wire can be used. The higher range of a particular coil is about ten times the lowest measurable current; that is the range may be from 10 to 100 milliamperes or from 100 to 1,000 milliamperes, etc.

The above described method of measuring electric currents has been used for the following purposes:

1. As a comparator for the calibration of alternating current instruments by comparison with direct current standards.
2. For measuring high frequency currents.
3. Experimental determination of the effect of coupling, tuning, etc., in the generation of electric waves.

UNIVERSITY OF TEXAS.

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[1] A. Einstein, Ann. der Phys., 22, p. 180, 1907.

[2] Sir J. J. Thompson, Phil. Mag., 30, p. 192, 1915.

[3] Abstract of paper read before the Columbus Meeting of the American Physical Society, December 28-30, 1915.

ON THE PROPERTIES OF MATTER AT LOW TEMPERATURES.[1]

BY JAKOB KUNZ.

IN a recent article on the present theory of magnetism I have shown that thê magnetic susceptibility follows the law $k(T + \Theta) = \text{const.}$ on the assumption of progressive conglomeration of the molecules of a substance with decreasing temperature. A. H. Compton has shown meanwhile that, as I had suggested, the specific heat curve can be accounted for in the same way. This hypothesis is now applied to electric, thermal, mechanical and optical phenomena, and the connection with Nernst heat theorem is demonstrated.

URBANA, ILLINOIS,
December 8, 1915.

A NEW LAW RELATING IONIZATION PRESSURE AND CURRENT IN THE CORONA OF CONSTANT POTENTIALS.[1]

BY EARLE H. WARNER.

THE "corona" is the glow which surrounds conductors when there exists high potential differences between them and neighboring conductors. Mr. Farwell has shown that at the instant the corona appears the pressure in the corona apparatus increases. It was the object of the experiments which have been performed to test the relationship between the ionization pressure and corona current.

The corona apparatus was of the wire and co-axial cylinder type. The continuous potential was obtained from a battery of forty continuous current, shunt wound, 500-volt generators connected in series. The corona current was measured with a D'Arsonval galvanometer. The increase in pressure was measured by a Bristol aneroid pressure meter. Experiments have been performed with dry air, hydrogen and nitrogen in the corona tube. In every case, with the wire positive, the ionization pressure is exactly proportional to the corona current. With the beads, which accompany the wire negative, the pressure varies with the arrangement of the beads and since these are not stable it is impossible to accurately determine the desired relationship.

UNIVERSITY OF ILLINOIS,
URBANA, ILL.

UNIPOLAR INDUCTION AND ABSOLUTE ROTATION.[1]

BY E. H. KENNARD.

IN order to fill out some gaps in the experimental evidence bearing on the problem of unipolar induction, an apparatus has been constructed consisting of a cylindrical condenser inside a solenoid, both condenser and solenoid being capable of independent rotation about their common longitudinal axis. The inner coating of the condenser is connected, by means of a needle dipping into mercury held in an insulated cup, to one pair of quadrants of an electrometer, the other pair being earthed; the outer coating of the condenser connects through the bearings to the frame of the apparatus and so to earth.

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[1] Abstract of a paper presented at the Columbus meeting of the Physical Society, December 28–30, 1915.

Let the condenser be set in rotation and let the solenoid be magnetized. Then according to the theory of Lorentz an electromotive force will be developed in all rotating radial parts of the apparatus; thus a difference of potential should be set up between the coatings of the condenser and the electrometer should show a deflection. Furthermore, the effect should be quite independent of the rotation of the solenoid.

Preliminary observations already obtained show the existence of an effect of the right order of magnitude and in the right direction. Simultaneous rotation of the solenoid has no effect. The exact quantitative comparison with theory will require considerable laborious observation and calculation, but it can hardly be doubted that the theory of Lorentz will be completely verified.

In experiments by Professor Barnett with a similar apparatus, only the solenoid could be set in rotation; he found that the condenser did not become charged, which agrees with present results.

Undoubtedly the most interesting case is when the condenser and solenoid are locked together and set in rotation: change of the magnetic field results in about the same deflection as if only the condenser were rotating. If we admit that the simultaneous rotation of the axial connecting wires, the electrometer, the room and the fixed stars would not alter the effect, as seems most likely to be true, then we may say that with the present apparatus a purely electromagnetic effect due to absolute rotation has actually been observed.

PHYSICAL LABORATORY,
UNIVERSITY OF MINNESOTA.

AN INVESTIGATION OF THE ACOUSTICAL PROPERTIES OF THE ARMORY AT THE UNIVERSITY OF ILLINOIS.[1]

BY F. R. WATSON.

THE armory at the University of Illinois presents an unusual case of defective acoustics because of its very large volume and comparatively small absorbing power. It was built to fulfil the usual requirements of an armory in regard to military drills; but, in addition, it has been used on several occasions for convocations and assemblies where the audiences have been very large. The acoustics proved to be impossible for speaking and music. In view of the proposed continued use of the building for such assemblies, an investigation was carried on to determine the possibilities of making it satisfactory in its acoustical properties.

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[1] Abstract of a paper presented at the Columbus meeting of the Physical Society, December 28–30, 1915.

The armory is 400 feet long, 212 feet wide and 93 feet to the highest point of the roof. Acoustically, it is defective because of echoes and reverberation. Echoes are set up by the distant walls, while the reverberation is caused by the undue prolongation of sound.

Several experiments were tried to determine the value of special devices for reinforcing and directing the sound. In one case, a huge parabolic reflector of special construction was used. This was based upon the known action of parabolic reflectors in directing sound along the axis of the parabola.[1]

A modified paraboloid was constructed, the parabolic ribs of which were arranged so as to spread the reflected sound over the entire area occupied by the audience. The framework was covered with oilcloth and mounted over the head of the speaker so that his mouth was at the common focus of all the parabolic ribs. Preliminary tests with the reflector showed that it admirably fulfilled its purpose in directing sound; but when used at an assembly with an audience, its action was practically drowned out by the excessive reverberation which prohibited any possibility of satisfactory acoustics. Another experiment of like nature involved the use of a special megaphone to distribute the sound of the speaker's voice. This megaphone was more efficient than the reflector, since it utilized all the sound sent out by the speaker instead of only the portion intercepted by the reflector. This device was also of little benefit because of the excessive reverberation. A third trial was made by using a number of loud-speaking telephones at different positions in the Armory. This attempt was also unsuccessful, although the telephones when used out in the open air were very effective in reinforcing and directing the sound.

These experiments showed the impossibility of using the entire armory for speaking purposes unless the reverberation could be materially reduced. Calculations made to ascertain the effect of introducing sound-absorbing material showed that the installation of 50,000 square feet of hair-felt would reduce the reverberation to 4.66 seconds, a value which would still be too large for satisfactory speaking. The only alternative was to reduce the volume. Calculations were then made for the acoustical properties of a room partitioned off by canvas curtains at one end of the armory so as to enclose a space 212 feet by 134 feet and 35 feet high. To do this it was first necessary to determine experimentally the action of the canvas in transmitting and absorbing sound. The time of reverberation for the room with an audience of 4,500 people present was then estimated to be 1.1 seconds, a value which has been found by repeated experience to be satisfactory.

On the basis of this calculation a room of the specified dimensions was enclosed at one end of the armory and used for the university commencement exercises. Auditors in all parts of this canvas-enclosed room could hear and understand the various speakers, so that the room was considered a success from the standpoint of acoustics.

University of Illinois, Urbana, Ill.

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[1] The Use of Sounding Boards in Auditoriums, Phys. Rev., Vol. 1 (2), p. 241, 1913; and The Brickbuilder, June, 1915.

THE TRIBO-LUMINESCENCE OF MANGANESE-ZINC COMPOUNDS.[1]

BY C. W. WAGGONER.

THIS paper is a preliminary report on the tribo-luminescence and X-ray fluorescence spectrum of some manganese-zinc sulphides. These compounds were loaned the writer by Mr. W. S. Andrews, Schenectady, N. Y.

The tribo-luminescence was excited by allowing a steel brush to bear lightly on the surface of a rotating disc over which had been painted the sample after mixing it with fish glue. The spectral distribution was determined by observing it visually with a Hilger spectrometer, the light intensity being too small when dispersed to be recorded photographically. The X-ray fluorescence was determined with the spectrometer when the sample was excited by X-rays. The extent of the spectrum is shown roughly in the following table:

MnS-ZnS (pure)	.645	.565 max.	.494	Tribo-
" " "	.583	.556 "	.523	X-Ray
MnS-ZnS+10 per cent. Willemite	.634	.564 "	.515	Tribo-
" " " "	.600	.550 "	.517	X-Ray
MnS-ZnS+20 per cent. Willemite	.623	.551 "	.504	Tribo-
" " " "	.593	.551 "	.525	X-Ray
MnS-ZnS+30 per cent. Willemite	.631	.560 "	.504	Tribo-
" " " "	.585	.546 "	.523	X-Ray
ZnS, No. X_1	.624	.553 "	.514	Tribo-
" " "	.584	.553 "	.525	X-Ray
ZnS, No. X_2	.663	.552 "	.515	Tribo-
" " "	.582	.555 "	.528	X-Ray
$ZnCo_3$, No. 4	.636	.554 "	.528	Tribo-
" "	.593	.552 "	.521	X-Ray

From the above table it will be seen that the maximum of the spectrum seems to be independent of the mode of excitation. The greater width of the tribo-luminescent spectrum may be due to the fact that the intensity of the light excited by mechanical means is greater than that excited by the X-rays and therefore observing the extent of the spectrum by a visual method it would appear wider at the higher intensity.

None of these compounds show any marked photo-phosphorescence when excited by the light from an iron spark.

The addition of Willemite to the compound seems only to reduce the intensity of the tribo-luminescent light.

It has been shown that the zinc salts are highly photo-active, and since the character of the light developed by mechanical means is similar to that developed by X-rays, it seems probable that photo-electric effects may be produced in these compounds by mechanical means. The writer hopes to test this last point with the compounds above, and also to develop some compounds which will show sufficient intensity to allow the spectrum to be determined photographically during the decay of the tribo-luminescent light.

WEST VIRGINIA UNIVERSITY.

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[1] Abstract of a paper presented at the Columbus meeting of the Physical Society, December 28–30, 1915.

THE EMISSION QUANTA OF CHARACTERISTIC X-RAYS.[1]

BY DAVID L. WEBSTER.

EXPERIMENTS have been tried with a rhodium target in a Coolidge tube, furnished by Dr. Coolidge, to determine the potential required for the production of each of its characteristic rays. For any wave-length, emitted as a part of the general radiation, the relation $e V = h\nu$ gives the minimum potential at which the rays are produced. Using Millikan's value $e = 4.77 \times 10^{-10}$ esu., the value of h appears to be 6.52×10^{-27} erg. sec., ± 1 per cent. Setting the spectrometer to receive any one of the characteristic lines, nothing more than general radiation is received up to a potential of 23.3 *kv*, 1 per cent. above the quantum potential of the γ line frequency. At this potential the lines all begin to appear simultaneously. Their absence below this point is confirmed definitely, for the case of the α-lines, by photographs at different potentials.

Above the critical potential, the intensities of the lines themselves, without general radiation, all increase in the same ratio for any given increase in potential up to 42*kv*, the limit of the present apparatus.

The critical potential is also just the potential required to excite the softest rays capable of producing fluorescent characteristic rays from the rhodium. This points to the conclusion that all characteristic rays are produced by the stimulation of higher frequency oscillators, either by X-ray absorption or by cathode rays, and that the same drop to the characteristic frequencies for emission occurs in either case.

HARVARD UNIVERSITY.

THE REFLECTING POWER OF ALKALI METALS IN CONTACT WITH GLASS, AS DETERMINED BY THE PHOTO-ELECTRIC CELL.[1]

BY J. B. NATHANSON.

AS a photometer, one of the most sensitive cells made by Dr. J. Kunz was employed. The cathode consisted of rubidium deposited by distillation on a film of silver, the rarified gas being argon. A metallic guard ring properly earthed, is located between cathode and anode.

Due to conflicting literature on the relation between the light intensity and the corresponding photo-electric current, it was decided to calibrate the cell in terms of known light intensities, as determined by the aid of crossed Nicol prisms. The source of light used was a Nernst glower, due precautions being taken to exclude extraneous light. The cell itself was in an earthed metallic box. A galvanometer, having a figure of merit of $.2 \times 10^{-10}$ amp. per mm., at a scale distance of 2½ meters was employed to measure the current. The resulting curves obtained for various voltages, between the light intensity and the photo-electric current were found to be ***not quite*** straight lines, but curves which were ***slightly*** concave towards the illumination axis. In all succeeding determinations of reflecting powers, proper corrections were made in accordance with the curves obtained.

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[1] Abstract of a paper presented at the Columbus meeting of the Physical Society, December 28–30, 1915.

In the determinations of the reflecting powers of the alkali metals, the photo-electric cell was mounted on the telescope of a spectrometer, thus facilitating the determination of the angles of incidence of the light on the alkali mirrors.

Great difficulties were encountered in making the mirrors. These were made both by repeated distillations and pourings. The metal was deposited on a glass plate which formed part of a small cell. The reflecting power of the alkali metals in contact with the glass is given by

$$R = \frac{O - r}{t^2(1 + Or' - r - r')}$$

O is the fraction of incident light reflected by the whole mirror (metal plus glass); r is the reflecting power of the glass surface from air to glass to air; r' is the reflecting power of the glass surface going from the glass to air to glass; and t is the transmission power of the glass plate for a single passage through the glass plate.

The values of t and r' for various angles of incidence are given by the equations

$$t = \frac{T^2 + (1 - R')(R' - r)}{T(1 - r)}; \quad r' = \frac{(R' - r)(1 - r)}{T^2 + (1 - R')(R' - r)}.$$

T is the fraction of incident light transmitted by the glass plate and incident on the photo-electric cell. R' is the reflecting power of the glass plate (both surfaces). There are no approximations in the derivation of these equations.

The investigations of the properties of the glass plates showed that r' is somewhat less than r. The values of r were obtained by abrading and blackening one surface of the glass plate, thus leaving only one surface effective.

Potassium gave reflecting powers of 88 per cent. at 9° incidence to 89 per cent. at 35°. A mirror formed by pouring the metal gave results coincident with those from one formed by distillation. Only one rubidium mirror has been investigated so far, its reflecting power at an angle of incidence of 9° being 76.3 per cent. This was increased to 78 per cent. at 35°. Sodium gave reflecting powers increasing from 89 per cent. at 9° to 91 per cent. at 35°. The reflecting power of the alkali metals therefore decreases as the atomic weight increases.

So far, nonpolarized light has been used. The work is to be continued with Na, K, Rb, and Cs. Polarized and monochromatic light will be used.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
December, 1915.

A POWERFUL SOURCE OF CONSTANT HIGH POTENTIAL.[1]

By Albert W. Hull.

THE apparatus to be described was developed for use in the study of X-ray spectra, and has proved so satisfactory that I have ventured to hope it may be of use to members of the Society who are doing similar work. In its present form it furnishes 5 kw. at any potential between 10,000 and 100,000 volts with a voltage fluctuation of less than 1 per cent. Much larger power with the same degree of constancy, or the same power with smaller fluctuation, can be obtained by increasing the size of the condensers.

The principle of the method used differs from those previously employed in the manner of using inductance and capacity and the use of kenotrons as rectifiers. Theoretically, the higher the frequency the easier the problem, since the amount of inductance and capacity necessary is smaller the higher the frequency, and the kenotrons operate equally well at all frequencies. 2,000 cycles has been found sufficiently high for the present purpose and is easy to generate and transform.

The arrangement is shown in Fig. 1. Alternating current of 2,000 cycles at 150 volts is stepped up to the desired potential by a transformer T, rectified by kenotrons K_1, K_2 and steadied by capacity C_1, C_2 (about 1/1000 microfarad each), and inductance L_1, L_2 (about 100 henries each) arranged in the manner shown. The constant potential is measured by an ordinary voltmeter V in series with a 10-megohm resistance R.

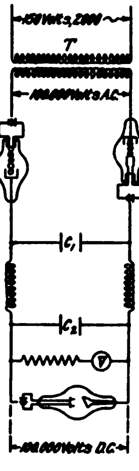

Fig. 1.

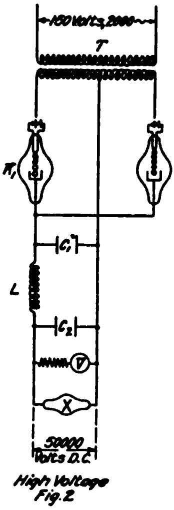

Fig. 2.

The function of condenser C_1 is to store sufficient energy, during the small fraction of a cycle when it is receiving current, to supply the system during the rest of the cycle. The inductances L_1, L_2 with the help of condenser C_2, feed this energy into the system at a nearly constant rate. The magnitude of condenser C_1 depends on how much power is to be used. If C_1 is very small, then in order to deliver a finite current throughout the cycle its potential variation must be very large. In order to test this, I attempted to make an oscillogram with C_1 removed, and broke down both inductances, according to theory.

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[1] Abstract of a paper presented at the Columbus meeting of the Physical Society, December 28–30, 1915.

The action of the inductance and capacity C_2 may be described as follows: The rectified voltage at the terminals of the condenser C_1 is an irregular wave which may be analyzed into a constant potential plus a Fourier's series of alternating potentials of which the lowest frequency component is 2,000. To each of these components the inductance $L_1 + L_2$ offers an impedance very high compared with that of the condenser C_2. Hence the amplitude which each component will have at the terminals of C_2 will be smaller than its amplitude at the terminals of C_1 in the ratio $1/(1 + LC_2\omega^2)$, where ω is the frequency, and may be made as small as desired by increasing L and C_2.

With the values of L and C given above this ratio is $\frac{1}{80}$ for the 2,000 ~ component, $\frac{1}{120}$ for the 4,000-cycle component, and so on. High frequency components, both of the rectified wave and "surges," will be so much reduced that they may be said never to reach the X-ray tube at all and this is true on open circuit as well as closed. This is very important, and is entirely dependent upon the use of capacity in the position C_2. In circuits of this kind it is frequently assumed that inductance in the circuit will of itself damp out high frequency surges. The present case is an excellent example to the contrary. Without the capacity C_2 the inductance would have no effect on oscillations of any frequency, since the impedance of an X-ray tube, that is, the ratio of increase of voltage to increase of current, is nearly infinite for all frequencies, even when the tube is carrying large currents.

It can easily be shown that the optimum distribution of capacity between C_1 and C_2 is given by the equation $C_1 = C_2 + (1/L\omega^2)$; and that the maximum voltage fluctuation for a given D.C. current i with a total capacity $C = C_1 + C_2$ is

$$\delta V = \frac{8\pi i}{L\omega^3(C + 1/L\omega^2)^2}.$$

For voltages below 50,000 it is convenient to connect the two kenotrons, one on either end of the transformer, in parallel instead of in series, thus rectifying both half cycles of the wave, and use the middle point of the transformer secondary as one terminal, as shown in Fig. 2. This makes the lowest frequency oscillation 4,000 cycles, which increases the efficiency of the condensers. If still larger currents are needed the condensers, which are made in 10,000-volt units, may be connected in series-parallel for the lower voltages so as to increase the capacity. In this way it is possible to obtain 5 kw. at any voltage in the form of constant D.C. potential, with fluctuation less than 1 per cent. With

3.2 kw. (65 milliamperes) at 50,000 volts the fluctuations are too small to measure on the oscillograms, certainly not over 1/10 per cent. These and other oscillograms showing the constancy of the voltage under varying conditions will be published elsewhere.

Research Laboratory,
General Electric Co.,
Schenectady, N. Y.

On the So-called Magnetic Rays of Righi.[1]

By James E. Ives.

IN 1908[2] Professor Righi, of the University of Bologna, described some peculiar effects in a Geissler tube of special form obtained when the discharge was acted upon by a strong magnetic field. To these effects he gave the name of magnetic rays, and upon their properties he built up an elaborate theory. In 1909 he published a book entitled "La Materia Radiente e i Raggi Magnetici," which in the same year was translated into German under the title "Strahlende Materie und Magnetische Strahlen." The form of the tube, constructed of glass, which he used is shown in Fig. 1.

BD is a long tube, from one to two meters in length, and about 5 cm. in diameter placed horizontally. *A* and *C* are the anode and cathode, respectively, both placed in a small side tube: the distance between them is about 8 cm. *R* is a solenoid with an iron core, capable of producing a magnetic field in the direction *CD* in the neighborhood of the cathode, of about 2,000 C.G.S. units. The current between *A* and *C*, of the order of a milliampere, was usually obtained with a battery of 2,640 storage cells. To prevent the current through the tube from becoming too great and burning out the electrodes, a non-inductive high resistance of half a million ohms or so is placed in series with the battery and the tube. The pressure of the air in the tube was a few tenths of a millimeter of mercury.

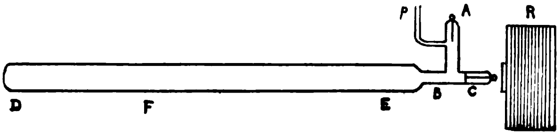

Fig. 1.

When no magnetic field is present, there is the usual Geissler tube appearance in the side tube *AC*, that is, there is a striated pink glow in the neighborhood of *A*, and a bluish violet glow in the neighborhood of *C*. As the strength of the magnetic field is gradually increased the number of pink striæ increases, and they move downwards until they fill the whole of the Faraday dark space. When the magnetic field reaches a certain strength, which depends upon the pressure of the air in the tube, the striæ coalesce and form a continuous pink column pressed by the magnetic field against the side of the tube. When this happens the discharge through the side tube becomes periodic, emitting a note. At the same time, a bluish cathode glow appears in the long tube *BD* extending from *C* towards *B*. Also at the end of the bluish glow, farthest from *C*, there now appears a pinkish glow, and beyond this again a bluish, or whitish, glow. So going from *B* to *D* we have first a short bluish column, then a long pinkish column, and finally a bluish white column extending from *F* to *D*. The short bluish column is either attracted to, or repelled from the north pole of an auxiliary magnet and produces a spot of fluorescence at the point where it touches the tube. We may therefore assume that it is made up of a stream of cathode particles. The pinkish column is bent upwards by the north pole of an auxiliary magnet, and the whitish or bluish column is bent downwards. The directions in which these columns are bent would indicate that a current flows from *F* to *E* on one side of *F* and from *F* to *D* on the other side of *F*. Righi therefore calls the point *F* a "virtual anode." Following More and Mauchly[1] he calls the luminous column from *E* to *D* the "induced column." I prefer to call it the Righi column as this name leaves out any assumption as to its origin. Both Righi and More and Mauchly assume that it is produced by the short bluish-violet column stretching from *B* to *E*. Righi assumes that this bluish-violet column consists of *magnetic rays*, and More and Mauchly that it consists of a stream of electrons.

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[1] Abstract of a paper presented at the Columbus meeting of the Physical Society, December 28-30, 1915.

[2] Rendiconti della R. Accad dei Lincei, 2 Feb., 1908; Mem. della R. Accad. dell. Sci. Bologna, 1908, 17, 87-90.

The portion of the Righi column from *E* to *F*, I shall call the "positive column" because it resembles in color the positive column in the ordinary Geissler tube. The portion from *F* to *D*, I shall call the "negative column."

Righi's theory of the action is as follows: He assumes that under the action of the magnetic field a positive ion and an electron unite to form a neutral doublet, or double star, with the electron revolving around the positive ion. These double stars under the action of the magnetic field move out along the lines of force, and so moving form his "magnetic rays." When they have moved out into the weak parts of the magnetic field, they are supposed to disintegrate, producing free positive and negative ions, and the positive ions so produced are assumed to form the virtual anode.

Such an action would of course give rise to the periodicity in the discharge which is found to exist, since the double stars must alternately form and break up.

In a recent paper Righi[1] has shown that, using a magnetic field to produce the Righi column, it is necessary to have (1) a magnetic field in the neighborhood of the cathode having the direction of the axis of the long tube, and (2) a magnetic field whose direction is at right angles to the anode. In the ordinary form of his apparatus these two conditions are fulfilled by one coil. The apparatus may, however, be so constructed that the two fields are supplied by separate coils.

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[1] Philosophical Magazine, 26, pp. 252–267, 1913.

[2] Physikalische Zeitschrift, 15, pp. 5, 6, 1914.

The "magnetic rays" have been investigated by Thirkell,[1] More and Mauchly, and others. Both Thirkell, and More and Mauchly have come to the conclusion that they are negatively charged particles moving in spirals along the lines of magnetic force, and they consider that all the peculiar phenomena of the Righi column, namely, the positive column, the negative column, the virtual anode, and the periodicity of the discharge can be explained by this stream of negatively charged particles moving in the direction of the axis of the long tube. In fact More and Mauchly have found that all the Righi effects can be obtained without a magnetic field by using a Wehnelt hot lime cathode. The Wehnelt cathode produces a copious discharge of electrons, and they assume that this discharge of electrons into the long tube produces the Righi effects.

I have repeated most of Righi's experiments and my experimental results are in agreement with his.

I finally modified the shape of the tube so that it had the form shown in Fig. 2. It will be noted that the form of the side tube is somewhat changed.

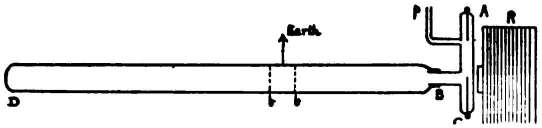

Fig. 2.

A is the anode and *C* the cathode. The anode is a platinum wire, and the cathode is either an aluminum or platinum wire. The cathode can if desired be covered with a small glass tube to within 10 mm. of the tip in order to confine the discharge to its end.

In the course of my researches on the Righi tube, I have discovered the following important facts:

1. I disconnected the long tube from the side tube at *B* (Fig. 2), and closed the aperture so formed with a glass plug. I then examined the action of the magnetic field upon the discharge in the side tube. Using a rotating mirror to observe the discharge, I found that for a certain strength of the magnetic field, the discharge through the tube became periodic, the frequency decreasing as the strength of the field increased, and varying through several octaves. When the field reached a certain strength the discharge in the tube ceased. These results show that the *periodicity* of the Righi effect is in the main *due to the action of the field upon the side tube* and does not depend upon the presence of the long tube *BD*.

2. I have found that the frequency of the note emitted by the tube when the Righi column is present in *BD* depends apparently upon the electrostatic

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[1] Proc. Roy. Soc., 83 A, pp. 324–334, 1910.

capacity of the tube. For instance, if the hand is placed upon the tube, the frequency of the note is decreased. If a brass band *bb* (Fig. 2), 10 cm. wide is placed around the tube and connected to earth, the pitch is considerably lowered. If the width of the band is made twice as great the pitch is reduced roughly to one half. These results show that the phenomena are dependent in some way on the capacity of the tube.

3. I examined the long tube throughout its length when the Righi column was present in it, by means of a pith-ball electroscope, and by other methods, and found that it apparently had a negative change when the Righi column was present.

4. My most important result is, that I have found that all the Righi effects *can be obtained without any magnetic field* by placing a suitable *spark gap* in the high potential battery circuit which includes the tube. With the circuit that I have been using, I have found that the gap must be placed between the positive pole of the battery and the anode. If it is placed between the negative pole of the battery and the cathode, the Righi effect is not obtained.

The spark gaps can be made of zinc, copper or brass. Those that I used were cylindrical with flat faces, and 12 mm. in diameter. If the gap is made too small the Righi column is not obtained. As the gap is increased in length the length of the Righi column increases. The frequency of the discharge decreases as the length of the gap increases. Using about 2,500 volts across the tube in series with a non-inductive resistance of about 800,000 ohms, I got the brightest column for a spark gap length of about .3 mm. To get the best results the high resistance must be placed between the negative pole of the battery and the cathode.

These results seem to show that for the circuit that I have been using, the thing essential for the production of the Righi effect is the periodic interruption of the high potential circuit on the anode side of the tube.

The effect of this would apparently be to produce a periodic charging of the tube negatively. After the spark jumps across the gap, outside of the tube, there will be a more or less complete neutralization of this negative charge within the tube, and a passage of electricity through it.

CLARK UNIVERSITY,
WORCESTER, MASS.,
December 8, 1915.

ARCS IN GASES BETWEEN NON-VAPORIZING ELECTRODES.[1]

BY G. M. J. MACKAY AND C. V. FERGUSON.

THE object of this note is to indicate the nature of some experiments, to be more fully described later, made during an investigation of the rectifying properties of gases, which tend to throw some light upon arc phenomena and gaseous conduction in general.

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[1] Abstract of a paper presented at the Columbus meeting of the Physical Society, December 28–30, 1915.

The work of Dr. Langmuir[1] upon the emission of electrons from an incandescent filament in vacuum has been extended and applied in the field of electrical conduction through gases at comparatively high pressures.

By the use of tungsten electrodes arranged as shown in figure 1, it has been found possible to construct enclosed arcs, which will act as rectifiers of alternating current and carry 10 or 20 amperes with such slight deterioration of the metal that a life of many hundreds of hours may be obtained. The electrodes operate at temperatures between 2600–3200° K. and there appears to be no more loss of material than can be accounted for by the normal rate of evaporation of the tungsten at these temperatures, which is extremely small as is shown by the life of the filament in the nitrogen-filled lamp which operates at about 2800° K. Absence of the so-called "cathode sputtering" is attained by (1) elimination of high cathode drop by the use of a uniformly highly heated electrode (2) a comparatively high pressure of gas which lowers the cathode drop by lessening the mean free path of the impinging positive ions, and also lowers the rate of evaporation of the hot metal as in the case of the filament of the gas-filled lamp.

By the use of a rugged tungsten filament as cathode, heated to approximately 2800° K., a ten- to thirty-fold reduction may be made in the arcing potential, so that an arc with a definite fixed gap may be started on 100 volts or less, thus obviating the bringing together of the electrodes in order to strike the arc.

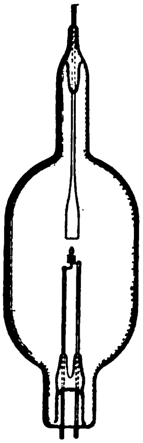

Fig. 1.

Remarkable results have been obtained with carefully purified argon at 5 cm. pressure. As shown by oscillograph on alternating current, the sparking potential from a hot cathode is no greater than the arc drop itself, and the latter has been reduced to less than 4 volts at 5 amperes and more. This is considerably less than the ionization potential found by Franck and Hertz.[2] The drop in the gas itself also appears to be less than ½ volt per centimeter.

While all the gases investigated show complete rectification under proper conditions those such as nitrogen, air, and others, which react chemically with the electrodes to produce more or less volatile products show on oscillograms no current during the first part of the half cycle and then a sharp drop from a comparatively high voltage to a lower when the current starts. This apparent inhibiting effect on electron emission is similar to that found by Langmuir for like gases in high vacuum.[3]

This effect of chemical reaction, and the slight loss of material from the electrodes, together with the fact that many arcs show no spectroscopic evidence of the presence of tungsten in the conducting stream, would indicate that the necessity, urged by some, of considering chemical reaction and rapid evaporation of the electrodes necessary factors in arc phenomena, is non-existent.

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[1] Phys. Rev., N. S., II., p. 450 (1913).

[2] Ber. d. deutsch. Phys. Ges., *15*, p. 34 (1913).

[3] Loc. cit.

More important factors would appear to be: (1) Thermionic emission from the hot cathode, (2) secondary electron emission from the cathode due to bombardment by positive ions, (3) thermionic emission from the hot gases, and (4) effect of temperature on the gas to increase the kinetic energy of the molecules so that ionization may be produced by impact as suggested by J. J. Thomson.

Dr. Langmuir has shown that Richardson's equation for the current emitted by an incandescent body holds for tungsten at all temperatures, and has determined the constants as follows:

$$i = 23.6 \times 10^6 \sqrt{T} e^{-\frac{52,500}{T}}.$$

Large currents cannot be obtained in high vacuum without employing very high voltage on account of the mutual repulsion of the similarly charged particles. This " space charge," however, is destroyed in gases by the positive ions produced by collision, so that the maximum electron emission may be realized at very low voltages.

In the following table the magnitude of some of these factors are indicated, the vapor pressure of tungsten, the rate of evaporation in grams per sq. cm. per sec., the thermionic current, and the number of molecules of hydrogen per cubic centimeter which possess a kinetic energy corresponding to the ionization potential of hydrogen, 11 volts, calculated from the average kinetic energy of the molecule and Maxwell's distribution law.

Temp. °K.	Vapor Press., Mm.	Rate Evap., G. Sq. Cm. S.	Thermionic Current.	Molecules per Cc. Capable of Ionizing.
2000	645.10^{-14}	114.10^{-15}	.004 amps.	1
2800	286.10^{-7}	429.10^{-9}	8.98 "	10^6
3540 M.P.	80.10^{-3}	107.10^{-5}	509 "	10^9
5100 B.P.	760		57025 "	10^{12}

The rate of evaporation of tungsten in an inert gas at atmospheric pressure is about 1 per cent. of the above value.

The currents obtained in practice are several fold the normal thermionic emission, but as the temperature of the gas is undoubtedly very high it may be highly thermionic, so that with secondary electron emission it is not surprising that high conductivity of the different media exists.

It is hoped in the near future to publish a quantitative study of these arcs and their application from the viewpoints of high frequency generators, rectifiers, and spectra of gases under different electrical conditions.

Research Laboratory,
General Electric Co.,
Schenectady, N. Y.

ON THE VELOCITY OF SOUND AND THE VALUE OF THE RATIO OF THE SPECIFIC HEATS FOR HYDROGEN.[1]

By Karl K. Darrow.

THE values heretofore obtained for the ratio of the specific heats of hydrogen range from 1.358 to 1.420, and the most recent is the most eccentric of all (1.332, obtained by Schweikert). The method here described was devised (largely following methods employed at Marburg) in order to test the inference, drawn from measurements upon the specific heat at constant volume made by Eucken and upon the specific heat at constant pressure made by Scheel and Heuse, that the value of the ratio, γ, rises from about 1.4 to about 1.6 as the temperature falls from about 0° C. to $-190°$ C.; the results here given are deduced from preliminary measurements made upon the velocity of sound in hydrogen about 21° C. The apparatus consists of a metal tube of about 5 cm. in diameter and 35 cm. length, one end of which is formed by the diaphragm of a telephone receiver, the other end by the diaphragm of a telephone transmitter or microphone. The diaphragm of the receiver is energized by an alternating current of frequency variable continuously between about 400 and 2,000 alternations per second, and thus emits a musical note of the same pitch (measured in vibrations per second) which at the proper frequencies forms stationary waves in the gas-column included between the two diaphragms; these diaphragms behave as open ends, so that the resonance-frequencies of the tube are in the ratios 1:2:3:4 . . . . These resonances are observed aurally by an observer who listens at a receiver connected in the customary way with the transmitter, and holds the frequency constant at the resonance-maximum while an assistant measures it (by measuring the speed of the motor whence the alternating current is generated). In this way it is possible to make as many as twenty successive determinations of ∝ resonance-frequency, the extreme values diverging by less than one per cent.

In the present determinations, air has been taken as standard gas. The ratio of the velocities of sound in the two gases (hydrogen and air) is taken as a ratio of the fundamental resonance-frequencies of the tube when filled with the two gases successively. As to the customary correction for the difference between the velocity in a tube and that in the free space, it is almost certainly negligible, since the radius of the tube is comparatively great and is the same in both cases, and the velocities may be compared at the same frequency for both gases, by interpolation between the overtones for air. In computing γ, I introduce the further assumption that both the gases behave practically as ideal gases, and the experimental data for the ratio of the densities of hydrogen and air and for the value of γ in air. For the first of these I have assumed $\rho : \rho_0 = 0.0696$, for the second $\gamma = 1.402$. It is probable that the experimental errors are inferior to the uncertainty in the value for γ in air.

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[1] Abstract of a paper presented at the Columbus meeting of the Physical Society, December 28–30, 1915.

Finally, it is important to note that, hydrogen being the rarest of gases, the effect of any impurity whatsoever is to reduce the observed value of the velocity of sound in the gas and therefore the computed value of γ. Hence, out of a group of measurements upon different samples of the gas, all equally good according to some other method of detecting impurities (*e. g.*, spectroscopic) that which gives the highest value is to be deemed the most reliable; conversely values notably smaller than reliable values already obtained can reasonably be ascribed to foreign gases. The purification of the hydrogen is a matter of considerable importance; and seems to have been somewhat neglected in previous determinations of γ, the authors for the most part either omitting to describe how it was done or giving intimations which indicate that it was insufficiently done. The hydrogen used in these tests was generated electrolytically from dilute sulphuric acid, desiccated by passing over calcium chloride and phosphorus pentoxide, freed of oxygen by passing through alkaline pyrogallol solution, and of condensible vapors by passing through a liquid-air trap. The upper limit of the values so far obtained (out of several dozen samples) for the velocity of sound in hydrogen relative to that in air taken as unity, and for γ in hydrogen, respectively, are **3.792** and **1.403**. Hydrogen, for which the velocity of sound relative to air was as low as 3.770, and the apparent value of γ as low as 1.388, showed very distinct traces of oxygen when examined spectroscopically.

RYERSON LABORATORY,
UNIVERSITY OF CHICAGO.

SOME DEVICES USED IN EVALUATING CERTAIN TRIGONOMETRIC AND EXPONENTIAL FUNCTIONS OCCURRING IN PHOTOMETRY.[1]

BY IRWIN G. PRIEST.

A DESCRIPTION of graphic and mechanical devices used at the Bureau of Standards to rapidly evaluate the following functions which are used in work with the Martens photometer and the König-Martens spectrophotometer, and in computing transmission of light:

$$\tan\phi_1 \cdot \cot\phi_2,$$

$$\tan^2\phi_1 \cdot \cot^2\phi_2,$$

$$t_1^{\,l}.$$

The apparatus described included two specially hand-ruled graphs and a special slide rule, which, although they involve no new principle, are of interest as affording great economy of time. Descriptions of this slide rule and these graphs may be had by applying to the Bureau of Standards.

NATIONAL BUREAU OF STANDARDS,
WASHINGTON, D. C.

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[1] Abstract of a paper presented at the Columbus meeting of the Physical Society, December 27–30, 1915.

THE RELATION BETWEEN THE PRESSURE EFFECT AND THE LIGHT EFFECT IN SELENIUM CRYSTALS.[1]

By E. O. Dietrich.

THE change in the conductivity of a selenium crystal due to a change of pressure exhibits a lag, similar to the lag present in the light effect. It is possible to measure the rate of change of conductivity with pressure by a method similar to that employed to measure the rate of change of conductivity due to the light.

The pressure effect was studied in a number of large selenium crystals, obtained by sublimation under different conditions of temperature and pressure. By applying the equation,[2]

$$\frac{di}{dt} = -\alpha i^2,$$

in which i is the conductivity produced by pressure and α is the coefficient of recombination, the value of the coefficient was determined for the following various conditions of the crystal:

1. When the crystal was in the dark and the conductivity was changed by pressure alone.

2. When the crystal was illuminated and the change of conductivity was produced by pressure alone.

3. When the crystal was illuminated and the change of conductivity was produced by the simultaneous change of pressure and light intensity.

When the conductivity of the crystal in the dark as produced by pressure is the same as that produced by light under less pressure, the rate of change of conductivity with pressure and the corresponding α are of the same order of magnitude as these same quantities due to light. For an increase of pressure a decrease in α is observed.

The same results are found when the crystal is illuminated and the change in conductivity results from a change in pressure alone.

The rate of change of conductivity due to a simultaneous change in the pressure and light intensity is greater than the sum of the rates of change due to these agencies alone, that is, the separate effects are not additive.

Physical Laboratory,
The State University of Iowa.

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[1] Abstract of a paper presented at the Columbus meeting of the Physical Society, December 28–30, 1915.

[2] F. C. Brown, Phys. Rev., Series II., Vol. V., p. 396, 1915.

NEW BOOKS.

Recueil de Constantes Physiques de la Société Française de Physique. Publié par Henri Abraham et Paul Sacerdote. Gauthier-Villars & Cie, Paris. (Collection of Physical Constants made by the French Physical Society, published by Henri Abraham and Paul Sacerdote. Gauthier-Villars & Cie, Paris. University of Chicago Press.)

Three volumes of this admirable publication have now appeared. It consists as is well known of a collection of physical constants made with the collaboration of a large number of scientific societies and professors of various universities scattered throughout the world. In the third volume, which has just come to hand, there are 49 chapters including practically every subject of interest to the students of physics, chemistry, zoölogy and to many engineers. The publishers have very wisely selected from the large volume of 592 pages certain chapters which have been bound and published separately. The first of these separate volumes is devoted to spectroscopy: the second to magnetism, electrical conductivity and electromotive forces; the third, to radioactivity and ionization; the fourth, to crystallography and mineralogy; the fifth, to biology and the sixth to the engineering sciences, including metallurgy. One cannot speak with too much enthusiasm of the excellences of these reports. The authors have searched the literature with the utmost care and have given not simply what may be called physical constants but also practically every experment which results in obtaining numerical values. Volume I. was devoted to the results obtained in the year 1910; Vol. II. to those obtained in 1911; and Vol. III., published in 1914, gives the results found in the literature of 1912. Thus one can judge of the admirable promptness with which the work is being done. The references are given in full for every statement published, and there is a most satisfactory and concise explanation of all the symbols. This last feature is worthy of special commendation as it makes the work of the student who is in search of the experimental work of others most easy.

It is impossible to compare this publication with any one bearing a title even remotely similar. It is superior to all the standard works in every respect, and should be in the library of every university and every laboratory of physics, chemistry, etc., where investigation is in progress. Each volume may be subscribed for separately, or the separate parts mentioned above may be bought individually. J. S. A.

Electrical Nature of Matter and Radioactivity. By Harry C. Jones. New York: D. Van Nostrand Company, 1915. Pp. lx+212. Price $2.00 net.

This work is the third edition of a volume originally published in 1906. It aims at giving in a semi-popular style an accurate but concise account of some of the leading phenomena in the conduction of electricity through gases and in radioactivity.

The author's treatment of electrons and their properties is good as far as it goes, and the portion of the book devoted to radioactive substances and their radiations is on the whole well written. The treatment of X-rays and gamma rays however is meager, weak and inadequate. It is regrettable that the author did not take time to add a chapter embodying the results of the recent work of Bragg and others on these two classes of radiations.

The chapter on atomic structure, too, should have embodied some consideration of the views which have been brought forward by Nicholson and by Bohr in connection with nuclear atoms and the radiations emitted by them.

J. C. M.

Smoke Abatement and Electrification of Railway Terminals in Chicago. Report of the Chicago Association of Commerce Committee of Investigation on Smoke Abatement and Electrification of Railway Terminals. By W. F. M. Goss, Chief Engineer. Chicago: Rand McNally and Company, 1915. Pp. lv+1,177.

A mass of detailed information, well discussed, is brought together under the following six heads: The Necessity for the Electrification of Chicago's Railroad Terminals; The Technical Feasibility and Cost of Electrifying the Railroad Terminals of Chicago; A Study of Results which are to be Anticipated from Electrification in Chicago; Financial Practicability of Electrification; Summary of Conclusions with reference to the Electrification of Railroad Terminals; Summary of Conclusions with reference to Atmospheric Pollution.

F. B.

How to Make Low-Pressure Transformers. Pp. 1+17, Price 40 cents; *Directions for Designing, Making and Operating High-Pressure Transformers.* Pp. 1+46. Price, 65 cents; *Examples in Magnetism.* Pp. 1+90. Price, $1.10; *Examples in Alternating Currents,* Vol. 1. Pp. 1+223. Price $2.40. By F. E. Austin. Hanover, N. H.

These booklets are intended for those who want " results " and who want to acquire a certain working knowledge of the subject in question by the repeated solution of numerical problems. The aim of the author is indicated by the following statement in a preface: " It is not the function of this book to derive or discuss equations. These are in general to be taken for granted as a matter of fact. The mathematical processes involved in the transposition of the members of equations, and the solution of equations expressing physical laws, to obtain useful numerical results, is the main object."

The volume on alternating currents gives in handy form the equations and graphical solutions for circuits containing R, L and C; a second volume on alternating currents is to follow.

F. B.

An Introduction to the Mechanics of Fluids. By EDWIN H. BARTON. New York: Longmans, Green and Co., 1915. Pp. xiv + 249. Price, $1.75.

The author prefaces his main topic of mechanics of fluids by about sixty pages devoted to elementary mechanics and in this small part of the book has very adroitly laid a perfectly adequate foundation for a treatment of the mechanics of fluids which is fuller and more rigid than is usually attempted in an elementary treatise.

Not a little of the book's clearness is due to the very excellent diagrams, of which there are a great many.

As the book is intended primarily for students preparing to pass the English university, naval or military examinations, it contains a great number of problems with answers and hints for the solution of some of the more difficult ones.

Any teacher giving a course in mechanics of fluids to engineering students or others who have had a year of general physics would do well to consider Dr. Barton's book as a possible text. A. T.

A Meteorological Treatise on the Circulation and Radiation in the Atmospheres of the Earth and of the Sun. By FRANK H. BIGELOW. New York: John Wiley and Sons, 1915. Pp. xi + 431. Price, $5.00.

Meteorology as a science is less than fifty years old, in fact the active scientific career of the author of Atmospheric Circulation and Radiation has been nearly as long as the subject he treats and whose progress he has done so much to further.

Meteorologists have naturally treated the atmosphere as a thermodynamic system similar in all respects to the smaller systems studied in the physical laboratory, but as experimental material has accumulated it has become evident that it may not be treated as an adiabatic system. Professor Bigelow proposes a system of thermodynamics which differs very fundamentally from the accepted system, the most striking difference being that he places the gas constant a function of the absolute temperature.

In the opinion of the reviewer Professor Bigelow's working formulæ are for the most part empirical in spite of the very good defense of them he makes on general theoretical grounds. The numerical data derived from them (or perhaps, more strictly speaking, interpolated by means of them) are in excellent agreement with the mass of experimental material he has collected.

With recognized existing fields of investigation like non-Euclidean geometry and non-Newtonian mechanics, there is no reason why one should not attempt non-Clausian thermodynamics and possibly this attempt of Professor Bigelow's is a forerunner of a new thermodynamics for a system on a very large scale which because of the absorption and reëmission of radiation and for other reasons may not be regarded as adiabatic. In spite of strong points in the author's arguments, one is left unconvinced.

The book deals with problems of great interest to the physicist, the expression is clear and the treatment such as to stimulate further inquiry.

A. T.

An Introduction to the Principles of Physical Chemistry. By Edward W. Washburn. New York: McGraw-Hill Book Company, 1915. Pp. xxv + 445. Price, $3.50.

This work is designed for the serious student who intends to specialize in physics or chemistry and who has a good working knowledge of the calculus. It deals with the principles rather than with the laboratory side of the subject. Considerable attention is given to the recent work which has made the atomic theory of matter and electricity more than a working hypothesis, and molecular kinetics and thermodynamics form the essential framework of the author's treatment. In addition to the subjects usually treated, one of the later chapters is devoted to radioactivity and another to the recent theories of atomic structure, X-ray spectra, and the chemical significance of the atomic numbers. The book contains many problems, some of them leading to the development of principles by the student, and also an abundance of references to both books and journal articles. An interesting feature is the presentation in the form of footnotes of brief biographical details concerning the investigators named in the text. It appears to be not only an excellent textbook for the student of physical chemistry, but also a useful book for the physicist whose principal interest lies in other directions. E. P. L.

The Telephone and Telephone Exchanges: their Invention and Development. By J. E. Kingsbury. New York: Longmans, Green and Co., 1915. Pp. x + 558.

Mr. Kingsbury's book is a history of the telephone and in no sense either a manual for the telephone engineer or a textbook.

The story of the inception of the idea of telephony and the practical working out of the many necessary technical and administrative details is told with extraordinary good judgment, for although the book contains a wealth of practical information one reads it with a pleasure rarely inspired by the textbook, however good it may be.

The reference material is easily obtainable by the use of the very good index at the end of the volume.

One is impressed by the fact that the subject gains greatly in interest by Mr. Kingsbury's having adopted the historical treatment and his marked success with this method may well make it serve as a model for others who write popular treatises on physical topics. A. T.

Dielectric Phenomena in High Voltage Engineering. By F. W. Peek. New York: McGraw-Hill Book Co., 1915. Pp. xv+265.

This book gives the properties of gaseous, liquid and solid dielectrics. The author's own extensive investigations, carried on in the engineering department of the General Electric Company on corona and spark-over, form a most valuable part of this work. The book opens with an admirable mathematical discussion of the dielectric field and dielectric circuit; it closes with chapters on practical corona calculation for transmission lines and practical considerations in insulation design. The book is commended to whoever is interested in high voltage phenomena, either as physicist or engineer. F. B.

Electrical Engineering. Vol. I. By T. C. BAILLIE. Cambridge: University Press, 1915. Pp. vii+236.

This book, which is designed as an elementary text, contains the following chapters: Historical Introduction, Currents of Electricity, Magnetism, Current Measurement, Electromotive Force, Measurement of Resistance, Potentiometer, Batteries, Electric Light. Elementary electrical measurements constitute the chief part of the book, and in these the potentiometer is particularly prominent. The 129 illustrations are for the most part trade cuts of British instruments. For this and other reasons the book will doubtless find its greatest usefulness within the British empire. F. B.

Magnetism and Electricity. By E. W. E. KEMPSON. New York: Longmans, Green and Co. Pp. viii + 240. Price, $.90.

This book is based on a year's course of lectures given in the upper forms of the Rugby School. Theory and historical origins are avoided throughout. The subject is developed from a well-selected series of experiments illustrating fundamental laws, many of which are suitable for class demonstrations, others for laboratory exercises. There are some surprising omissions—for example, there is no mention of current induction or of familiar applications of electrical principles; nor does the author attempt to initiate beginners into the mysteries of wireless telegraphy, X-rays, and electrons, although it may perhaps do the elementary student no harm to be told that there are such subjects awaiting his more mature study. E. P. L.

Relativity and the Electron Theory. By E. CUNNINGHAM. New York: Longmans, Green and Co., 1915. Pp. vii + 96. Price, $1.10.

In his Principle of Relativity the author gave a comprehensive account of this subject, including detailed mathematical developments. The present volume is intended for the use of the experimental rather than of the mathematical physicist. It emphasizes the close relation between the principles of relativity and electron theory, and those aspects of the subject which are most closely related to experimental investigations. Mathematics is not altogether avoided, but is reduced to the minimum necessary to clear understanding. An account is given of the various experiments, the failure of which seemed to demand some revision of traditional notions regarding time and space. The topics treated in successive chapters are: The origin of the principle; the relativity of space and time; the relativity of the electromagnetic vectors; mechanics and the principle of relativity; Minkowski's four-dimension vectors; the new mechanics; relativity and an objective ether. In the latter chapter the author gives reasons for the belief that the existence of an ether is not necessarily inconsistent with the principles of relativity. E. P. L.

Second Series. *April, 1916* *Vol. VII., No. 4.*

THE

PHYSICAL REVIEW.

A STUDY OF APPARENT SPECIFIC VOLUME IN SOLUTION.

By LeRoy D. Weld.

I. Introduction.

THE research herein set forth had for its origin a desire to obtain further information as to the physical properties of dissolved substances when in very concentrated aqueous solution, that is, near the saturation point and even in the region of supersaturation. The percentage of ionization in such solutions being relatively low, it is reasonable to expect that the molecular properties of the solute will either resemble in large degree those of the pure substance in the liquid state, or else will be unique in some way and differ from those of the same solute in dilute solution.

The matter is here approached in one way only, perhaps the simplest. The immediate subject of the present paper relates to the volume of the solution as compared to the normal volume of the contained water, or what will be called the *apparent specific volume* of the dissolved solute; by which is meant, *the volume of the solution containing one gram of the solute, minus the natural volume of the pure water entering into it, at the same temperature.* The quantity just defined will be designated by Δ.

Δ will, of course, be generally less than the specific volume of the solid substance, sometimes very much less. In the case of potassium chlorate in saturated solution at ordinary temperatures, Δ is about 84 per cent. of the specific volume of the crystals, a ratio, it may be remarked, not differing very widely from that of the specific volume of liquid water and solid ice.

The values and variations of this quantity Δ under different circumstances and for different substances are here to receive attention. The first investigations relate to the variations of Δ with temperature at fixed concentration (what may be called the coefficient of expansion in solution), and the variations of Δ with concentration at fixed temperature.

II. EXPERIMENTAL ARRANGEMENTS FOR OBSERVING CONTINUOUS CHANGE IN DENSITY OF SOLUTION.

Determinations of Δ will of course depend upon measurements of the density of the solution. It so happens that the excellent standard tables of physico-chemical data which are so useful for general purposes, such, for example, as Landolt and Börnstein, Castell-Evans, etc., do not give values of sufficient minuteness and precision for this purpose. In most of the work herein described, the probable errors of the measured densities are in the fifth decimal place, while the solution densities published in standard tables are usually given to the third or fourth place only.

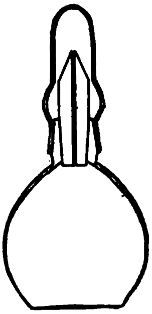

Fig. 1.

After considering and discarding in turn various forms of dilatometers and hydrometers, on account of the experimental difficulties involved in adapting them to highly precise absolute measurements with saturated solutions of accurately known concentration, the writer has adopted, as a standard, a specially designed pyknometer of the specific gravity bottle form and of about 25 c.c. capacity, shown in Fig. 1. The special features are (1) a tight-fitting glass cap, ground upon the neck of the bottle, which effectually prevents any measurable evaporation from neck and bore even after twenty-four hours in a desiccator; and (2) the conically pointed stopper, employed instead of the usual flat-topped stopper in order to prevent the formation of the overflow drop, which is so readily sucked in as the bottle cools after filling. The overflow immediately runs down the steep sides of the cone and cannot be sucked in.

In order to ensure still greater constancy of filling, the force with which the stopper is inserted is regulated and rendered uniform by means of a special instrument shown in Fig. 2. This is pressed down upon the conical stopper with the finger, against the spring, until the sharp point touches the finger, the overflow escaping through the slot in the side ofthe tube. The pressure on the stopper at this moment is about eight ounces.

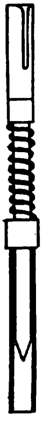

Fig. 2.

In the process of filling, the liquid is placed in a closed jar or glass cell about 5 cm. in diameter, which is kept under precise thermostatic control. The open bottle is completely immersed in the liquid, where it remains for some time before being closed and removed. The stopper, held by a special clamp or handle, is also immersed in the liquid, so that all can come to the common temperature, indicated by a thermometer inserted in the bottle, and reading to tenths of a degree. The conditions having become stable and uniform, the cell is uncovered and the bottle quickly stoppered. The stopper projects above the liquid surface, so that the pressure regulator can be applied. The bottle is now removed with another special forceps, quickly dried with desiccated gauze, and capped. When cool, it is ready for weighing.

The probable error of such a filling and weighing with this capped, cone-stoppered pyknometer has been found from a series of observations to be about 0.00025 g., or one-thousandth of one per cent., and is only half that with a flat-stoppered bottle similarly capped, the observations being therefore four times as reliable. The determination of Δ requires great absolute precision in the measurements of the density of the solution, and some of the earlier results, obtained with an ordinary specific gravity bottle loosely capped, were found later to be misleading.

Unusual care has to be exercised, also, in the weighing, since absolute results are required. To this end the balance resides permanently on a heavy pier in the center of a small room. The case is blanketed and otherwise shielded from temperature inequalities, as is very necessary,[1] and the pointer readings made at a moderate distance by means of a large collimating lens. Double weighings are always employed, the sensibility constants of the balance being determined with great care from time to time with standardized weights. The probable error of a single weighing has been found from several series of observations to be less than one millionth of the load. The vacuum correction is applied in the usual manner.

The dilatation constants of the pyknometer are also periodically determined by filling with water at different temperatures and weighing, as described above. Thus any gradual "creeping" of the glass with continued use is detected and eliminated from the results. Newly blown bottles are especially likely to exhibit such variations.

The stock solutions are carefully synthesized, of such strength as to become saturated at a predetermined temperature. They are made up one or two liters at a time and kept in tightly corked flasks. When desired for use, the flask is heated while still tightly closed until the crystals dissolve, and then cooled to about the required temperature, the specimen being then quickly removed to the closed thermostatic cell. The stock solutions are not exposed to evaporation for more than a few seconds during their entire history, and therefore remain very nearly constant.

All the earlier work was done directly by the pyknometer method

[1] Weld, Iowa Academy of Science, Proc., 1909, Vol. 16, p. 181.

above described, and it is hardly necessary to say that it was very tedious. Every point on the Δ curve represented a separate specimen from the stock solution and a filling and weighing of the pyknometer (an operation requiring from half to three-quarters of an hour), so that only a few points could be obtained in an afternoon's work.

In order to replace this slow process by a sufficiently reliable one that would be quicker and at the same time more nearly continuous in its indications of Δ, the writer has resorted to an Archimedes principle method, which may be called the *secondary* method, dependent upon the absolute pyknometer method as a *primary* standard of accuracy. The sinker consists of a loaded glass bulb of about 34 c.c. volume and weighing about 50 g. suspended from the balance beam by a very fine platinum wire. When immersed in the solution, and counterpoised with weights, this arrangement acts as a very sensitive hydrometer, whose scale is represented jointly by the weights and the balance pointer scale. A change of temperature of one degree is accompanied by an alteration of as much as ten to fifteen milligrams in buoyancy. This change is of course neutralized to the nearest milligram by the addition or removal of weights, the fractional milligram being then read off as a slight deflection on the pointer scale from the known sensibility constants. The rider has not been resorted to in any of the work, except as a means of adjusting the pointer to zero at the outset of each experiment. The pointer has to be read stationary, not by oscillations, as the viscosity of the solution quickly damps out the oscillations.

The sinker, like the pyknometer, has been accurately calibrated for dilatation by immersing in pure water of varying temperature. The standardization presently to be explained may possibly render this unnecessary, but it has been thought best to make the secondary method as nearly absolute as practicable without sacrificing its convenience.

A more immediate reason, even than that of labor-saving, for adopting the secondary method, was the fact that it involves much less disturbance of the liquid under examination, and is therefore better adapted to use with supersaturated solutions, which are likely to begin crystallizing out if agitated. The sinker hangs very quietly in the liquid, which is examined at short intervals in a strong beam of light; and when crystals begin to form, they are immediately detected and the work stopped.

The principal sources of discrepancy between the secondary and absolute measurements lie (1) in the impossibility of double weighings in the former, and hence the entrance into the results of the balance arm-ratio (which differs on different days and often shows a slow progressive variation even in the course of a few hours' work); and (2) in the uncertain capillary force acting upon the platinum suspension wire at the surface of the liquid. It is also difficult to get the depth of solution above the sinker quite constant, so that the amount of the fine wire immersed is slightly variable in different experiments.

The reduction of the secondary results to absolute measure is made by means of corresponding values of the specific volume *of the solution*, this being the first stage reached in common by the two methods. This process may be illustrated by the following example.

A near-saturated solution of potassium chlorate was being experimented upon. The specific volumes of the solution obtained from a typical series of sinker observations are here tabulated along with corresponding pyknometer determinations for certain temperatures.

Temp.	Sinker Method.	Pyknometer Method.	Calculated Absolute.
°			
23.0	0.952750		0.952839
23.5	0.952879		0.952982
24.0	0.953003		0.953121
24.5	0.953177	0.953322	0.953316
25.0	0.953339	0.953488	0.953497
25.5	0.953505		0.953683
26.0	0.953678		0.953876
26.5	0.953840	0.954059	0.954057
27.0	0.954012		0.954249
27.5	0.954187		0.954446
28.0	0.954366		0.954646
28.5	0.954526		0.954824
29.0	0.954701		0.955020
29.5	0.954872		0.955212
30.0	0.955075		0.955438
31.0	0.955426		0.955831

The values in the second column constitute the secondary series of specific volumes taken with the sinker, and involve all the persistent errors inherent in the method. Each of the three determinations in the third column is the mean of three careful measurements with the pyknometer, and these may be considered as "tie points," to be used in reducing the secondary series to absolute measure. This "tying in" process is as follows.

Let X = a secondary result at any temperature,

Y = the corresponding absolute result desired.

Assume the simple linear relation $Y = AX + B$ as the basis of the observation equations which are to connect the series. There will be as many of these equations as there are absolute data available, in this case three.

Least square adjustment (which need be here applied only to the last four decimal places), now gives the constants A and B, and the transformation formula with these values inserted becomes for this case

$$Y = 1.1183\,X - 0.112622.$$

The adoption of such a simple linear relation is justified by the small values of the discrepancies to be eliminated. The absolute specific volumes thus calculated are given in the last column of the table, and the whole process is shown graphically on the accompanying diagram, Fig. 3. The "tie points" in this particular case were taken too near together. They have usually been chosen as far apart as possible, but they cannot enter the region of supersaturation on account of the formation of crystals in the pyknometer.

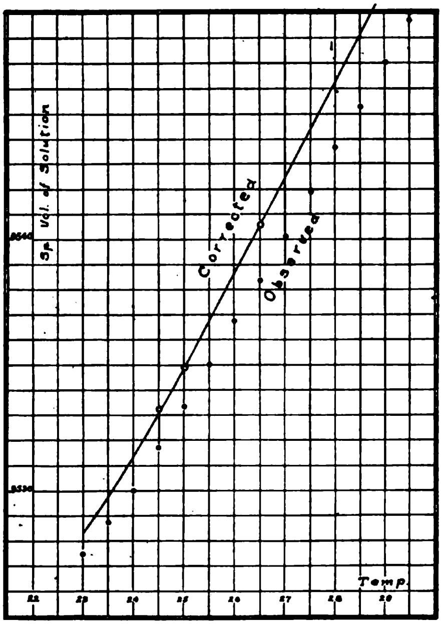

Fig. 3.

For the study of the behavior of Δ at given concentration over any particular temperature range, several such series as the above may be made in a short time (each series taking but one specimen from the stock solution), the series weighted, averaged and standardized, and the corresponding Δ series then calculated from the known concentration and the known specific volumes of pure water.

III. RESULTS WITH POTASSIUM CHLORATE.

The only substance with which complete series have been obtained up to the present time is potassium chlorate, but experiments are now in progress along similar lines, in which copper sulphate, sodium carbonate, sodium sulphate and possibly other solutes are to be examined, the results to be published later in a separate paper. The results with potassium chlorate are, however, in certain respects so singular, as to lead the writer to present them at this stage.

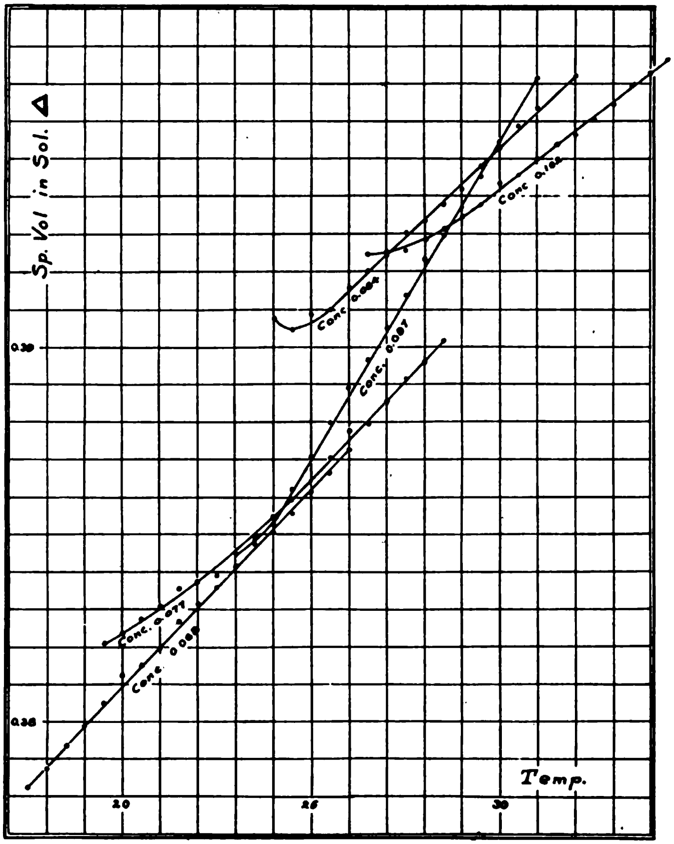

Fig. 4.

Potassium chlorate is rather sparingly soluble in water, about 7 per cent. dissolving at 20° C. The solubility rapidly increases with temperature, the heat of solution and hence the constant *B* of the solubility equation being large. This gives a sharply defined saturation, which is advantageous in some ways and in others not so. The other salts mentioned are much more soluble and saturation is less distinct.

In Fig. 4 are shown typical curves for potassium chlorate, in which abscissas represent temperature and ordinates the apparent specific volume in solution, Δ. The concentration is indicated for each curve. Concentrations are expressed in grams of solute per gram water.

In each of the curves except that for the most dilute solution, there is a distinct upward concavity and an unmistakable suggestion of approaching a minimum as the temperature is lowered. A minimum actually appears in the curve for concentration 0.094, and there is indication that it would be reached in the others if the experiment could be extended far enough into the region of supersaturation, which was successfully accomplished only in the one case. The saturation temperature for this concentration is about 27°.5, or three degrees above the minimum indicated by the curve.

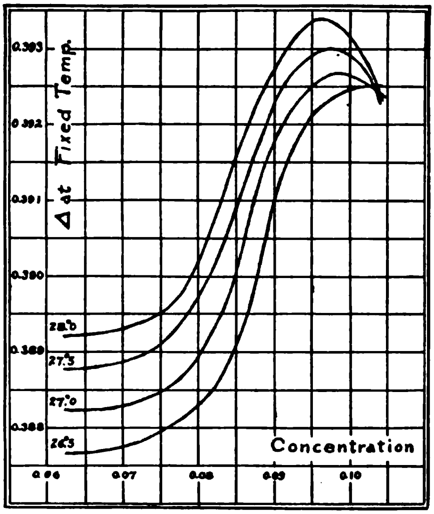

Fig. 5.

The significance of this concavity in the curves, and of the minimum if it really exists, is very interesting, for it suggests at once the behavior of pure water just before it crystallizes on cooling, and of impure water (as a weak solution of salt) when supercooled. It is well known that the minimum volume point, which is at 4° for pure water, is lowered by the addition of a solute, and soon passes below the freezing point of the solution, so as not to be attained except by supercooling. Now if we could interchange the relations of solute and solvent, and regard a concentrated solution as merely a mixture of two substances both in the fused condition (though below their pure melting points), we might regard this observed behavior of potassium chlorate mixed with water as analogous to that of water mixed with salt, and explain it by something akin to Roentgen's ice molecule hypothesis. This would also be in agreement with the fact that, the more impurity (water) there is mixed with the substance under examination (potassium chlorate), the farther below the crystallizing (saturation) point of the latter does the minimum specific volume appear to be, which is evident from an examination of the curves.

The second interesting feature of these results is the variation of Δ with concentration at a fixed temperature, obtained from the same series of experiments. This is shown typically by the curves in Fig. 5, each of which pertains to a single temperature.

It was expected that there would be, for each temperature, a more or less uniform change in Δ with concentration. On the contrary, we observe at 26°.5, for example, a sudden increase in the neighborhood of concentration 0.087, giving a point of inflection; while for the higher temperatures, furthermore, Δ goes through a very pronounced maximum for concentrations between 0.095 and 0.100. For this phenomenon the writer has, as yet, no explanation to suggest. It is interesting to compare these results with those obtained by J. Holmes[1] on the volume of a mixture of *n*-amyl alcohol in chloroform. As the percentage of the alcohol, as compared with the chloroform, increases, the volume of the mixture goes through a minimum and then a maximum with a point of inflection between (Fig. 6), the curve being somewhat suggestive of some of the curves of Fig. 5. Probably a separate research, arranged along somewhat different lines of procedure, might be profitably directed toward this question.

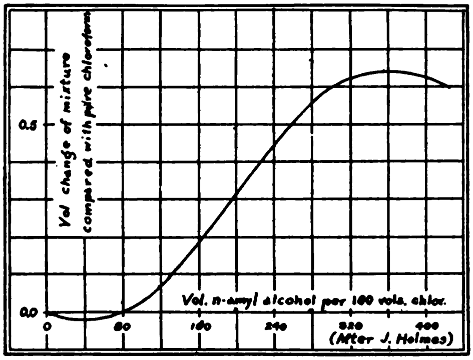

Fig. 6.

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[1] J. Holmes, Intermiscibility of Liquids, Chem. Soc. Jour., 103, Dec., 1913.

In conclusion, attention may be called to the very large apparent coefficient of expansion of potassium chlorate in solution, that is, the rapid fractional increase of Δ with temperature, as indicated by the slopes of the curves in Fig. 4. This is seen to be especially large for concentration 0.087, and the average apparent volume coefficient per centigrade degree for all the concentrations tested, taken outside the neighborhood of the minimum volume, is about 0.0029, a value more than fifteen times the volume coefficient of expansion of mercury and about four-fifths that of a perfect gas.

Very little physical significance can, however, be attached to these latter figures, inasmuch as the quantity Δ is defined in an arbitrary manner, without any reference to the possible shrinkage of the water itself when the salt is dissolved in it,[1] which would make the actual value of the volume of the salt in solution larger than Δ by an unknown amount. The point is that, while Δ itself represents something quite uncertain, the *changes* in Δ here observed undoubtedly do represent some physical process peculiar to the solute and probably connected with its molecular structure. In what manner this process depends upon the dissolved substance, may appear more clearly when other substances have been similarly experimented upon.

I desire to acknowledge my indebtedness to Mr. M. H. Teeuwen, of the University of Iowa, for assistance in constructing certain parts of the apparatus, and to several of my students, whose painstaking cooperation proved very helpful in the experimental work.

COE COLLEGE,
CEDAR RAPIDS, IOWA,
June, 1915.

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[1] W. F. Magie, PHYS. REV., Vol. 18, p. 449.

THE HEAT LOSSES FROM INCANDESCENT FILAMENTS IN AIR.

By L. W. Hartman.

IN a former paper by the writer[1] the heat losses from a hot black body in air were determined and on the assumption that the Stefan-Boltzmann law held for the radiation, the law governing the conduction and convection losses was deduced. During the past two years several new determinations of the constant of radiation, σ, in the relation

$$E = \sigma(T^4 - T_0^4)$$

of the Stefan-Boltzmann law have been published. The value of 5.61×10^{-12} watt cm.$^{-2}$ degree^{-4} for σ, given by Coblentz in a recent communication,[2] seems to be the best determination made thus far, although Gerlach[3] defends with considerable vigor his determination, where $\sigma = 5.90 \times 10^{-12}$ watt cm.$^{-2}$ degree^{-4}. It appeared therefore that a new computation and study of the heat losses from small hot rods in air and the graphs obtained from these data would prove of interest and value, and might serve to indicate which was the most probable value of the constant mentioned.

The method of obtaining the data was as follows: Wires of platinum sponge varying in size from 0.2 mm. to 0.7 mm. in diameter served as black bodies. On the ends of these were fused terminals of silver wire. These wires were then mounted in front of an electric furnace at the center of which was mounted a piece of magnesium oxide. This furnace consisted of an unglazed porcelain tube, 10–15 cm. long and 1 cm. in diameter, over which was wrapped, as the heater, a layer of platinum wire, and the whole was surrounded by a thick layer of plaster paris so as to fill a large glass cylinder in which this combination was mounted with the axes of the tube and cylinder coincident.

The temperature of the furnace was measured with a thermo-couple embedded in the magnesium oxide and with a Wanner pyrometer. These two series of temperature measurements were found to be in close accord. The wires serving as black bodies were then mounted in front of the electric furnace so that they had the piece of magnesium oxide for a background. By means of a current from a storage battery the wires were heated until they seemed to have the same temperature as the interior of the furnace. This was determined by observing the interior of the furnace through a telescope when the wire could be seen against the heated block of magnesium as a background. As the current through the wire was varied a point was finally attained where the incandescent wire disappeared from view. On the principle on which the Holborn-Kurlbaum pyrometer is based, the wire and the oxide at this point were assumed to have the same temperature. The temperature of the furnace was then determined by means of the pyrometer, and the current supplied to the black body was read on an ammeter in series with the wire and the voltage across its terminals was determined by means of a voltmeter. Thus, readings of power and temperature were obtained for temperatures ranging approximately from 700° C. to 1700° C.

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[1] Physikalische Zeitschr, 5, 576, 1904.

[2] Bull. Bureau Standards, 11, 87, 1914.

[3] Ann. der Phys. (4), 38, 1, 1912; 40, 701, 1913; 41, 99, 1913; 42, 1163, 1167, 1913.

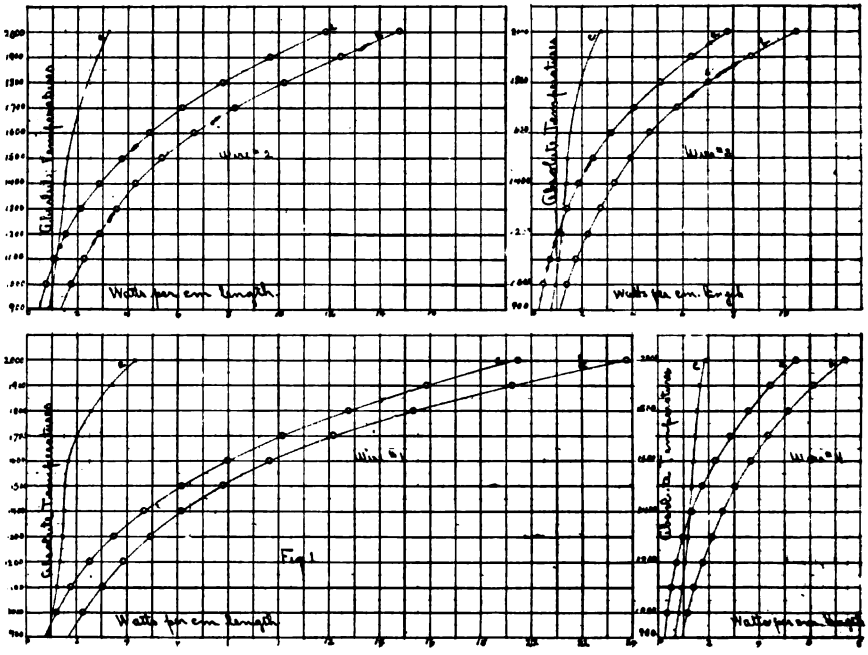

Fig. 1.

With the data thus obtained curves were then plotted with absolute temperatures as ordinates and power, expressed in watts per centimeter length, supplied to the wire as abscissæ. From these curves the values of the watts supplied to the wires to produce absolute temperatures of 900, 1000, . . . 2000° Absolute could be determined. Using the Coblentz value of σ, indicated above, and the Stefan-Boltzmann law, the energy radiated by a black body having the same dimensions as those of the wire used, was computed. The difference between these two values for a given wire represents the loss due to convection and conduction at the given temperature. These values are shown in Table I. In a similar manner the same quantities based upon Gerlach's determination of σ, were computed, but owing to lack of space are not shown here.

If now one should plot for each wire, from the data contained in Table I., a series of curves with absolute temperatures for ordinates and computed watts per centimeter, measured watts per centimeter, and watts per centimeter lost in air, respectively, as abscissæ he would obtain the curves *a*, *b*, *c*, shown in Fig. 1. Plotting the curves marked *c* for all four wires to a larger scale upon one sheet it will be found that the curves based upon Coblentz's determination of σ are more symmetrical than those based upon Gerlach's value. The losses per square millimeter for each wire, based on the respective values of σ mentioned above were then computed and the curves obtained from these data are shown in Fig. 2.

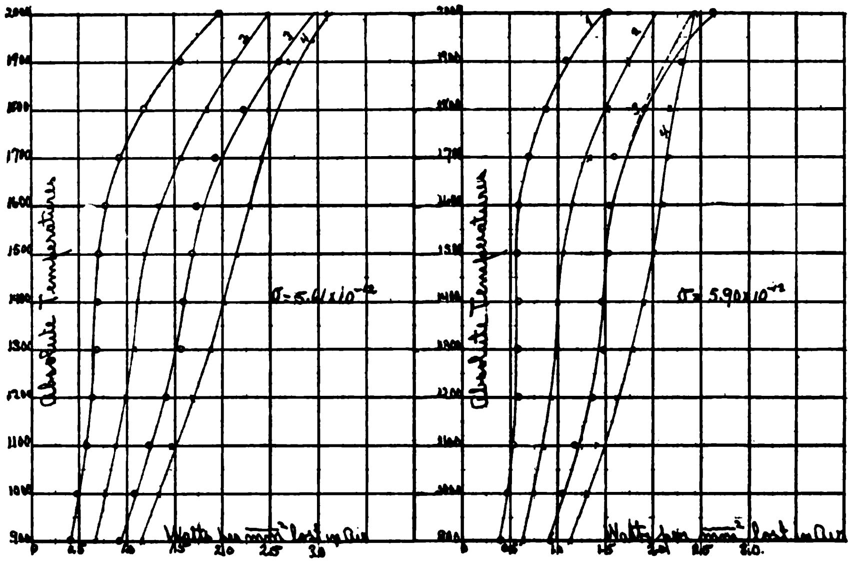

Fig. 2.

Again the curves based on Coblentz's determination of the value of the radiation constant are more symmetrical and uniform than those based on Gerlach's determination of this constant. In the fifth division of Table I. are given the values of the ratio of the watts per centimeter length of wire to the absolute temperature. It will be observed that

TABLE I.

$\sigma = 5.61 \times 10^{-12}$ watt cm.$^{-2}$ deg.$^{-4}$

Computed Radiation in Watts per Centimeter.

Diameters in Cm.	Absolute Temperatures.											
	900	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
1. .0690	.79	1.21	1.78	2.52	3.47	4.67	6.16	7.97	10.15	12.77	15.85	19.45
2. .0420	.48	.73	1.09	1.53	2.11	2.84	3.74	4.84	6.17	7.77	9.65	11.85
3. .0275	.32	.48	.71	1.01	1.38	1.86	2.45	3.17	4.05	5.09	6.32	7.75
4. .0194	.22	.34	.50	.71	.97	1.31	1.73	2.24	2.85	3.59	4.46	5.47

Watts per Centimeter Observed.

	1.70	2.26	3.01	3.88	4.92	6.18	7.70	9.63	12.15	15.33	19.25	23.75
	1.35	1.75	2.26	2.84	3.53	4.29	5.33	6.60	8.25	10.20	12.45	14.75
	1.12	1.40	1.76	2.23	2.73	3.23	3.91	4.67	5.72	7.00	8.64	10.45
	.92	1.15	1.39	1.74	2.12	2.54	3.04	3.64	4.32	5.10	6.10	7.35

Watts per Centimeter Lost in Air.

	.91	1.05	1.23	1.36	1.45	1.51	1.54	1.66	2.00	2.56	3.40	4.30
	.87	1.02	1.17	1.31	1.42	1.45	1.57	1.76	2.08	2.43	2.80	3.26
	.80	.92	1.05	1.22	1.35	1.37	1.46	1.50	1.67	1.91	2.32	2.70
	.70	.81	.89	1.03	1.15	1.23	1.31	1.40	1.47	1.51	1.64	1.88

Watts per Sq. Mm. Lost in Air.

	.42	.48	.57	.63	.67	.69	.71	.77	.92	1.18	1.57	1.98
	.66	.77	.89	.99	1.08	1.10	1.18	1.33	1.58	1.84	2.12	2.47
	.92	1.06	1.22	1.41	1.56	1.58	1.69	1.73	1.94	2.21	2.58	3.12
	1.15	1.33	1.46	1.69	1.89	2.02	2.16	2.30	2.41	2.48	2.69	3.08

(*Watts per Centimeter Lost in Air ÷ Absolute Temperature*) $\times 10^6$.

Mean.[1]	900	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
1,092	1,011	1,050	1,118	1,133	1,115	1,080	1,027	1,038	1,178	1,420	1,789	2,150
1,083	967	1,020	1,062	1,091	1,091	1,035	1,047	1,100	1,222	1,350	1,472	1,630
976	889	920	955	1,017	1,039	979	973	938	983	1,060	1,220	1,350
854	780	810	810	858	887	879	874	875	865	840	863	940

(*Watts per Sq. Mm. Lost in Air ÷ Absolute Temperature*) $\times 10^6$.

498	466	480	520	525	515	493	474	480	530	655	826	990
811	735	770	810	825	830	785	786	831	930	1,022	1,118	1,235
1,115	1,030	1,060	1,109	1,173	1,200	1,130	1,125	1,080	1,140	1,230	1,360	1,560
1,394	1,280	1,330	1,327	1,408	1,455	1,445	1,440	1,439	1,420	1,380	1,417	1,540

between 900–1700° Absolute these ratios for a given wire are approximately constant. Hence we may say that the energy loss per centimeter length of wire through conduction and convection, (*L*), within this range

of temperature, is proportional to the absolute temperature, (T). This law may be expressed by the equation

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[1] These means are taken between 1000–1700° Absolute.

$$L = kT,$$

where k has the value shown in the table. The last division of Table I. shows the values of the ratio watts per square millimeter lost in air to absolute temperature. As one would expect between 900–1700° Absolute these ratios for a given wire are constant and in each case the mean value of this constant within the range specified is shown in the column to the left. It may be noted here, however, that the energy loss per centimeter length of wire due to convection and conduction can be expressed approximately by the relation

$$L = 0.0010T,$$

where the wires range between 0.2–0.7 mm. in diameter.

In order to separate the conduction and convection losses, the effect of enclosing the wires in a vacuum was then tried. In a glass vessel of suitable form having a ground glass cork provided with a mercury seal each wire in turn was mounted so that the wire was in front of the electric furnace as mentioned above. With a mercury pump the air was exhausted from this vessel, a current of electricity was passed through the wire for some time and the vessel again pumped out until after several repetitions of this process a relatively high vacuum was obtained. The vessel was then sealed off. The energy and temperature measurements previously indicated were then repeated. The difference between the watts per centimeter length as observed and the watts per centimeter length as computed from the Stefan-Boltzmann law for each wire gives the losses due to conduction. The watts per centimeter length supplied to each wire for given temperatures were determined when the wires were mounted within the vessel but with the seal broken and then when the wires were mounted in the open air outside the vessel. From the curves plotted for these two series of observations, correction can be made for the absorption of the glass of the vessel. These curves showing the absorption by the glass vessel are given in Fig. 3. It will be observed that the loss increases with the rise of temperature, and of course increases with the increase in diameter of the wire. The difference between the total energy loss and the sum of the radiation and conduction losses, the two being corrected for the absorption of the glass vessel, gives the convection loss. This loss for each wire at the temperature specified is given in Table II., the data in the first division of which are based on the lower value of σ and in the second division on the higher value of this constant. Fig. 3 shows graphically the results for the first division of Table II.

In the second section of Division II. of Table II., it may be noted that the conduction losses in watts per centimeter decrease with rise of temperature for the smallest wire. This is not true in the corresponding section of Division I. of the same table. From this one might infer that 5.90×10^{-12} watt cm.$^{-2}$ degree^{-4} is too high a value for the constant σ; in fact, one is even inclined to believe that 5.61×10^{-12} watt cm.$^{-2}$ degree^{-4} is as high as, or even higher than, these data justify.

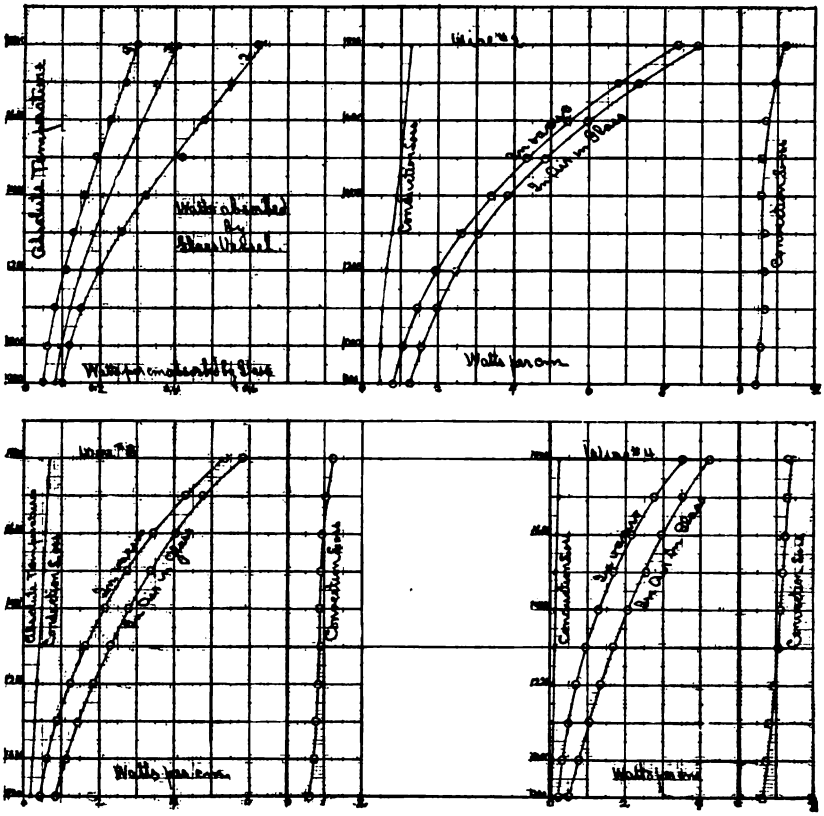

Fig. 3.

Lorenz[1] has developed a formula for the energy absorbed by a wire when heated in air. His equation takes the form

$$E' = A(T^4 - T_0^4) + B(T - T_0),^{5/4}$$

where E' is the energy absorbed, T is the absolute temperature of the wire, T_0 is the room temperature on the absolute scale, and A and B are constants. If we use the method of least squares and apply it to the

[1] Wied. Ann., 13, 582, 1881.

Table II.

$\sigma = 5.61 \times 10^{-12}$.

Division I. Computed Radiation in Watts per Centimeter.

Wire No.	Diameter in Cm.	Absolute Temperatures.									
		900	1000	1100	1200	1300	1400	1500	1600	1700	1800
2.	.0420	.48	.73	1.09	1.53	2.11	2.84	3.74	4.84	6.17	7.77
3.	.0275	.32	.48	.71	1.01	1.38	1.86	2.45	3.17	4.05	5.09
4.	.0194	.22	.34	.50	.71	.97	1.31	1.73	2.24	2.85	3.59
Observed Watts per Cm. in Vacuum in Glass Vessel.											
		.80	1.07	1.43	1.94	2.60	3.40	4.32	5.43	6.75	8.37
		.42	.60	.87	1.21	1.63	2.13	2.73	3.42	4.30	5.35
		.23	.36	.50	.69	.95	1.27	1.68	2.16	2.76	3.50
Observed Watts per Cm. in Air in Glass Vessel.											
		1.25	1.55	1.95	2.46	3.05	3.83	4.82	5.96	7.30	8.85
		.82	1.11	1.44	1.83	2.30	2.79	3.36	4.03	4.76	5.80
		.50	.75	1.02	1.33	1.67	2.04	2.55	2.94	3.50	4.22
Watts Absorbed by Glass Vessel.											
		.10	.12	.15	.20	.26	.32	.42	.48	.55	.62
		.08	.09	.12	.15	.18	.21	.27	.32	.35	.41
		.05	.06	.08	.11	.13	.16	.19	.23	.27	.30
Conduction Loss in Watts per Cm.											
		.42	.46	.49	.61	.75	.88	1.00	1.07	1.13	1.22
		.18	.21	.28	.35	.43	.48	.55	.57	.60	.67
		.06	.08	.08	.09	.11	.12	.14	.15	.22	.23
Convection Loss in Watts per Cm.											
		.45	.56	.68	.70	.67	.57	.59	.69	.95	1.21
		.62	.71	.77	.87	.92	.89	.91	.93	1.07	1.24
		.64	.73	.81	.94	1.04	1.11	1.17	1.25	1.29	1.30
$\sigma = 5.90 \times 10^{-12}$. *Division II. Computed Radiation in Watts per Centimeter.*											
		.51	.77	1.14	1.61	2.22	2.98	3.94	5.10	6.49	8.19
		.33	.50	.75	1.06	1.46	1.96	2.58	3.34	4.25	5.35
		.23	.36	.53	.75	1.03	1.38	1.82	2.36	3.00	3.78
Conduction Loss in Watts per Cm.											
		.39	.42	.44	.53	.64	.74	.78	.81	.81	.80
		.13	.18	.24	.30	.35	.38	.42	.40	.40	.41
		.05	.06	.05	.05	.05	.05	.05	.03	.03	.04
Convection Loss in Watts per Cm.											
		.45	.56	.68	.70	.67	.57	.57	.69	.95	1.21
		.66	.71	.77	.87	.92	.89	.91	.93	1.07	1.24
		.64	.73	.81	.94	1.04	1.11	1.17	1.25	1.29	1.30

data for wire No. 1 as contained in the second part of Table I., we obtain $A' = 1.12 \times 10^{-12}$ and $B' = 3.035 \times 10^{-4}$. Tereschin[1] makes the assumption that the constant A in the above equation is proportional to the surface of the wire and therefore for a unit length of wire is proportional to the diameter of the wire. On this basis one can compute the values of A and B for each wire. Thus one obtains:

TABLE III.

T Abs.	Wire No. 1.						Wire No. 2.					
	E Obs.	E' Comp.	S Comp.	Energy Losses.			E Obs.	E' Comp.	S Comp.	Energy Losses.		
				$E'-S$	L Obs.	$L=kT$ Comp.				$E'-S$	L Obs.	$L=kT$ Comp.
900	1.70	1.66	.74	.92	.91	.98	1.35	1.20	.45	.85	.87	.97
1,000	2.26	2.22	1.11	1.11	1.05	1.09	1.75	1.71	.68	1.03	1.02	1.08
1,100	3.01	2.94	1.63	1.31	1.23	1.20	2.26	2.21	.99	1.22	1.17	1.19
1,200	3.88	3.84	2.32	1.52	1.36	1.31	2.84	2.82	1.41	1.41	1.31	1.30
1,300	4.92	4.93	3.20	1.73	1.45	1.42	3.53	3.56	1.95	1.61	1.42	1.41
1,400	6.18	6.24	4.30	1.94	1.51	1.53	4.29	4.43	2.62	1.81	1.45	1.52
1,500	7.70	7.72	5.66	2.06	1.54	1.64	5.33	5.45	3.44	2.01	1.57	1.63
1,600	9.63	9.75	7.35	2.40	1.66	1.75	6.60	6.69	4.47	2.22	1.76	1.73
1,700	12.15	11.97	9.35	2.62	2.00	1.86	8.25	8.13	5.69	2.44	2.08	1.84
1,800	15.33	14.60	11.75	2.85	2.56	1.97	10.20	9.85	7.17	2.68	2.43	1.95
1,900	19.25	17.69	14.60	3.09	3.40	2.08	12.45	11.76	8.90	2.86	2.80	2.06
2,000	23.75	21.24	17.90	3.34	4.30	2.18	14.75	13.99	10.89	3.10	3.26	2.17
	Wire No. 3.						Wire No. 4.					
900	1.12	1.05	.29	.76	.80	.88	.92	.84	.21	.63	.70	.77
1,000	1.40	1.46	.44	.92	.92	.98	1.15	1.08	.31	.77	.81	.85
1,100	1.76	1.73	.65	1.08	1.05	1.07	1.39	1.38	.46	.92	.89	.93
1,200	2.23	2.18	.93	1.25	1.22	1.17	1.74	1.70	.65	1.05	1.03	1.02
1,300	2.73	2.71	1.28	1.43	1.35	1.27	2.12	2.10	.90	1.20	1.15	1.11
1,400	3.23	3.32	1.72	1.60	1.37	1.37	2.54	2.55	1.21	1.34	1.23	1.19
1,500	3.91	4.04	2.26	1.78	1.46	1.46	3.04	3.09	1.59	1.50	1.31	1.28
1,600	4.67	4.90	2.93	1.97	1.50	1.56	3.64	3.72	2.06	1.66	1.40	1.37
1,700	5.72	5.90	3.74	2.16	1.67	1.66	4.32	4.44	2.63	1.81	1.47	1.45
1,800	7.00	7.05	4.70	2.35	1.91	1.76	5.10	5.28	3.31	1.97	1.51	1.54
1,900	8.64	8.37	5.82	2.55	2.32	1.86	6.10	6.25	4.11	2.14	1.64	1.62
2,000	10.45	9.90	7.15	2.75	2.70	1.95	7.35	7.36	5.05	2.31	1.88	1.71

$A' = 1.120 \times 10^{-12}$ $B' = 3.035 \times 10^{-4}$

$A'' = .682 \times 10^{-12}$ $B'' = 2.820 \times 10^{-4}$

$A''' = .447 \times 10^{-12}$ $B''' = 2.500 \times 10^{-4}$

$A'''' = .315 \times 10^{-12}$ $B'''' = 2.100 \times 10^{-4}$

Lorenz's Formula:—$E = A(T^4 - T_0^4) + B(T - T_0)^{\frac{5}{4}}$

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[1] Physikalische Zeitschr., 6, 217, 1905.

$$A'' = 0.682 \times 10^{-12},$$
$$AA''\,' = 0.447 \times 10^{-12},$$
$$A''\,'' = 0.315 \times 10^{-12}.$$

Having thus obtained the values of A for the different wires one can compute for each of the absolute temperatures given in the table the values of the first term in the second member of Lorenz's equation, which may be designated by S, as follows:

$$S = A(T^4 - T_0^4).$$

The difference between the observed energy supplied to each wire and the value of S gives us

$$E - S = B(T - T_0)^{5/4}.$$

These values are given in Table IV. The mean values between 900–1700° Absolute and 900–2000° Absolute are also indicated in this table.

Table IV.

Computation of B in Lorenz's Formula.

T	E Obs.	$E-S$	$B''\times10^{-4}$ Comp.	E Obs.	$E-S$	$B'''\times10^{-4}$ Comp.	E Obs.	$E-S$	$B''''\times10^{-4}$ Comp.
900	1.35	.90	2.96	1.12	.83	2.74	.92	.62	2.05
1,000	1.75	1.07	2.92	1.40	.96	2.62	1.15	.84	2.29
1,100	2.26	1.27	2.94	1.76	1.11	2.57	1.39	.93	2.15
1,200	2.84	1.43	2.86	2.23	1.30	2.60	1.74	1.09	2.18
1,300	3.53	1.58	2.78	2.73	1.45	2.74	2.12	1.22	2.14
1,400	4.29	1.67	2.61	3.23	1.51	2.36	2.54	1.33	2.08
1,500	5.33	1.89	2.65	3.91	1.65	2.32	3.04	1.45	2.04
1,600	6.60	2.13	2.70	4.67	1.74	2.21	3.64	1.58	2.00
1,700	8.25	2.56	2.96	5.72	1.98	2.29	4.32	1.69	1.96
1,800	10.20	3.03	3.22	7.00	2.30	2.44	5.10	1.79	1.90
1,900	12.45	3.55	3.48	8.64	2.82	2.76	6.10	1.99	1.95
2,000	14.75	3.86	3.51	10.45	3.30	3.00	7.35	2.30	2.09

Mean (900–1,700) 2.82……………………2.50……………………2.10
Mean (900–2,000) 2.97……………………2.55……………………2.07

A'', $A''\,'$, $A''\,''$, respectively, have the values given in Table III.

Taking the mean value of B between 900–1700° Absolute and the corresponding value of A shown above, from Lorenz's equation one can compute for each wire the energy (E') required to produce a given temperature. These values are also shown in Table III. The quantity S, in this table represents, as mentioned above, the first term in the second member of Lorenz's equation. Hence the columns labeled $E'-S$ should equal the losses due to conduction and convection. Under the heading Energy Losses in Table III. are given the quantities just mentioned, the observed values of the conduction and convection losses as given in the third section of Table I., and the losses as computed from the equation

$$L = kT.$$

It will be observed that the deviation between 900–1700° Absolute from the observed loss as computed from this latter equation is less than is the quantity $(E' - S)$ between the same limits of temperature computed by Tereschin's method.

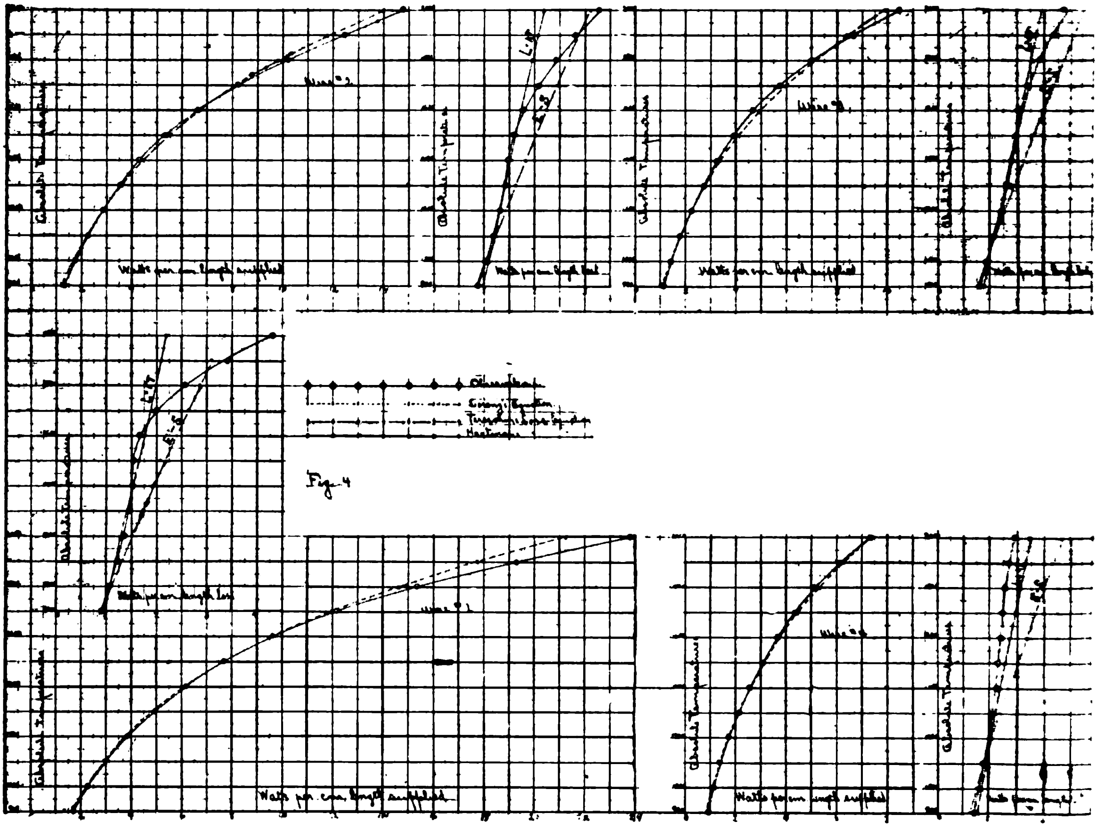

Fig. 4.

These results are all shown graphically in Fig. 4. It will be noted that the values of E' as computed from Lorenz's equation coincide with the observed curve of energy supplied to each wire with great exactness up to 1700 or 1800° Absolute, but that in the case of the convection and conduction loss curves the equation

$$L = kT$$

represents the loss actually observed much more closely than the values of $E' - T$ obtained by Tereschin's method.

By applying the method of least squares to the observed data for each wire for temperatures ranging between 1000°–1700° Absolute, as given in the second part of Table I., the values of A and B in Lorenz's equation can be obtained for each wire. With these values one can then compute the quantity of energy required to heat the given wire to any desired temperature. If one should plot the values of energy thus computed from Lorenz's equation, together with the observed values of energy supplied, as given in Table I., as ordinates and absolute temperatures as abscissæ, the curves thus obtained would apparently coincide with the corresponding curves of Fig. 4. Accordingly, the above-mentioned assumption of Tereschin receives the support of this additional evidence.

CERTAIN CASES OF THE VARIATION OF SOUND INTENSITY WITH DISTANCE.

BY G. W. STEWART.

IT is not possible to secure a source of sound without the presence of reflecting surfaces. In quantitative considerations one desires to know the variation of intensity with distance from the source and is thus led to inquire as to the deviations from the inverse square law caused by the reflectors. A case of practical interest is that in which a small vibrating area is located on a rigid sphere, for it is possible to construct a source which conforms very closely to these theoretical conditions.[1] An additional interest arises from the fact that an investigation of such a source leads to an estimate of the deviations from the inverse square law in the case of a person speaking or singing. Furthermore, by utilizing the results of the same theoretical investigation, we can obtain the relative intensities on a sphere when the distance of the sphere from a simple source is varied. These values give an estimate of the deviations from the inverse square law in the case of a person listening. The deviations in the case of a speaker are of interest in architectural acoustics: those occurring with the listener are of importance in the psychological laboratory.

With a small vibrating area located on a rigid sphere, the geometrical figure of the reflector makes a mathematical investigation possible. Lord Rayleigh[2] was the first to obtain an expression for the sound intensity in the various directions at a great distance from such a sphere. Subsequently the writer[3] extended the investigation in order to obtain similar results for distances that are not great.

A brief statement of the theory will doubtless prove helpful to the reader. Let the source be confined to a small area on the surface of the sphere within which $P_n(\mu)$ of Legendre's series approximates unity. Assume that the velocity of this area is represented by Ue^{ikat}, and that it has the same magnitude at all points. Assume that

ψ is the velocity potential,

a is the velocity of sound,

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[1] Stewart and Stiles, PHYS. REV., N. S., Vol. I., No. 4, 1913, p. 309, and PHYS. REV., N. S., Vol. III., No. 4, 1914, p. 256.

[2] Rayleigh, Theoiy of Sound, Vol. II., p. 254.

[3] Stewart, PHYS. REV., XXXIII., No. 6, p. 467, December, 1911.

r is the distance from the center of the sphere,
c is the radius of the sphere,
dS is an element of surface of the sphere,
k is 2π/wave length,
γ is $k(at - r + c)$,
F is $\Sigma \frac{2n\ 1+}{2} P_n(\mu) \frac{\alpha\alpha' + \beta\beta'}{\alpha^2 + \beta^2}$,
and that G is $\Sigma \frac{2n + 1}{2} P_n(\mu) \frac{\alpha\beta' - \alpha'\beta}{\alpha^2 + \beta^2}$,
where $f_n{}^1(ikr) = \alpha' + i\beta'$
and $F_n{}^1(ikc) = \alpha + i\beta$.

Then $\dot\psi = ka/2\pi r(F \sin\gamma + G \cos\gamma)\iint Uds$. The mean potential energy, which is the "intensity" we desire, proves to be

$$\tfrac{1}{2}\rho_0 \frac{\dot\psi^2}{a^2} = \tfrac{1}{2}\rho_0(F^2 + G^2)\left(\frac{k}{2\pi r}\iint UdS\right)^2.$$

In this formula $\left(\iint UdS\right)^2$ measures the intensity of the source. For a constant source, a constant sound velocity and a fixed wave-length, the relative intensities are proportional to

$$\frac{(F^2 + G^2)}{r^2}. \qquad (1)$$

F and G are functions of k, r, c, and $\cos\theta$ or μ. The accompanying Fig. 1 will make clear the meaning of r, c and $\cos\theta$. The source is located at the point (c, $\theta = 0°$). P is the point at which the intensity is desired and is located at a distance r and in the direction θ. Obviously P is any point on a circle whose radius is $r \sin\theta$ and whose circumference is everywhere at a distance r from the center of the sphere.

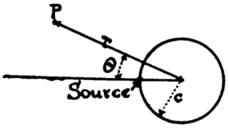

Fig. 1.

In view of (1), the deviations of the intensities from the inverse square law are indicated by relative values of $F^2 + G^2$. The computations involved in securing the numerical values of $F^2 + G^2$ are very laborious. Fortunately, certain numerical results are available, these having been obtained in previous investigations. As already indicated, the values of F and G depend upon those of ikr and ikc,[2] as well as upon μ.

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[1] f_n (ikr) and F_n (ikc) are defined in Rayleigh's Theory of Sound, Vol. II., p. 238, and by Stewart, loc. cit.

[2] For the terms from which F and G can be computed in the two cases $kc = 0.5$, $kr = 25$ and $kc = 1.0$, $kr = 50$, see Phys. Rev., XXXIII., No. 6 (1911), Table I., p. 473, and Table II., p. 475.

Table I. indicates the variation in the relative values of $F^2 + G^2$ with r and with θ, the former being expressed in terms of c, the radius of the sphere. In order to show the percentage deviations in $F^2 + G^2$ which represent the percentage deviations from the inverse square law, the value of this sum is assumed to be unity at a distance of 50 c. Table I. utilizes the values of F and G when $k \times c$ is 1 and when $k \times r$ has the values 2, 3, 4 and 50. The values enclosed in parentheses are computed by substituting in each case for the distance r in (1) the distance from P the source on the sphere. In other words, the values in parentheses indicate the deviations from the inverse square law when the distances are measured from the actual source rather than from the center of the sphere.

Table I.

Relative Values of $(F^2 + G^2)$, those at 50 c being assumed Unity.

$$\text{Radius of sphere, } c = \frac{\text{wave length}}{2\pi}.$$

Distance.	$\theta = 0°$.	30°.	60°.	90°.	120°.	150°.	180°.
2 c	4.24 (1.10)	2.58 (.98)	1.14	.622	.396	.329	.270
3 c	2.26 (1.05)	1.81 (.98)	1.18	.728	.564	.446	.420
4 c	1.75 (1.03)	1.53 (.98)	1.15	.862	.661	.544	.520
50 c	1.00 (1.00)	1.00 (1.00)	1.00	1.00	1.00	1.00	1.00

Table II.

Relative Values of $(F^2 + G^2)$, Those of 200 c Being Assumed Unity.

$$\text{Radius of sphere} = 0.5 \times \frac{\text{wave-length}}{2\pi}.$$

Distance.	$\theta = 0°$.	30°.	60°.	90°.	120°.	150°.	180°.
2 c	6.28 (1.59)	3.54 (1.30)	1.28	.652	.655	.765	.797
50 c	1.04 (1.01)	1.03 (1.01)	1.02	1.00	.978	.968	.957
200 c	1.00 (1.00)	1.00 (1.00)	1.00	1.00	1.00	1.00	1.00

Table II. contains the values obtained when $k \times c$ is 0.5 and when $k \times r$ has the values 1, 25 and 100.

The tabulated results show that when distances are measured from the center of the sphere the direction of minimum variation from the inverse square law is found to depend upon the wave-length. With the longer wave there is less deviation in the rear of the sphere and with the

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For values of F and G for $kc = 1$ and $kr = 2, 3, 4$ and 50 see Phys. Rev., N. S., IV., No. 3, Table I., p. 255, and Table II. (60 cm.), p. 256.

The values of $F^2 + G^2$ for $kc = 0.5$ and $kr = 1.0$ and $kr = 100$ appear only in the form of curves, Fig. 2, Phys. Rev., XXXIII., No. 6, December, 1911.

shorter, less deviation in front. Both of these facts are in accord with anticipation based upon elementary considerations. If the distances in the directions 0° and 30° are measured from the source on the sphere there is less deviation from the inverse square law, but it is yet large. Whether distances be measured from the source or from the center of the sphere, the differences obtained by changing wave-length are very marked. This suggests that, in any practical case, a closer approximation to the inverse square law in front of the source can be secured by increasing the frequency of the tone. It also suggests the well-known fact that the relative intensities of the components of any sound will change with distance and direction from the source.

In order to utilize these numerical values for an estimate of the deviation from the inverse square law in the case of a speaker, it is necessary to assume that the head acts as a rigid sphere and that the source occupies but a small area on that sphere. If we choose as a circumference, 60 centimeters, the results in Table I. and Table II. refer to wave-lengths of 60 and 120 centimeters respectively. Although the lack of conformity to the theoretical conditions is obvious, yet we can consider that the above results furnish a fairly satisfactory estimate of the deviations from the inverse square law in the case of a speaker or singer using tones approximately 60 and 120 centimeters in wave-length, or approximately 575 and 287 in frequency. With the sphere circumference 60 centimeters, the distances are approximately 19.1, 28.6, 38.2 and 477 centimeters in Table I., and 19.1, 477 and 1,910 centimeters in Table II. In the former table, with a frequency 575, the deviations from the inverse square law at the above distances are over 400 per cent. If the distance to the source of sound instead of to the center of the head is substituted in the inverse square formula, then the deviations are 10 per cent. or less. The deviations in various directions are readily read in the tabulation. Table II. should be used for a frequency approximating 287. Neither table gives the relations between intensities at points having a constant r and varying values of θ, for such a comparison has already been published.[1]

In order to apply the results to the case of hearing, we must have recourse to the reciprocal theorem of Helmholtz,[2] which permits us to interchange positions of the source and the points at distances r where the relative intensities are desired. Then we can consider a simple source at a distance r from the center of the head (or, using the parenthetical results, from the ear) and can obtain an estimate of the variation of intensity with distance from either ear. For the frequency 575, the maximum deviation from the inverse square law with distances from the center of the head to the source of approximately 19.1 and 477 centimeters (9.5 to 468 centimeters from the ear) does not exceed 10 per cent. (see Table I., 0°). For the frequency 287 and the same distance from the ear, the deviation is almost 60 per cent. (see Table II., 0°). If with the latter frequency distances of 477 to 1,910 centimeters are selected, the deviation is only 1 per cent. There is a distinct advantage in using the higher frequency in cases where an estimate of relative intensities is obtained by using the inverse square law.

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[1] Stewart, PHYS. REV., XXXIII., No. 6, December, 1911.

[2] Rayleigh, Theory of Sound, Vol. II., p. 294.

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STATE UNIVERSITY OF IOWA.

DISTRIBUTION OF CURRENT BETWEEN A GAS COLUMN AND A METALLIC SHUNT.

By Robert F. Earhart.

THESE experiments were made for the purpose of studying some of the conditions under which a current of electricity passing through a gas might be diverted through a metallic circuit. It was desired also to study the quantitative distribution between a gas column and a metallic circuit under several conditions.

An article by Lehman[1] deals with this topic in a qualitative way. Among other experiments Lehman sent a discharge produced by a static machine through a long tube across which perforated metallic diaphragms were placed. These could be connected externally. The changes in appearance of the luminous discharge which resulted when cross connections were made is described and these are illustrated by colored plates.

The general form of apparatus used in the present experiment consisted of a cylindrical glass tube closed at each end by a flat electrode. The tube was provided with pump connection. Within the tube perforated metallic partitions were placed parallel to the end electrodes. These partitions were provided with terminals sealed through the walls of the tube. They could be connected through a switch and low resistance galvanometer. A current from a high potential storage battery sent through the tube was measured either with a milliammeter or shunted galvanometer.

A considerable number of experiments were made with tubes of different size, under varying pressure conditions and for currents varying from a few microamperes to fifty milliamperes.

When a current flows through a metallic circuit terminated by diaphragms in the manner described, the diaphragms must continually acquire charges from the gas column. One of the first points to be investigated was the probable mechanism by means of which these partitions acquire charges. For this purpose several pairs of diaphragms were made in which the ratio between area of aperture and area of metal partition was kept constant. A pair containing a large number of small holes was found to be more effective in diverting current than a pair having a few larger openings. Moreover this effectiveness was increased by increasing the thickness of the metal walls. This indicated that the diaphragms acquire charge as the streams of ionized gas pass through the openings rather than by direct impingement of ions on the metal partition.

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[1] Ann. der Phys., VII., p. 1, 1902.

A fine meshed copper gauze is effective in diverting current from the gas column but two layers of gauze separated a few millimeters and with the edges connected is many times more effective. This indicates again that the terminals of the shunt circuit acquire charge by diffusion from the gas.

If the diaphragms are separated a distance commensurate with the length of the tube, the effect of closing the shunt circuit is to diminish the total current traversing the tube. Here offering an alternative path diminishes the total current. In fact when the gas pressure is low (1/20 mm.) closing the external circuit quenches the discharge entirely. It appears that the conducting gas is impoverished by diversion of some of the ions.

While the proportion of total current diverted through the shunt depends to a large extent on the terminal conditions of this circuit, certain characteristics are exhibited which are common to all of the cases studied. A typical case is represented in Fig. 1.

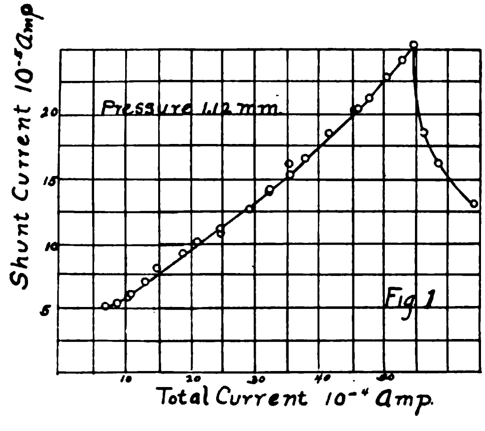

Fig. 1.

The discharge tube in this case was 52.8 mm. long and 14 mm. in diameter. The copper partitions were .4 mm. thick and were placed so as to divide the tube transversely into three equal compartments. Each partition had 16 apertures of 1 mm. diameter. Total area of aperture = .123 sq. cm. Total area metal partition = 1.415 sq. cm. If one notes the difference in value of ordinates and abscissae it will be observed that approximately 10 per cent. of the total current passes through the shunt circuit for small values of the current and that this proportionality is constant with increase in current up to a limiting value, when further increase in total current is accompanied by decrease in shunt current.

Figs. 2 and 3 show members of a family of curves taken with the same tube under other pressure conditions. The slope of the lower portion of the curves indicates the same proportionality between shunt and total current for the small values of current. They indicate that the limiting value is determined by the pressure. A similar experiment was made with this tube, one partition being of double thickness. When the thinner partition was opposite the anode the results gave rise to a family of curves precisely similar to the cases shown in Figs. 1, 2 and 3. With the thicker partition opposite the anode the same slope of curves were obtained for small values but larger currents could be sent through the tube before the limiting value was attained.

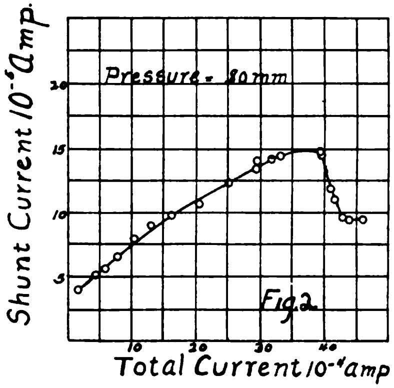

Fig. 2.

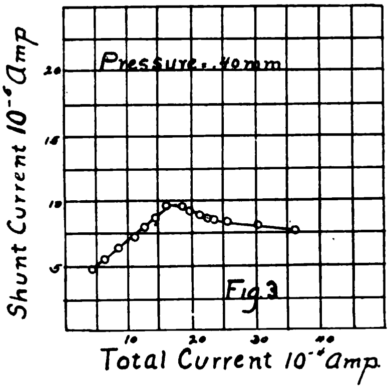

Fig. 3.

This lack of symmetry obtained upon reversal of the direction of current through the tube was studied in a rather extended series of measurements where in one diaphragm was altered in size, number of openings, and thickness while the configuration of the other was maintained constant. This group of experiments may be summarized by stating that a decided lack of symmetry exists upon interchanging anode and cathode and that the terminal collecting the positive charges from the gas places the limitation on the carrying capacity of the shunt circuit.

The experiments hitherto described were made with small currents. It was desired to alter the conditions so that measurements of current could be made directly with a milliammeter. A tube 3 cm. in diameter and 12 cm. long was provided with a pair of double gauze partitions arranged so as to divide the tube into three approximately equal sections. The battery used would produce a current of 50 milliamperes in the tube when the air pressure was equal to .5 mm. of Hg. Starting with a small current, the entire current could be lead through the shunt, the space between the diaphragms being nonluminous. Currents up to 8 milliamperes could be thus diverted. A further small increase above 8 milliamperes produced an unstable condition, the shunt current varying from 2 to 8 milliamperes, the remainder flashing through the gas. Passing this unstable state the current through the shunt decreased to 2 m.a. and remained at that value while the total current was increased to 50 m.a. The values when plotted give rise to a curve possessing the same general features represented in Fig. 1.

SUMMARY.

1. The effectiveness of electrodes in taking current out of a conducting gas column depends more upon the state of division of the electrodes than upon their superficial area.

2. The effect of offering an alternative path through an external metallic conductor is to decrease the total current.

3. For small currents an increase through the shunt circuit occurs when the total current increases. Such increases are proportional to the total current but the value of the factor depends on the pressure of the gas and form of the electrodes.

4. The proportionality factor referred to in (3) holds up to a limiting value which depends on the pressure employed. Further increase in total current results in a decrease in current through the shunt circuit.

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OHIO STATE UNIVERSITY,
December, 1915.

SELECTIVE RADIATION FROM OSMIUM FILAMENTS.

By Ernest F. Barker.

I. Introduction.

The problem of selective radiation from metals is of especial interest in connection with the study of incandescent lamps because of its direct bearing upon the question of luminous efficiency. If all incandescent sources exhibited the properties of a black body it is clear that luminous efficiency would be a function of temperature only, and any choice between materials for high efficiency illumination would be upon the basis of their relative abilities to withstand high temperatures. Experiment shows, however, that this is not the case. In many, if not all radiators the emissivity is found to vary with the wave-length, the energy curve for pure metals being generally somewhat depressed in the region of longer wave-lengths. The ratio of luminous to total energy radiated is therefore higher than for a black body. The existence of a general effect of this type was anticipated some time ago by Aschkinass[1] in the approximate displacement law for metals

$$\lambda_M T = 2{,}666,$$

derived from Maxwell's equations, in which the constant, and hence the value of λ_M, is lower than for the black body. It has also been shown experimentally for many different filaments, but its magnitude is not readily measurable owing to the lack of a satisfactory method for determining operating temperatures. In 1910 Hyde[2] published a table showing the relative efficiencies of various filaments when operated, not at the same temperature, but at a "color match" with a given standard lamp. This probably arranges them in the proper order, since if there were no selectivity a color match would mean the same temperature of operation. It throws no light, however, upon the question of a possible temperature variation.

If the emissivity E is independent of temperature and the Wien equation is assumed for black body distribution, it should be possible to represent the spectral energy curve of any metallic radiator by the equation

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[1] Aschkinass, Ann. d. Phys., 17, p. 960, 1905.

[2] Hyde, Jour. Franklin Inst., 170, p. 32, 1910.

$$J = \frac{c_1 E_\lambda}{\lambda^5 e^{\frac{c_2}{\lambda T}}}. \tag{1}$$

Attempts to fit this relation to experimental data for various spectral energy curves, assuming E to be some simple function of λ, as, for example, $E = k\lambda^a$, have, however, not been entirely successful.[1] Whether the difficulty lies in the choice of the function E_λ, or in a variation of selectivity with temperature, remains to be determined, but Hyde[2] has demonstrated by means of a simple criterion obtained directly from equation (1) that such variations probably do exist in the case of tungsten filaments. Bidwell[3] has found evidence of a similar effect by direct measurements upon a mass of nickel whose temperature could be observed and controlled over a considerable range.

One of the objects of the present investigation is to determine whether or not the osmium filament exhibits a temperature variation of selectivity. Osmium was chosen for study in this connection because, in the first place, its selectivity is probably higher than that of most other metals so that the effects sought will be most easily measurable, and secondly, its characteristics have not been very fully described so that any data obtained from it will be of interest. The lamps, which were obtained through the courtesy of Dr. Hyde, of the Nela Research Laboratory, consume normally about 39 watts at 42 volts, giving approximately .55 candle power per watt. The filaments consist of two separate loops in series, each anchored to the glass bulb at the outer end. The investigation consists of (1) a study of the behavior of three osmium lamps under various operating conditions, with a test for selectivity; (2) a qualitative study, by comparison with carbon lamps, of the characteristic radiation from these filaments, showing variations of emissivity with temperature, and indicating the shape of the energy curve for at least a part of the visible spectrum; (3) a similar investigation of the near infra-red region of the spectrum.

II. EXPERIMENTAL RESULTS IN THE VISIBLE SPECTRUM.

1. *Lamp Characteristics.*

As a preliminary study the three osmium lamps were compared with each other and with an untreated carbon lamp for current and power consumption, candle power, and color at various voltages. The photometer measurements were against a secondary standard which was several times carefully compared with a lamp furnished by the Bureau of Standards. This lamp had been calibrated at each of eight different voltages, thus making it possible to almost completely eliminate errors due to color differences. All measurements were by the method of substitution, but they represent candle power only in a single direction, since the osmium filaments at incandescence soften sufficiently to preclude the possibility of rotating them. The three lamps were not found to differ materially, and the data for filament No. 1 appears in Table I. and Fig. 1.

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[1] Coblentz, Bull. U. S. Bureau of Standards, 5, p. 339, 1909.

[2] Hyde, Astrophysical Jour., 36, p. 131, 1912.

[3] Bidwell, PHYS. REV., 3, p. 439, 1914.

Table I.

Osmium Lamp No. 1.

Volts.	Amperes.	Watts.	Candle Power.	Watts per Candle.
5	.242	1.21		
10	.377	3.77		
13	.441	5.74		
16	.503	8.05		
20	.580	11.60	.91	12.74
23	.637	14.65	1.70	8.62
26	.683	17.76	2.99	5.92
30	.755	22.65	5.55	4.09
33	.800	26.40	8.20	3.22
36	.847	30.52	11.69	2.61
40	.908	36.32	18.00	2.02
42	.938	39.40	21.48	1.835
Normal values. 40.25	.910	36.63	18.30	2.00

It would seem of interest to represent these relations for all the lamps by single algebraic functions, if possible. Middlekauff and Skogland[6] have shown that the variations of amperes, watts, candle power and watts per candle power with impressed voltage, for tungsten lamps of any type, may be expressed by an equation of the form,

$$y = Ax^2 + Bx + C,$$

where x is the logarithm of the ratio of any voltage to a chosen normal voltage, and y is the logarithm of the corresponding ampere, watt or candle power ratio, or the logarithm of the actual watts per candle. The value of C is, then, the logarithm of the normal watts per candle for the last equation, and zero for the first three, since they each represent curves passing through the origin.

Following this suggestion, equations of the same form were tried for osmium lamps for the volts-watts and volts-candle-power variations. It is easily seen that if such a relation holds for the two cases $y_1 = \log w/w_0$ (log. watt ratio) and $y_2 = \log c/c_0$ (log. candle power ratio), it necessarily holds for $y_3 = \log a/a_0$ (log. ampere ratio) and for $y_4 = \log w/c$ (log. watts per candle), for

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[6] Middlekauff and Skogland, Phys. Rev., 3, p. 485, 1914.

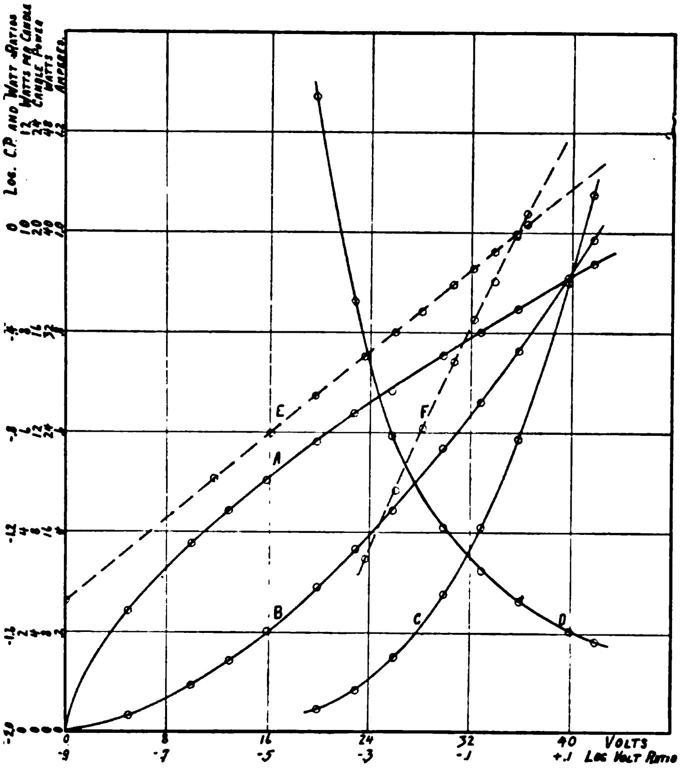

Fig. 1.

Characteristic Curves for Osmium Lamp No. 1. *A*, amperes; *B*, watts; *C*, candle power; *D*, watts per candle; *E*, log. watt ratio; *F*, log. candle power ratio.

$$\frac{w}{w_0} = \frac{a}{a_0} \cdot \frac{v}{v_0},$$

or

$$\log \frac{w}{w_0} = \log \frac{a}{a_0} + \log \frac{v}{v_0},$$

$$y_1 = y_3 + x.$$

But if

$$y_1 = Ax^2 + Bx,$$

then

$$y_3 = Ax^2 + (B - 1)x,$$

a function of the same form. Similarly, taking the logarithm of

$$\frac{w}{w_0} \div \frac{c}{c_0} = \frac{w}{c} \div \frac{w_0}{c_0},$$

we have

$$y_1 - y_2 = y_4 - (y_4)_0,$$

or

$$\begin{aligned} y_4 &= y_1 - y_2 + (y_4)_0 \\ &= (A_1 - A_2)x^2 + (B_2 - B_1)x + C. \end{aligned}$$

The normal consumption chosen was 2 watts per candle power, and the corresponding normal values of watts and candle power as they appear in Table I., were taken from the curves. The coefficients were determined by the method of least squares from the data taken upon filament no. 1, and the resulting equations are as follows:

(A) $$y_1 = 0.026609x^2 - 1.65072x$$

where x = log volt ratio, and y_1 = log watt ratio, and

(B) $$y_2 = - 1.24490x^2 - 3.91898x$$

where x = log volt ratio, and y_2 = log c.p. ratio.

Table II. shows how nearly these equations represent the desired relations. It is to be noticed that the percentage of error to be expected in the candle power determinations is comparatively large because of the faint illumination at lower voltages.

The three available osmium lamps are all of the same type, so it is not possible to show that these equations are of general applicability. However, having been derived from the data for filament no. 1 only, they were found to apply almost as well to filaments no. 2 and no. 3 when the proper normal values were inserted.

Table II.

(A) Volts-Watts Relation for Filament No. 1.

Volts.	y_1 = Log (Watts + 36.63)		Watt Ratio.		Per Cent. Difference.
	Computed.	Observed.	Computed.	Observed.	
10	−.98857	−.98750	.1027	.1029	+.20
13	−.80380	−.80493	.1978	.1973	−.25
16	−.65708	−.65804	.2203	.2198	−.23
20	−.49894	−.49938	.3170	.3167	−.09
23	−.39962	−.39800	.3985	.3990	+.13
26	−.31235	−.31440	.4871	.4849	−.45
30	−.21028	−.20877	.6162	.6183	+.34
33	−.14219	−.14224	.7208	.7207	−.01
36	−.07995	−.07926	.8512	.8526	+.16
40	−.00447	−.00369	.9898	.9915	+.17
42	+.03051	+.03166	1.0728	1.0756	−.28

(*B*) *Volts-Candle-Power.*

Volts.	$y_2 = \text{Log (C.P.} + 18.30)$		Candle Power Ratio.		Per Cent. Difference.
	Computed.	Observed.	Computed.	Observed.	
20	−1.30518	−1.30341	.0495	.0497	− .40
23	−1.02600	−1.03200	.0942	.0929	+1.40
26	− .78867	− .78678	.1627	.1634	− .43
30	− .52055	− .51816	.3016	.3033	− .56
33	− .34731	− .34864	.4495	.4481	+ .31
36	− .19290	− .19464	.6563	.6537	+ .40
40	− .01063	− .00718	.9758	.9836	− .80
42	+ .07199	+ .06958	1.1803	1.1815	− .10

The curves corresponding to (A) and (B) are plotted with a broken line upon Fig. 1. It will be noted that (A) represents very nearly a straight line. Assuming it to be actually straight, with a slope of $m = 1.65$, the relation could be written in the form

$$W = c_1 V^m,$$

where W = watts radiated and V = volts. Now if

$$W = IV = \frac{V^2}{R}$$

be substituted, where R is the resistance, this becomes

$$W = c_2 R^{\frac{m}{2-m}} = c_2 R^\beta,$$

and if the resistance be proportional to absolute temperature, as it is very nearly for tungsten at high temperatures,[7]

$$W = c_3 T^\beta = c_3 T^{4.71}.$$

The value of this exponent, even though only a rough approximation, is of interest. For a black body its value is 4, and Hyde[8] has found experimentally $\beta = 4.7$ for tantalum and $\beta = 6.0$ for tungsten.

Direct evidence of the selectivity of the osmium filaments is obtained by comparing the watts per candle power which they radiate with the watts per candle power radiated by a carbon lamp when the two are at a color match, *i. e.*, when the ordinates of the spectral energy curves are proportional throughout the visible spectrum. Under these conditions, if there were no selectivity, both would be at the same temperature and both would have the same luminous efficiency. As appears from Table III., however, the osmium filament is much more efficient than the carbon, indicating that when the energy distribution is the same for both in the visible part of the spectrum, the curve for the osmium must lie considerably below that of the carbon at longer wave-lengths. This relation will be discussed further in connection with the study of the infra-red radiation.

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[7] Pirani, Phys. Zeitsch., 13, p. 753, 1912.

[8] *Loc. cit.*

TABLE III.

Carbon and Osmium Filaments at Color Match.

Carbon No. 1.		Osmium No. 1.		Ratio Watts per Candle.
Volts.	**Watts per Candle.**	**Volts.**	**Watts per Candle.**	
180	9.35	26.0	5.92	.633
190	7.15	28.3	4.75	.665
200	5.61	30.8	3.81	.679
210	4.51	33.3	3.15	.697
220	3.72	35.8	2.65	.713

2. *Variations of Emissivity with Temperature.*

Under the caption of "Criterion I" Hyde[9] has described a comparatively simple method for investigating qualitatively the temperature variations in emissivity from any source, as follows: Equation (1) may be considered as defining emissivity E, which we will assume to be independent of temperature.

$$J = \frac{c_1 E}{\lambda^5 e^{\frac{c_2}{\lambda T}}}. \tag{1}$$

Suppose a black body B, and any other substance A having an emissive power E and radiating in accordance with this equation, are at such temperatures that for a chosen pair of wave-lengths each has the same relative intensities, *i. e.*,

$$\left[\frac{J_{\lambda_1}}{J_{\lambda_2}}\right]_A = \left[\frac{J_{\lambda_1}}{J_{\lambda_2}}\right]_B. \tag{2}$$

Let the temperature of B be increased by a given small amount, and the temperature of A by such an amount that the same condition again holds, and

$$\left[\frac{J_{\lambda_1} + dJ_{\lambda_1}}{J_{\lambda_2} + dJ_{\lambda_2}}\right]_A = \left[\frac{J_{\lambda_1} + dJ_{\lambda_1}}{J_{\lambda_2} + dJ_{\lambda_2}}\right]_B. \tag{3}$$

Differentiating (1) and combining with (2) and (3) we obtain

$$\left[\frac{dJ_{\lambda_1}}{J_{\lambda_1}}\right]_A = \left[\frac{dJ_{\lambda_1}}{J_{\lambda_1}}\right]_B \quad \text{and} \quad \left[\frac{dJ_{\lambda_2}}{J_{\lambda_2}}\right]_A = \left[\frac{dJ_{\lambda_2}}{J_{\lambda_2}}\right]_B,$$

i. e., for either wave-length the relative change of intensity due to the increase in T is the same for A as for the black body B. This may be written

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[9] *Loc. cit.*

$$\left[\frac{J_{\lambda_1}+dJ_{\lambda_1}}{J_{\lambda_1}}\right]_A = \left[\frac{J_{\lambda_1}+dJ_{\lambda_1}}{J_{\lambda_1}}\right]_B \tag{4}$$

with a similar relation holding for wave-length λ_2. Equations (2), (3) and (4) constitute criterion I., (4) following from (2) and (3) necessarily if (1) properly represents the energy distribution in the spectrum of A, *i. e.*, if the emissivity is independent of the temperature. Equation (4) is a necessary, though not a sufficient condition for this constancy of E with respect to T. Hence if the radiation from any substance does not obey criterion I., its emissivity is a function of temperature. The application of criterion I. may be made directly with a spectrophotometer by measuring the various intensities at two wave-lengths for the radiators which are to be compared.

If, for given temperatures of A and B, equation (2) holds, not simply for two particular wave-lengths, but for any arbitrarily chosen pair of wave-lengths in the visible spectrum, then each source radiates the same proportionate amount of energy for each spectral region. Equation (2) is thus the condition for what may be called an integral color match. With a Lummer-Brodhun contrast photometer it is possible to adjust two sources quite accurately for such a color match, and this affords a satisfactory and somewhat less laborious method of applying criterion I., by measuring the relative variations in total intensity.

Data are here presented for the comparison of the osmium filaments with two untreated carbon filaments. The latter, though not black bodies in the strict sense, form very convenient reference standards, and are sufficiently non-selective to be quite acceptable. Direct measurements against a standard electrically heated black body are, in fact, very difficult owing to the impossibility of obtaining temperatures comparable with those of the incandescent filament, and also because the highest temperatures available can scarcely be kept constant while the comparison is being made. Criterion I. has been tested, both with the photometer and with the spectrophotometer, using a range of temperatures for the carbon lamps corresponding to impressed voltages from 180 to 220. In every case it has been shown that there is actually a change of emissivity with temperature.

Photometric Data.—A Lummer-Brodhun contrast photometer of the ordinary type was used, with an adjustable bench about seven meters in length. The two sources to be compared were carefully screened, and two diaphragms with apertures about ten by fifteen centimeters were mounted between the photometer and each lamp. All light not coming directly from the lamps was excluded as completely as possible, and in order to eliminate any effects due to construction and arrangement of the system, a part of the observations were taken with the reference standard at each end of the bench. The three test lamps were each carefully color matched against both carbon filaments at voltages for the latter ranging from 220 to 180 volts in ten volt intervals, the corresponding relative intensities being recorded (Table IV., *B*). *Criterion I. is fulfilled if these relative intensities are constant for each set of observations,*

Table IV.

Voltages and Relative Intensities for Integral Color Match.

A, Mean color match voltage of test lamp. *B*, Mean relative intensities of test lamp and comparison lamp. *C*, Ratio of relative intensities in terms of values corresponding to normal voltage of comparison lamp.

Series I. Photometer length 400 cm. Comparison lamp carbon No. 2.

Test Lamp.		Comparison Lamp Voltages.				
		180	190	200	210	220
Osmium no. 1	A	26.0	28.6	30.9	33.1	35.8
	B	.837	.837	.824	.781	.776
	C	1.08	1.08	1.06	1.01	1.00
Osmium no. 2	A	26.0	28.4	30.8	33.6	36.2
	B	.817	.815	.800	.794	.779
	C	1.05	1.05	1.03	1.02	1.00
Osmium no. 3	A	26.1	28.7	31.4	33.5	36.2
	B	.851	.826	.837	.791	.783
	C	1.09	1.05	1.07	1.01	1.00

Series II. Photometer length 500 cm. Comparison lamp carbon No. 1.

Osmium no. 1	A	26.0	28.3	30.8	33.3	35.8
	B	.798	.767	.755	.743	.733
	C	1.09	1.05	1.03	1.01	1.00
Osmium no. 2	A	26.5	28.8	31.3	33.9	36.3
	B	.819	.789	.788	.783	.764
	C	1.07	1.03	1.03	1.02	1.00
Osmium no. 3	A	26.6	28.7	30.9	33.4	36.1
	B	.851	.814	.772	.755	.753
	C	1.13	1.08	1.03	1.00	1.00

Series III. Photometer length 500 cm. Comparison lamp tantalum No. 1.

		70	80	90	100	110
Osmium no. 1	A	32.1	36.8	41.3	45.7	49.8
	B	1.619	1.530	1.480	1.405	1.358
	C	1.19	1.13	1.09	1.03	1.00

as follows from equation (4), or, in other words, if the ratio of the relative intensities corresponding to two different temperatures (Table IV., C) is unity. Since the carbon lamps cannot be operated at a temperature sufficiently high to match in color the osmium filaments near their normal temperature, a comparison was also made between osmium filament no. 1 and a tantalum lamp, tantalum having been found to very nearly fulfill criterion I.[1] This appears in the table as Series III.

Considerable difficulty was encountered at the lower temperatures in making the color match because of the faint illumination, so that the first series of readings was taken with the photometer bench at 400 cm. Series II., with a length of 500 cm., is probably more accurate for the higher voltages, but less accurate for the lower, as appears from the variations in C, Table IV. While the results are not quite as consistent as might be desired, they show without exception that *there is a considerable departure from the conditions upon which criterion I. depends.*

It must be understood that there is a certain assumption underlying these conclusions, viz., that when a color match is obtained as above the ordinates of one spectral energy curve are actually proportional to the corresponding ordinates of the other curve, not simply in the brighter portion, but throughout the visible spectrum. This is the condition represented by equations (2) and (3), which must be satisfied before equation (4) is to be applied. Further investigation with a spectrophotometer, therefore, seemed desirable.

Spectrophotometric Data.—The instrument used was of the Lummer-Brodhun type, manufactured by Schmidt and Haensch, and was equipped with a variable sectored disc as designed by Hyde, giving a continuous range of transmissions from about 60 per cent. down to 0 per cent. This was calibrated in position by comparison with several standard discs. The slit width used on each collimator was .4 mm., and before each slit diffusing screens of ground glass were mounted. The method of substitution was employed throughout, two lamps being compared by successive balancing against a fixed comparison source, and the latter frequently tested for changes in temperature and color by comparison with a standard. During the course of some months the variation of the comparison source was almost imperceptible. Under these conditions no slit width corrections are necessary for the relations expressed by equations (2), (3) and (4), so long as the width of the collimator and ocular slits is not changed, because the correction factor, depending only upon the wave-lengths and the shape of the energy curve under investigation, cancels out in each case. The conclusions to be deduced are, therefore, quite independent of the purity of the spectra compared.

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[1] Hyde, *loc. cit.*

The preliminary observations involved a comparison of the spectral energy curves for osmium and carbon at various color match temperatures, through as wide a range of wave-lengths as possible. No marked differences between them were noted, the agreement being sufficiently close to indicate that no particular region of the spectrum demands special attention. Hence the conclusion seemed justified that if the curves are matched at four or five wave-lengths about equally spaced across the visible region, they will show satisfactory agreement throughout. As a matter of fact, however, there was some tendency, scarcely greater than the necessary experimental error, toward larger ratios in the middle region of the spectrum, indicating an elevation of the osmium curve with respect to the carbon, and later observations show the same effect. This conclusion is not in agreement with a statement published by Hyde,[1] for he found "slight evidence in the case of osmium that the curve of the latter when showing the same relative distribution between the energy emitted at 0.7 μ and that emitted at 0.5 μ as compared with carbon, dropped a little below that of the carbon at intermediate wave-lengths." These variations are not, however, of sufficient magnitude to noticeably affect the results here presented.

Test for Variations of Emissivity with Temperature.—In order to avoid the tedious process of spectrophotometric color matching by repeated trials with slightly varying voltages in each of the temperature regions

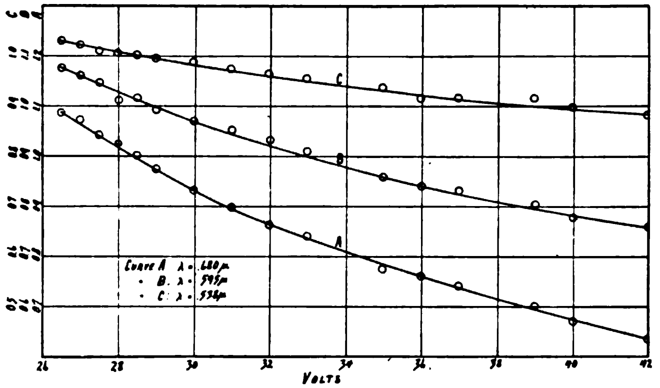

Fig. 2.
Color Ratios for Osmium No. 1.

to be studied, the following method was adopted. Measurements upon each lamp of the intensities for wave-lengths .680 μ, .595 μ, .538 μ and .500 μ in terms of the corresponding intensities of the comparison source were taken for a number of different voltages at arbitrary intervals. A set of ratios, which will be referred to as "color ratios," were then computed and plotted. These represent the quotient of the intensity for the shortest observed wave-length, $\lambda = .500\ \mu$, by the intensities at the same temperature for the other three wave-lengths. The three numbers so obtained may be used as an index of the color of the test

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[1] Hyde, Jour. Franklin Inst., 170, p. 31, 1910.

TABLE V.

Lamp.	Voltage.	Color Ratios.				Observed Transmission.			
		$\lambda = .680\mu$	$.595\mu$	$.538\mu$	$.500\mu$	$.680\mu$	$.595\mu$	$.538\mu$	$.500\mu$
O no. 1	35.97	.759	.844	.925	1.00	.1839	.1658	.1520	.1396
O no. 2	36.12	.759	.843	.917	1.00	.1597	.1437	.1320	.1211
O no. 3	35.70	.759	.849	.925	1.00	.1610	.1431	.1325	.1227
	mean	.759	.845	.922	1.00				
C no. 1	220.2	.759	.850	.925	1.00	.1000	.0906	.0831	.0761
C no. 2	222.2	.759	.846	.925	1.00	.1216	.1090	.0991	,0910
	mean	.759	.848	.925	1.00				
O no. 1	33.50	.820	.891	.946	1.00	.2404	.2210	.2086	.1985
O no. 2	33.45	.820	.888	.938	1.00	.2105	.1960	.1852	.1743
O no. 3	33.35	.820	.895	.950	1.00	.2074	.1888	.1775	.1690
	mean	.820	.891	.945	1.00				
C no. 1	210.0	.820	.890	.937	1.00	.1309	.1214	.1145	.1091
C no. 2	212.8	.820	.886	.941	1.00	.1580	.1458	.1380	.1303
	mean	.820	.888	.939	1.00				
O no. 1	31.00	.895	.947	.971	1.00	.3234	.3042	.2974	.2900
O no. 2	30.90	.895	.946	.965	1.00	.2850	.2700	.2650	.2555
O no. 3	31.00	.895	.945	.975	1.00	.2707	.2553	.2491	.2422
	mean	.895	.946	.970	1.00				
C no. 1	200.0	.895	.940	.960	1.00	.1774	.1684	.1634	.1579
C no. 2	202.4	.895	.936	.965	1.00	.2150	.2055	.1990	.1926
	mean	.895	.938	.962	1.00				
O no. 1	28.93	.978	.996	.995	1.00	.4160	.4080	.4088	.4060
O no. 2	28.78	.978	1.002	.992	1.00	.3750	.3675	.3667	.3650
O no. 3	28.76	.978	1.000	.998	1.00	.3597	.3504	.3535	.3520
	mean	.978	.999	.995	1.00				
C no. 1	190.0	.978	1.002	.994	1.00	.2459	.2341	.2404	.2409
C no. 2	191.6	.978	.991	.990	1.00	.3075	.3045	.3032	.3015
	mean	.978	.997	.992	1.00				
O no. 1	26.75	1.075	1.069	1.028	1.00	.5500	.5555	.5785	.5947
O no. 2	26.50	1.075	1.067	1.024	1.00	.5040	.5040	.5325	.5425
O no. 3	26.50	1.075	1.066	1.024	1.00	.4900	.4935	.5087	.5225
	mean	1.075	1.067	1.025	1.00				
C no. 1	180.0	1.075	1.073	1.016	1.00	.3497	.3538	.3706	.3744
C no. 2	182.4	1.075	1.058	1.017	1.00	.4200	.4278	.4428	.4527
	mean	1.075	1.065	1.016	1.00				

lamp, since two lamps will match in color only when these ratios are the same for each. Fig. 2 shows the variations in color ratios for osmium filament no. 1, and makes it possible to find the voltage corresponding to any desired color.

The voltage of each lamp when at color match with carbon no. 1 at 180, 190, 200, 210 and 220 volts may be read from these curves immediately, and the exactness of the color match determined by the agreement of the corresponding color ratios. The figures appear in Table V. The average ratios for carbon and for osmium at each temperature show that a very nearly perfect match has been obtained, the differences between corresponding values for different filaments being not much greater than those for two similar filaments. The osmium ratios for yellow and green (.595 μ and .538 μ) seem to run slightly higher than those for carbon, which agrees with the previous conclusion regarding the relative shapes of the curves. The agreement of these pairs of ratios constitutes the fulfillment of equation (3).

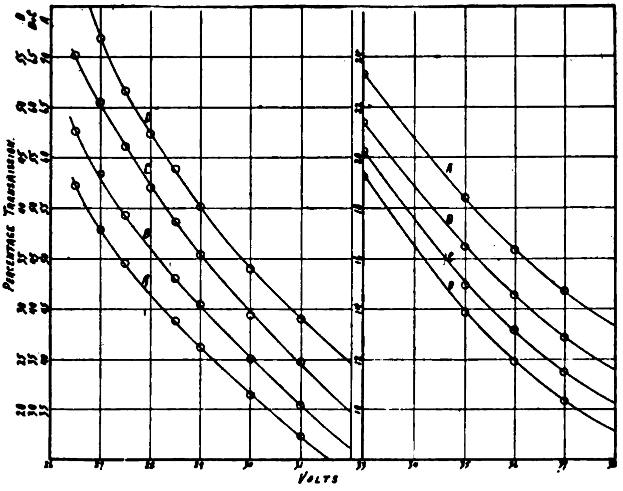

Fig. 3.

Isochromatic Curves for Osmium No. 1. Curve *A*, .680 μ; Curve B, .595 μ; Curve C, .538 μ; Curve D, .500 μ.

The same observations from which the above color ratios were computed may also be plotted in the form of isochromatic transmission-voltage curves, as in Fig. 3, the disc transmissions varying inversely as the intensities relative to the comparison source. From Fig. 3 we may determine the intensities for each of the four wave-lengths considered, at such voltages for osmium no. 1 (found from Fig. 2) as correspond to a color match with carbon no. 1 at the chosen voltages. These intensities, the reciprocals of the disc transmissions appearing in Table V., *are precisely the data necessary for the application of criterion I.*

TABLE VI.

Intensity Ratios for Each Lamp when at a Color Match with Carbon Lamp No. 1.

$I_{v_1}/I_{v_2} = T_{v_2}/T_{v_1}$, I = intensity of emission, $v_1 < v_2$.
T = fractional transmission of disc.

Lamp.	**Color Match Voltages.**		**Wave-lengths.**			
			.680μ	.595μ	.538μ	.500μ
A. Color match with carbon No. 1 at 180 and 220 volts.						
O no. 1	26.75	35.97	.334	.298	.263	.235
O no. 2	26.50	36.12	.317	.285	.248	.223
O no. 3	26.50	35.70	.328	.290	.260	.234
		mean	.326	.291	.257	.231
C no. 1	180.0	220.0	.286	.256	.224	.203
C no. 2	182.4	222.2	.289	.255	.224	.201
		mean	.287	.255	.224	.202
B. Color match with carbon No. 1 at 190 and 220 volts.						
O no. 1	28.93	35.97	.441	.406	.372	.343
O no. 2	28.78	36.12	.425	.391	.360	.332
O no. 3	28.76	35.70	.447	.409	.375	.348
		mean	.438	.402	.369	.341
C no. 1	190.0	220.0	.407	.387	.346	.316
C no. 2	191.6	222.2	.395	.358	.327	.302
		mean	.401	.372	.336	.309
C. Color match with carbon No. 1 at 180 and 210 volts.						
O no. 1	26.75	33.50	.436	.397	.361	.332
O no. 2	26.50	33.45	.419	.390	.348	.321
O no. 3	26.50	33.35	.424	.382	.349	.324
		mean	.426	.390	.353	.326
C no. 1	180.0	210.0	.374	.344	.309	.291
C no. 2	182.4	212.8	.376	.340	.312	.288
		mean	.375	.342	.310	.289

The ratio of two intensities, or the inverse ratio of two transmissions, at any wave-length and a chosen pair of voltages for a given lamp, forms one member of equation (4). The other member is a similar intensity ratio for the other lamp at color match voltages and the same wave-length. These ratios for each lamp for various wave-lengths and various pairs of voltages appear in Table VI.; to satisfy equation (4) the values for the osmium lamps at any wave-length should equal those for the carbon lamps at the same wave-length, in any one of the three groups. The mean values show a very wide deviation from equality, however, and always in the same direction. *Hence criterion I. is not fulfilled*, and there is a change of emissivity with temperature, even at the short wave-lengths within the visible spectrum.

III. Comparison of the Infra-red Spectra of Carbon and Osmium.

The marked depression of the energy curve of osmium with respect to that of carbon filaments in the longer wave-lengths suggests at once the application of criterion I. in the infra-red. This has been done, using a fixed arm mirror spectrometer with a quartz prism, set up according to Wadsworth's[1] method for minimum deviation. The Cornu quartz prism, with faces 74 X 110 mm., was supplied by A. Hilger, Ltd., and its indices of refraction for various wave-lengths were determined from measurements by Paschen[2] upon a similar prism. The detector was a sensitive thermopile constructed by Coblentz, with a Leeds and Northrup high sensitivity galvanometer. With the scale at 128 cm. it

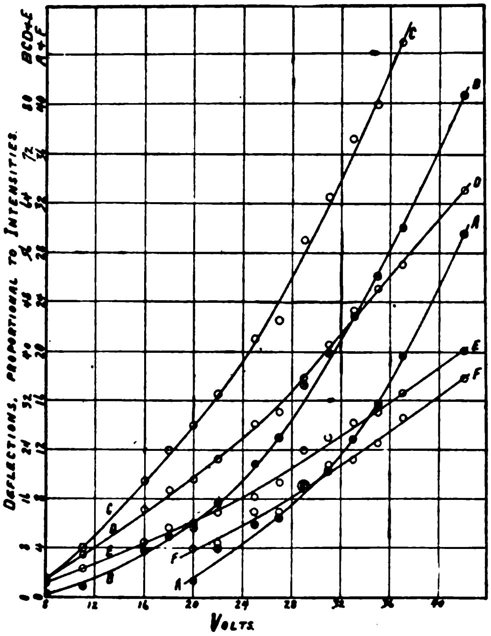

Fig. 4.

Isochromatic Curves for Osmium No. 1. Curve *A*, .608 μ; Curve *B*, .726 μ; Curve *C*, 1.068 μ; Curve *D*, 1.750 μ; Curve *E*, 2.098 μ; Curve *F*, 2.400 μ.

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[1] Wadsworth, Phil. Mag., 38, p. 337, 1894.

[2] Paschen, Ann. d. Phys., 35, p. 1005, 1911.

gave a deflection of 80 mm. for a standard candle at one meter. The thermopile slit was .38 mm. in width, and so placed that it received an average spectral range of about 185 Ångstrom units.

The data consists of observations of the energy distributions at various temperatures for osmium lamp no. 1 and carbon lamp no. 1, from the extreme red of the visible spectrum to the longest wave-lengths giving appreciable deflections. The results for repeated trials were quite consistent, and are shown graphically for osmium no. 1 in Fig. 4. From such curves the mean galvanometer deflections of Table VII. were obtained. This table also shows the ratios of the intensities at various wave-lengths to those at $\lambda = 2.098\ \mu$ for each different temperature, which correspond to the color ratios for the visible spectrum. The intensity ratios are plotted in Fig. 5.

Table VII.

Galvanometer Deflections, and Intensity Ratios.

Deflections at Wave-lengths.						Volts.	$I_{2.098\mu} + I_\lambda$					
λ=.608 μ	.726 μ	1.068 μ	1.750 μ	2.098 μ	2.400 μ		.608μ	.726μ	1.068μ	1.750μ	2.098 μ	2.400 μ
						Osmium No. 1.						
	.6	3.1	3.0	2.3		8		3.83	.742	.766	1.00	
	3.3	10.7	8.2	5.6		12		1.70	.521	.683	1.00	
	7.1	19.0	13.7	9.0		16		1.27	.474	.656	1.00	
1.4	12.1	27.8	19.6	12.7	3.5	20	9.08	1.05	.457	.649	1.00	3.63
4.3	19.3	38.4	26.1	16.5	5.8	24	3.79	.855	.430	.632	1.00	2.84
7.6	29.2	52.0	33.5	21.2	8.2	28	2.79	.726	.408	.633	1.00	2.58
11.5	41.9	68.0	42.4	26.3	10.7	32	2.29	.629	.387	.621	1.00	2.46
17.4	56.5	85.5	51.8	31.6	13.3	36	1.82	.560	.370	.610	1.00	2.38
25.0	72.6	104.8	61.2	37.2	16.2	40	1.49	.512	.355	.608	1.00	2.30
29.5	81.5	114.5	66.0	40.2	17.8	42	1.36	.494	.351	.610	1.00	2.26
						Carbon No. 1.						
12.2	51.0	145.5	109.0	76.3	31.3	180	6.25	1.50	.525	.700	1.00	2.43
14.2	63.6	168.5	122.8	84.7	34.1	190	5.96	1.33	.503	.690	1.00	2.48
18.3	78.2	191.5	136.5	93.0	36.8	200	5.09	1.19	.485	.681	1.00	2.53
23.1	96.2	214.4	150.5	101.3	39.6	210	4.39	1.05	.473	.674	1.00	2.56
29.7	119.0	240.0	166.0	109.6	42.4	220	3.68	.935	.456	.660	1.00	2.58

The variation of any intensity ratio with increasing voltage is an indication of the rate of increase of intensity with temperature for the given wave-length, relative to that for $\lambda = 2.098\ \mu$. Thus, in the case of the carbon lamp, the ratios for each wave-length shorter than $2.098\ \mu$ decrease (Table VII.), showing that the corresponding I_λ increases more rapidly than does $I_{\lambda=2.098\mu}$, and the rate of decrease becomes more rapid with shortening wave-lengths. For $\lambda = 2.400\ \mu$ the ratios increase, showing that here I_λ increases more slowly than at $2.098\ \mu$. This is exactly as would be expected for black body radiation, according to Wien's displacement law. In the case of osmium, I_λ increases more rapidly for each shorter wave-length than for $2.098\ \mu$, but the last column shows a more rapid increase for $2.400\ \mu$ than for $2.098\ \mu$, which is quite an unexpected result. This cannot be explained by absorption effects, since they would influence the carbon radiation equally; neither does it seem to be a mere observational error since there is a definite and smooth progression. Possibly the explanation is to be found in a peculiarity of the energy curve such as an emission band in this neighborhood, but further investigation will be necessary before definite conclusions can be reached.

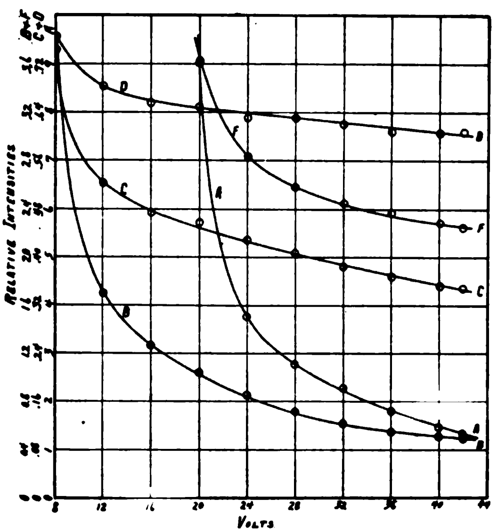

Fig. 5.

Energy Distribution, Osmium No. 1. Intensities relative to $I_{\lambda=2.098\mu}$. Curve A, .608 μ; Curve B, .726 μ; Curve C, 1.068 μ; Curve D, 1.750 μ; Curve F, 2.400 μ.

Application of Hyde's Criterion.—From Fig. 5 showing the variations of intensity ratios for the osmium lamp it is at once apparent that no voltage can be found at which the energy curve from this lamp will be of the same shape as that from the carbon lamp at any of the measured voltages. Hence criterion I. cannot be applied to this spectral region as a whole, but only at particular pairs of wave-lengths and temperatures.

Choosing $\lambda = 2.098\ \mu$ for comparison with each of the other wave-lengths, and, as before, temperatures corresponding to voltages for

TABLE VIII.

Isochromatic Intensity Ratios for Wave-Lengths and for Pairs of Voltages at which the Ratios ($I_{\lambda=2.098\mu} \div I_{\lambda'}$) are the Same for Both Lamps.

Wave-length.	Intensity Ratios ($I_{\lambda=2.098\mu} \div I_{\lambda'}$)		Carbon Lamp Voltages.		Osmium Lamp Voltages.		$I_{\lambda'}$ for Carbon No. 1.			$I_{\lambda'}$ for Osmium No. 1.		
	At Lower Voltage.	At Higher Voltage.	Lower.	Higher.	Lower.	Higher.	Lower.	Higher.	Ratio.	Lower.	Higher.	Ratio.
.608	6.25	3.68	180	220	21.20	24.30	12.2	29.7	.410	1.55	4.60	.337
	5.96	3.68	190	220	21.35	24.30	14.2	29.7	.478	1.60	4.60	.348
	6.25	4.39	180	210	21.20	22.90	12.2	23.1	.528	1.55	3.50	.443
.726	1.50	.935	180	220	13.60	22.20	51.0	117.0	.436	4.7	15.7	.299
	1.3	.935	190	220	15.45	22.20	63.6	117.0	.544	6.5	15.7	.414
	1.50	1.05	180	210	13.60	19.90	51.0	96.2	.531	4.7	12.0	.392
1.068	.525	.456	180	220	11.90	19.20	145.5	240.0	.606	10.4	26.0	.400
	.503	.456	190	220	13.50	19.20	168.5	240.0	.702	13.7	26.0	.527
	.525	.473	180	210	11.90	16.80	145.5	214.4	.678	10.4	20.7	.502
1.750	.700	.660	180	220	10.85	15.40	109.0	166.0	.657	6.6	13.0	.508
	.690	.660	190	220	11.55	15.40	122.8	166.0	739	7.5	13.0	.577
	.700	.674	180	210	10.85	13.12	109.0	150.5	.725	6.6	9.8	.674
2.400	2.43	2.58	180	220	33.30	28.25	31.3	42.2	.740	10.6	8.4	1.26
	2.48	2.58	190	220	31.25	28.25	34.1	42.4	.805	10.	8.4	1.21
	2.43	2.56	180	210	33.30	28.80	31.3	39.6	.790	10.6	8.7	1.22

The fulfillment of Criterion I would be indicated by the agreement of the ratios in columns 10 and 13.

carbon no. 1 of 180 and 220, 180 and 210, and 190 and 220, the intensity ratios which form the right hand members of equations (2) and (3) respectively may be found in Table VII. These are rearranged for convenience in Table VIII., columns 2 and 3. From Fig. 5 the voltages of the osmium lamp for which the same ratios hold may be determined, so that equations (2) and (3) are satisfied at these voltages (Table VIII., columns 6 and 7). Fig. 4 and similar curves give the intensities for the two lamps for these voltages at the chosen wave-lengths, and the isochromatic ratios of these intensities for any two temperatures form the two members of equation (4), which constitutes criterion I. These ratios appear in columns 10 and 13 of Table VIII., which should agree, according to equation (4). The wide discrepancy shows marked changes of emissivity with temperature, the greatest relative differences occurring in the neighborhood of the maximum of the energy curve, which is about 1.0 μ.

The peculiarity of osmium radiation at 2.400 μ already noted is here again evident, for at this wave-length when the carbon voltage is increased, that of osmium must be decreased in order to maintain the same relative intensities for wave-lengths 2.098 μ and 2.400 μ.

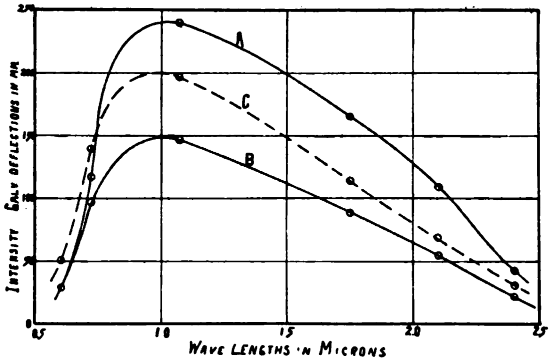

Fig. 6.

Approximate Spectral Energy Curves for Osmium and Carbon. Curve *A*, Carbon No. 1 at 220.0 volts; Curve *B*, Osmium No. 1 at 35.97 volts; Curve *C*, Osmium No. 1 at 42.00 volts; Curves *A* and *B* at color match temperatures.

Relative Selectivity of Osmium and Carbon.—The isochromatic curves of Fig. 4 supply the data necessary for a comparison of the energy curves for osmium and carbon when the two are at a color match in the visible region. Table IX. shows the deflections at various wave-lengths for both lamps corresponding to a color match with the carbon at 220 volts. All the deflections for the observations on osmium were reduced to the same scale as the carbon deflections by a constant factor, viz., the ratio of the deflections at λ = .608 μ, since, the visible spectra matching, these ordinates should be equal. The deflections for carbon and the reduced

Table IX.

Comparison of the Spectral Energy Distribution in the Infra-red for Carbon No. 1 and Osmium No. 1.

Lamp.	Volts.		Deflections at Wave-lengths.					
			.608μ	.726μ	1.068μ	1.750μ	2.098μ	2.400μ
C no. 1	220		29.7	117.0	240.0	166.0	109.6	42.4
O no. 1	35.97	Observed:	17.25	56.4	85.3	51.7	31.6	13.3
		Reduced:	29.7	97.0	147.0	89.0	54.4	22.9
O no. 1	42.00	Observed:	29.5	81.5	114.5	66.0	40.2	17.8
		Reduced:	50.8	140.5	197.2	113.8	69.3	30.6

deflections for osmium are plotted in Fig. 7. The curve for osmium at 42 volts is also plotted to the same scale. No attempt has been made to locate exactly the maxima of these curves, nor have corrections been made for absorption, but the very large effect of selectivity is at once apparent in the depression of the osmium curves relative to those of carbon throughout the infra-red region. Even at 42 volts the osmium radiates less than the carbon for wave-lengths greater than 0.8 μ, although the former is far above the temperature for color match in the visible spectrum.

IV. SUMMARY.

Three incandescent lamps with osmium filaments have been studied. Data are presented and equations derived for the variations with voltage of the power consumed and the candle power radiated, and the functional dependence of current and watts per candle power upon voltage is indicated.

The luminous efficiencies of osmium and carbon lamps at color match voltages are compared, showing that osmium as a radiator is highly selective. Hyde's criterion for the detection of variations of emissivity with temperature has been applied both photometrically and spectrophotometrically, carbon lamps being used as comparison sources. Without exception all the results show that the emissivity is a function of both wave-length and temperature. A method is described for determining the relative shapes of the energy curves for two filaments when the best possible color match has been obtained, and the color ratios computed under various conditions indicate that, when the extremes are matched, the curve for osmium is somewhat depressed relative to that for carbon in the middle portion of the visible spectrum.

Energy curves for both radiators have been obtained in the infra-red region out to wave-lengths of 2.4 μ, and the observations show that the two cannot be brought into coincidence at any voltage. An application of criterion I. to these data again shows marked changes of emissivity with temperature. The relations of the energy curves in the infra-red at voltages corresponding to a color match in the visible spectrum are shown, the osmium radiation being very noticeably deficient in energy of the longer wave-lengths.

It has been shown that, within the range of observations presented, the rate of increase of intensity with temperature increases from the longer toward the shorter wave-lengths for the carbon filament, but that for the osmium in the neighborhood of 2.4 μ the rate of increase is greater than for wave-lengths 2.098 μ or 1.750 μ. An explanation of this effect as indicating an emission band is suggested, though this has not been verified.

In conclusion the writer wishes to express his indebtedness to Dr. K. E. Guthe for his continual interest and helpful suggestions; also to Dr. E. P. Hyde for loaning the osmium lamps used and the standard sectored discs for calibration purposes.

Physical Laboratory,
University of Michigan,
May 1, 1915.

THE OPTICAL ACTIVITY OF LIQUIDS AND GASES.

By Frank Gray.

THE natural activity of isotropic substances has been an interesting subject both to the chemist and to the physicist. Each has attacked the problem according to his own particular method; but unfortunately, the two lines of attack have not met. On the chemical side Pasteur, Le Bel and van't Hoff developed the present conception of molecular structure and proposed the theory connecting optical activity with an asymmetric structure of the molecule, a theory confirmed by the enormous amount of experimental work carried out in recent years. In the domain of physics the subject has been treated more from the optical view point and Drude and Voigt have shown just what form of electro-magnetic equations must hold in an active medium in order to produce rotation. However, the two theories have not been tied together; it has not been shown why the asymmetric molecule should give rise to these particular equations.

It was for this reason that the present study of optical activity was taken up. Part I. of the present paper is a brief report of a theoretical study of a few types of molecules, a study which has led to a very natural and satisfactory explanation of both the chemical and the physical facts of optical activity. Part II is a report of an experimental search for phenomena predicted from the theory.

Part I. Theoretical.

In the three-dimensional formulæ of chemistry, a symmetrical molecule is one that can be divided into two similar halves by a single plane, an asymmetrical molecule cannot so be divided. An enormous amount of work has been done on molecular structure by other methods than those of optical activity and the results have supported the theory that only an asymmetrical molecule can cause rotation.

If we accept, in general, the three-dimensional formulæ as representing molecular structure, then the experimental facts of optical activity may be stated as follows:

1. Any substance whose molecules are asymmetrical causes rotation. This is true for all carbon compounds, internal compensation and indirect asymmetry giving rise to only apparent exceptions.[1] Rotation has also

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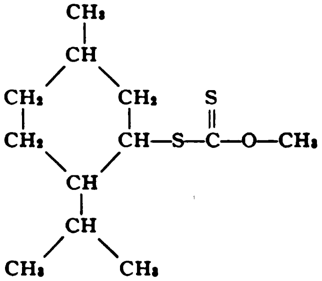
[1] "Stereo Chemistry," A. W. Stewart, 44, 83, 105.

been observed in compounds of nitrogen, sulphur, selenium, tin, and silicon.

2. Antipodes, a molecule and its mirror image, rotate in opposite directions.

3. Symmetrical molecules are not active,[1] pseudo-asymmetry and a few other cases are only apparent exceptions.

4. Absorption systems in an active substance cause anomalies in the rotary dispersion, although the origin of the band may lie in a part of the molecule distant from the central carbon atom.[2] Thus for example, in the menthyl ester of methyl dithio carbonic acid the asymmetric atom is in the ring group, while the absorption center is in the group $S = C$.



5. The magnitude of the activity is profoundly influenced by the molecular structure and is roughly dependent on the magnitude of the asymmetry of the molecule.

6. Activity varies with temperature[3] and pressure, and with the concentration of active molecules.

In attempting to account for the general facts of optical activity, new terms have been added to the field equations.[4] Thus Dude, Voigt, and Lorentz write the first equation for an active medium

$$\text{curl}\, H = \frac{1}{C}(\epsilon\dot{E} - f\,\text{curl}\,\dot{E}), \tag{1}$$

according to Lorentz the second field equation remains unchanged,[5]

$$\text{curl}\, E = -\frac{1}{C}\dot{H}. \tag{2}$$

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[1] "Stereo Chemistry," A. W. Stewart, 21.

[2] Cotton, Ann. Chim. et d. Phys., 8, 374, 1896. Tuchugaff and Ogordnikoff, Zeit. für Phys. Chem., 74, 503, 1910, and 85, 481, 1913.

[3] Landolt, "Optical Rotation of Organic Substances," 206.

[4] Landolt, "Optical Rotation of Organic Substances," 169.

[5] Lorentz, Versuch einer Theor. der elekt. und optik. Erscheinungen.

Drude and Voigt have decided from energy considerations that the second equation should be

$$\text{curl } E = -\frac{1}{C}\left(\dot{H} + \frac{f}{C}\ddot{E}\right). \tag{3}$$

The solution of 1 with either 2 or 3 for a plane polarized wave shows that the electric vector E is rotated as the wave passes through the medium, the amount of the rotation being half as great when the Lorentz equations are used.

For the case of a plane polarized light wave these equations may be interpreted as stating that a space rate of change of E in the direction of propagation produces a component of electric polarization normal to both E and the direction of propagation, a component that does not exist in a non-active medium. In the following work we shall denote this component by P_n.

Drude has shown that the modified field equations would result if the paths of some of the electrons in an active medium were short helices all twisted in the same direction and with axes oriented at random in space, but he made no attempt to show how the asymmetrical structure of the molecule causes these spiral tracks, and thus his theory does not tie up with molecular structure, nor does it account for the anomalous rotary dispersion of the metallic tartrates and such cases as the one mentioned above. It was for this reason that the following study of the interaction between atoms was made in the hope that such interactions might afford a more satisfactory explanation of optical activity.

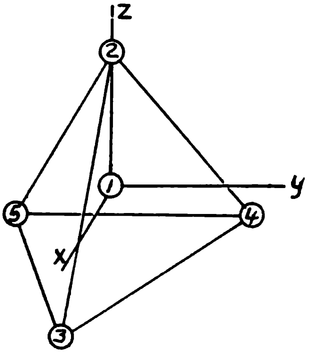

Fig. 1.

A Molecule of Five Atoms.

Consider a molecule consisting of a central atom with four other atoms grouped about it. The origin of a coördinate system (Fig. 1) will be taken at the central atom, and the axes so oriented that atom 2 will lie on the z axis and atom 3 in the xz-plane. The position of the nth atom is determined by x_n, y_n, z_n, and its distance from the origin by r_n. The distance from the nth to the mth atom is d_{nm}.

It is assumed that the atoms may be electrically polarized in any direction by an electric force and that

$$p_{xn} = K_n E_x, \text{ etc.,} \tag{4}$$

where p_{xn} is the x component of polarization of the nth atom and K_n is a quantity depending on the atom, the molecule in which the atom occurs, and on the frequency of the wave. K_n is to be thought of as the restoring coefficient that would exist if the nth atom could be made to oscillate electrically while all the other atoms remained unpolarized. It, therefore, does not contain any terms depending on the interactions of the electrical oscillations of the atoms, but it might contain terms depending on the static forces that hold the atoms together. For mathematical simplicity it is also assumed that the atoms are small compared to the distances between them; the atoms may thus be treated as electric doublets.

When a train of light waves passes over such a molecule it will tend to polarize the atoms in the E direction. The polarized atoms will then interact in a complicated manner, and as a result the molecule as a whole will not be polarized directly in the direction of the original E of the wave and the plane of vibration of the wave will be twisted a little out of its original direction. A molecule with some other orientation may twist the plane of vibration in the opposite direction. The problem is to determine if there will be any resultant rotation when plane polarized light passes through an aggregate of such molecules oriented at random in space.

The method of attacking the problem will be to find the component of polarization of a single molecule in the N direction, *i. e.*, normal to E and the direction of propagation, and then find the average of this component for all orientations of the molecule. The polarization P_n of the medium will be found by multiplying this average value by the number of active molecules per unit volume. The polarization of the medium in the E direction will be found in the usual manner. Thus the polarization of the medium will be completely known and it should be possible to write out the field equations which will hold in the medium for a plane polarized light wave. It is evident that if P_n is different from zero the medium will be active.

The vector expressions[3] for the forces at a distance r from an oscillating doublet are

$$\begin{aligned} (5) \qquad E &= \frac{1}{r^3}\{3p_r r_1 - p\} + \frac{1}{r^2}\{3\dot{p}_r - \dot{p}\} - \{\ddot{p} r_1 - p\} \\ H &= -\frac{1}{r^2}[r_1\dot{p}] - \frac{1}{r}[r_1\ddot{p}], \end{aligned}$$

where p_r is the polarization in the r direction and r_1 is a unit vector in the

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[1] Drude, Gött. Nachr., 1904, I., 1.

[2] Drude, "Lehrbuch der Optik," 338.

[3] Abraham, "Theorie der Elektrizatät," II., 63.

r direction. Since r is small compared to the wave-length of light, all terms but the $1/r^3$ terms will be neglected. This term is only the ordinary electro-static force of a doublet. Let p_{xm}, p_{ym}, p_{zm} be the components of polarization of the mth atom whose coördinates are x_m, y_m, z_m, and let d_{nm} be the distance to the nth atom whose coördinates are x_n, y_n, z_n. Then, the components of force at atom n due to the polarization of atom m are

$$E_x = \frac{1}{d_{nm}^3}\left\{\frac{3}{d_{nm}^2}(x_n - x_m)[p_{xm}(x_n - x_m) + p_{ym}(y_n - y_m) \right.$$
$$\left. p_{zm}(z_m - z_n)] - p_{xm}\right\},$$

$$(6) \qquad E_y = \frac{1}{d_{nm}^3}\left\{\frac{3}{d_{nm}^2}(y_n - y_m)[\cdots] - p_{ym}\right\},$$

$$E_z = \frac{1}{d_{nm}^3}\left\{\frac{3}{d_{nm}^2}(z_n - z_m)[\cdots] - p_{zm}\right\}.$$

Let x', y', z' be a fixed coördinate system to which the plane polarized light wave is referred. The z' axis will be taken in the direction of propagation, the x' axis in the E direction, and the y' axis in the N direction. The x, y, z system in which the molecule is fixed, will be referred to the x', y', z' system by the angles ϕ, β, θ of Fig. 2.

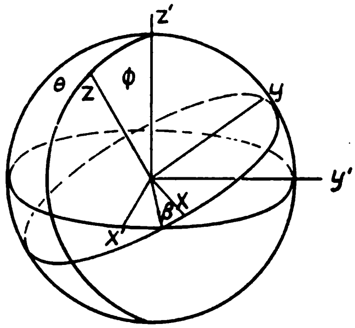

Fig. 2.

Let the electric force of the wave be given at the origin by

$$E_{x'} = E_{x'} \cos \nu t, \text{ etc.}$$

Then the external force on the nth atom is

$$(7) \qquad E_{x'} = E_{x'} \cos\left(\nu t - \frac{\nu}{c} z_n'\right), \text{ etc.}$$

The total force acting on an atom has the same period as the wave; so the polarization of the mth atom may be written

$$p_{xm} = P_{xm} \cos(\nu t - \delta_{xm}),$$
$$(8) \qquad p_{ym} = P_{ym} \cos(\nu t - \delta_{ym}),$$
$$p_{zm} = P_{zm} \cos(\nu t - \delta_{zm}).$$

The electro-static action of one atom upon another will be retarded, thus the force exerted upon atom n by atom m at the time t will be due to the retarded state

$$(9) \qquad p_{xm} = P_{xm} \cos\left(\nu t - \delta_{xm} - \frac{\nu}{c} d_{nm}\right), \text{ etc.}$$

In the equation for the polarization of an atom,

$$\frac{1}{K_n} p_{xn} = E_x, \text{ etc.,} \tag{4}$$

the electric force includes that of the wave and also the forces arising from each component of polarization of the other four atoms. Using equations 6, 7 and 9 to determine the total force acting on an atom *n*, equation 4 becomes

$$\begin{aligned} \frac{1}{K_n} \cos(\nu t - \delta_{xn}) &= E_x \cos\left(\nu t - \frac{\nu}{c} z_n'\right) \\ &+ \sum_{m=1}^{5} \frac{1}{d_{nm}^3} \left[\frac{3(x_n - x_m)^2}{d_{nm}^2} - 1\right] \cos\left(\nu t - \delta_{xm} - \frac{\nu}{c} d_{nm}\right) P_{xm} \\ &+ \frac{3}{d_{nm}^2} (x_n - x_m)(y_n - y_m) \cos\left(\nu t - \delta_{ym} - \frac{\nu}{c} d_{nm}\right) P_{ym} \\ &+ \frac{3}{d_{nm}^2} (x_n - x_m)(z_n - z_m) \cos\left(\nu t - \delta_{zm} - \frac{\nu}{c} d_{nm}\right) P_{zm}, \end{aligned} \tag{10}$$

where the summation excludes the *n*th atom. After expanding the cosine terms, the equation reduces to

$$(\cdots) \cos \nu t + (\cdots) \sin \nu t = 0.$$

The quantities inside the bracket are independent of *t*; therefore each must be equal to zero. If we let

$$P_{xm} \cos \delta_{xm} = A_{xm},$$
$$P_{xm} \sin \delta_{xm} = B_{xm},$$

these conditions are expressed by the equation

$$\begin{aligned} \frac{1}{K_n} A_{xn} = E_x \cos \frac{\nu}{c} z'_n &+ \sum_{m=1}^{5} \frac{1}{d_{nm}^3} \cos \frac{\nu}{c} d_{nm} \left\{ \left[\frac{3(x_n - x_m)^2}{d_{nm}^2} - 1\right] A_{xm} \right. \\ &\left. + \frac{3}{d_{nm}^2} (x_n - x_m)(z_n - z_m) A_{ym} + \frac{3}{d_{nm}^2} (x_n - x_m)(z_n - z_m) A_{zm} \right\} \\ &- \frac{1}{d_{nm}^3} \sin \frac{\nu}{c} d_{nm} \left\{ \left[\frac{(3x_n - x_m)^2}{d_{nm}^2} - 1\right] B_{xm} + \frac{3}{d_{nm}^2} (x_n - x_m)(y_n - y_m) B_{ym} \right. \\ &\left. + \frac{3}{d_{nm}^2} (x_n - x_m)(z_n - z_m) B_{zm} \right\}, \end{aligned}$$

and a similar one for $1/K_n(B_{xn})$ in sine terms but with the *A*'s and *B*'s interchanged. There are thus thirty linear equations to solve for the thirty *A*'s and *B*'s.

In forming the determinant the unknowns may be numbered as follows:

$$\begin{array}{cccccc} 1 & 2 & 3 & 4 & \cdots & 15 \\ A_{x1} & A_{y1} & A_{z1} & A_{x2} & & A_{z5}, \\ 16 & 17 & 18 & 19 & \cdots & 30 \\ B_{x1} & B_{y1} & B_{z1} & B_{x2} & & B_{z5}, \end{array}$$

and the equations may be numbered

$$(1) \qquad \frac{1}{K_1} A_{x1} = E_x \cos \frac{v}{c} z_1' + \Sigma$$

$$(2) \qquad \frac{1}{K_1} A_{y1} = \text{———————}$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$(15) \qquad \frac{1}{K_5} A_{z5} = \text{———————}$$

$$(16) \qquad \frac{1}{K_1} B_{x1} = \text{———————}$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$(30) \qquad \frac{1}{K_5} B_{z5} = \text{———————}.$$

The following relations hold among the minors of the general determinant. If D_r^h is the minor formed by suppressing the rth row and the nth column, then

$$D_r^h = D_h^r,$$

$$D_r^h = D_{h+15}^{r+15},$$

$$D_r^h = - D_{r+15}^{h-15},$$

$$D_{r+15}^h = - D_{r+5}^h,$$

$$D_{r-15}^{h+15} = D_r^h.$$

The value of any unknown, number m, may be found by substituting the constants for the mth column and dividing by D. The following is the solution for A_{x1} after expanding the numerator by minors with respect to the column of constants.

$$A_{x1} = \frac{1}{D} \Big\{ E_x D_1^1 - E_y D_1^2 + E_z D_1^3 - E_x D_1^4 \cos \frac{v}{c} z_2' + E_y D_1^5 \cos \frac{v}{c} z_2' + \cdots E_x D_1^{16} + E_y D_1^{17} - E_z D_1^{18} + E_x D_1^{19} \sin \frac{v}{c} z_2' - E_y D_1^{20} \sin \frac{v}{c} z_2' + \cdots \Big\}.$$

There will be similar solutions for all the other A's and B's.

If the solution A_{xm} be multiplied by cos νt and that of B_{xm} by sin νt and combined using the relation between minors given above and also letting

$$R_r^h = \sqrt{(D_r^h)^2 + (D_r^{h+15})^2},$$

then

$$P_{xm} = \frac{1}{D}\Big\{\sqrt{2}[\pm E_x R_m{}' \cos(\nu t - \epsilon) \pm E_y R_m{}^2 \cos(\nu t - \epsilon) \pm E_z R_m{}^2 \cos(\nu t - \epsilon)] \pm E_x R_m{}^4 \cos\left(\nu t - \frac{\nu}{c} z_2' - \epsilon\right) \pm \cdots\Big\}, \tag{11}$$

where all the ϵs are different. An example of one ϵ is

$$\epsilon = \tan^{-1}\frac{D_1{}^5}{D_1{}^{20}}.$$

It is found that the ϵs do not occur in the final result, and they are thus of little interest.

The undetermined signs of equation 11 could be determined in an actual case by going back to the physical meaning of equation 3. Even in the present general case the sign of a great many of the terms may be found. There are fifteen such equations, one for each component of polarization of the five atoms.

The total polarization of the molecule in the N direction, *i. e.*, $P_{y'}$ may be found by projecting the components of polarization of each atom on the y' axis and taking the sum for all atoms in the molecule. The average of this polarization $\overline{P_{y'}}$ for molecules oriented at random, may be shown to be

$$\overline{P_{y'}} = \frac{1}{8\pi^2}\int_0^{2\pi} d\phi \int_0^{2\pi} d\beta \int_0^{\pi} P_{y'} \sin\theta d\theta,$$

where, of course, $P_{y'}$ will be a function of ϕ, β, θ. Before integrating it is necessary to express E_x, E_y, E_z occurring in $P_{y'}$ in terms of $E_{y'}$ and also to express z_n' in terms of x_n, y_n, z_n.

In order to carry out the integration the following approximation is made

$$E_{x'} \cos\left(\nu t - \frac{\nu}{c} z_m' - \epsilon\right) = E_{x'} \cos(\nu t - \epsilon) - \frac{\partial E_{x'}}{\partial z'} z_n', \text{ etc.}$$

It may be shown that ϵ expresses the lag of the retarded interaction behind the E of the wave and that it is very small compared to 2π. So $\partial E_{x1}/\partial z'$ will be practically constant for such a short time and the approximation is justified.

There will be 225 terms in $\overline{P_{y1}}$ similar to

$$\frac{1}{8\pi^2}\int_0^{\pi} d\varphi \int_0^{\pi} d\theta \int_0^{2\pi} \frac{R_m{}^4}{D} (\cos\phi\cos\theta\cos\beta - \sin\beta\sin\theta)(\cos\beta\cos\phi\sin\theta$$
$$+ \sin\phi\cos\theta)\left\{E_{x'}\cos(\nu t - \epsilon) - \frac{\delta E_{x'}}{\partial z'}(-x_2\cos\beta\sin\varphi\right.$$
$$\left. + y_2\sin\beta\sin\varphi + z_2\cos\varphi\right\}\sin\varphi\, d\beta.$$

In the integration all terms will vanish except certain ones containing $\partial E_{x1}/\partial z'$. The final result is

$$\overline{P_y}' = \frac{1}{12D}\frac{\partial E_{x'}}{\partial z'}[z_2\{(\pm R_1{}^5 \pm R_2{}^4) \pm (R_7{}^5 \pm R_8{}^4) \pm (R_{10}{}^5 + R_{11}{}^4)$$
$$\pm (R_{13}{}^5 + R_{14}{}^4) + x_3\{\cdots\} + z_3\{\cdots\} + \cdots + z_5\{\pm (R_1{}^{14} + R_2{}^{13})$$
$$\pm (R_4{}^{14} + R_5{}^{13}) \pm (R_7{}^{14} + R_8{}^{13}) \pm (R_{10}{}^{14} + R_{11}{}^{13})\}]. \tag{12}$$

It may be shown that this expression is not identically equal to zero. Thus $\overline{p_{y'}}$ will, in general, be different from zero and the medium will have a component of polarization in the N direction. As stated previously, this means that the medium will be active.

In equation 12 let

$$b = \frac{1}{12D}[\cdots],$$

then

$$\overline{P_{y'}} = b\frac{\partial E_{x'}}{\partial z'}.$$

If n is the number of active molecules per unit volume, then the polarization in the N direction will be

$$P_H = nb\frac{\partial E_{x'}}{\partial z'}.$$

This is for the special case of axes x', y', z' taken in the direction of propagation of the wave and in the H and E directions respectively. For any set of axes this equation may be shown to be

$$P_H = nb \operatorname{curl} E.$$

The polarization in the E direction is

$$P_E = E\Sigma\frac{e^2/m}{\nu_0{}^2 - \nu^2} = E\Sigma,$$

Where ν_0 is the natural frequency of an electron in the medium, e is the electronic charge, and m is the electronic mass. The summation is taken over all electrons in unit volume. It is to be noted that this summation is the same one that occurs in the theory of refractive indices and that it is taken over both active and non-active molecules in the medium. The interaction between atoms in the same molecules has been taken care of in ν_0, for ν_0 means the frequency of an electron as it occurs in the molecule and not the frequency it would have in the isolated atom.

The total polarization is

$$P = P_N + P_E = E\Sigma + nb \operatorname{curl} E.$$

In this equation we have neglected the fact that the force acting on an electron in a material medium is really not E but $E + aP$, where a is usually considered to be about $4\pi/3$. Putting in this additional term

$$P = (E + aP) + nb \operatorname{curl} (E + aP).$$

Then,

$$P = E\frac{\Sigma}{1 - a\Sigma} + \frac{nb}{1 - a\Sigma}\operatorname{curl} (E + aP).$$

Put

$$P = E\frac{\Sigma}{1 - a\Sigma}$$

in the curl term; then

$$P = E\frac{\Sigma}{1 - a\Sigma} + \frac{nb}{(1 - a\Sigma)^2}\operatorname{curl} E.$$

The first field equation then becomes

$$\operatorname{curl} H = \frac{1}{c}\left(1 + 4\pi\frac{\Sigma}{1 - a\Sigma}\right)\dot{E} + \frac{4\pi}{c}\frac{nb}{(1 - a\Sigma)^2}\operatorname{curl} \dot{E},$$

and according to equation 3, the second equation should be

$$\operatorname{curl} = -\frac{1}{c}\dot{H} - \frac{8\pi}{c^2}\frac{nb}{(1 - a\Sigma)^2}\ddot{E}.$$

The solution of the two equations gives for the rotation per unit length

$$\delta = -\frac{4\pi nb}{(1 - a\Sigma)^2}\frac{\nu^2}{c^2}. \tag{13}$$

According to Lorentz, equation 2, the second field equation remains unchanged and the rotation should be half as great.

Antipodes.

It may be shown that antipodes rotate in opposite directions. The antipode of the present general molecule of five atoms may be easily treated by interchanging the positions of atoms 4 and 5 with respect to the xz-plane but letting each atom keep the same absolute valueof coördinates. If the present system of coördinates is changed into a left-handed system by reversing the y and y' axes and measuring θ and β in a left-handed sense, then all equations will hold identically in the left-handed system; so $\overline{p_y'}$ will be given by equation 12. But, with respect to the light wave, y' is now in the opposite direction to what it was for the first molecule, and the rotation will be opposite in sign but of the same magnitude as before.

The Symmetrical Molecule.

The symmetrical molecule may be most easily considered by letting atoms 4 and 5 be identical; then

$$x_4 = x_5$$

$$|y_4| = |y_5|$$

$$z_4 = z_5$$

$$K_4 = K_5.$$

From a consideration of the general determinant it may be shown that a great many of the Rs in Eq. 12 vanish and the remainder cancel out leaving $\overline{p_y'} = 0$. So a symmetrical molecule cannot cause rotation.

Other Molecules.

The case of a molecule containing less than five atoms is obtained by setting $K = 0$ for one or more atoms. It is found that:

A molecule of only two atoms cannot be active.

A molecule of three atoms cannot be active.

An asymmetrical molecule of four atoms would be active.

However, a molecule of this type is not known. Every known molecule of four atoms consists of a central trivalent atom with three other atoms attached to it; thus all four atoms probably lie in the same plane or at least oscillate back and forth through the same plane. In either case the molecule would be inactive. This is supported by the failure to resolve such compounds into isomeric forms.

A treatment similar to the present could be extended to molecules of more than five atoms; and from analogy with the present case it seems probable that an asymmetric molecule will be active and a symmetrical molecule inactive.

Rotatory Dispersion.

The K for an atom is the polarization of the atom by unit electric field and it may be shown that

$$K_n = \Sigma \frac{e^2/m}{\nu_0^2 - \nu^2},$$

where the summation is taken over all electrons in the nth atom. Thus K will vary with the frequency of the wave and cause rotatory dispersion. At an absorption band whose origin is in an active molecule, one of the Ks will pass through anomalous values and change sign. The factor b in equation 13 is an algebraic function of the Ks; thus we might, in general, expect the rotation to be anomalous at a band, but not necessarily in the simple manner demanded by the Drude theory. This is in agreement with the rotatory dispersion of the metallic tartrates and malates. It is conceivable that the general rotatory dispersion of a substance might prevent an absorption band from producing an anomaly just as in the case of ordinary dispersion a band does not always produce anomalous dispersion.

Bruhat[1] has shown that the term $(1 - \Sigma)^2$ in Eq. 13 cannot cause an appreciable anomaly in ordinary cases. The absorption coefficient must be of the order of that for metallic absorption before this term becomes important.

Rotation and Atomic Refractivity.

The Ks are closely connected with atomic refractivities. The molecular refractivity of a substance is defined as

$$M = \frac{m}{d}\,\frac{n^2 - 1}{n^2 + 1} = \text{constant.}\ \Sigma\frac{e^2/m}{\nu_0^2 - \nu^2},$$

where m is the molecular weight; d, the density; and n is the refractive index. The atomic refractivity is the contribution of each atom to this sum. In many organic compounds the atomic refractivities are approximately additive and, in such cases, the Ks are proportional to the atomic refractivities. In complex molecules it often happens that the atomic refractivities are not additive, but the summations over closely built groups are additive, and in such cases we may think of K for the whole group as being proportional to its group refractivity. There should thus be a close connection between atomic or group refractivity and optical activity; a radical with a large group refractivity should exert a greater influence upon the activity of a molecule than a radical with a smaller group refractivity.

This is well shown by the influence of the double ethylene bonding $C = C$ upon the activity of a molecule. It is well known that an ethylene group introduced into a molecule produces an abnormally high molecular refractivity, and Walder and Zilnsky[13] have shown that such a group also tends to increase the molecular activity when introduced into a molecule.

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[1] "Stereo Chemistry," Stewart, 98.

[2] Bruhat, Phil. Mag., 28, 302, 1914.

These considerations bring out clearly the reason for the failure of the Guye hypothesis[1] in so many cases. Roughly, this hypothesis states that the optical activity of a molecule is measured by its mass asymmetry. From the present theory it is evident that it is not the mass asymmetry that is important but the asymmetry in terms of atomic or group refractions that is important.

Change of Rotation with Temperature.

The activity of most substances varies rapidly with the temperature;[4] for some substances it increases; for others, it decreases and may even change sign. No attempt has been made to explain these bewildering changes, because of the lack of any adequate theory of rotation itself. There can be little doubt that, on account of thermal agitations, the atoms are vibrating about points of equilibrium inside of the molecule. However, on the Guye hypothesis, it was difficult to see how such oscillations could affect the rotation, for on the average the atom would act as if it were fixed at the position of equilibrium.

This is not true from the present point of view. The electrostatic action of two atoms varies as $1/d^3$. For such a law of force the average interaction will change with the amplitude of the oscillations and thus cause the rotation to vary with the temperature.

Without doubt other factors such as loose chemical changes also enter into the temperature effect.

PART II. EXPERIMENTAL.

On the Drude theory, only those electrons which vibrate along spiral paths give rise to anomalous rotation; on the present theory, any absorption band belonging to an electron in an active molecule might be accompanied by an anomaly. The work of Tuchugoff and Ogordnikoff referred to in the first part of this paper may be looked upon as supporting this view. It is well known that certain atomic groups called chromophores act as absorbing centers and that a colored compound is produced when one is introduced into the molecule of an otherwise colorless substance. These two experimenters have united non active radicals containing the chromophore $C = S$ with complex active radicals, and they find that the absorption bands of the resulting compounds show anomalous rotation. Although the absorption in such cases is always connected with the chromophores, the appearance and position of the band is characteristic of the molecule. So it was decided that a more severe test of the theory was desirable, and this test was carried out in the following manner.

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[1] [1]Landolt, "Optical Rotation of Organic Substances," 299.

The group of neodymium absorption bands near 5,800 and 5,200 are characteristic of the neodymium atom and are not greatly influenced by the molecule in which the atom is present. The electrons causing these bands do not vibrate along spiral paths, for such compounds as neodymium nitrate are inactive. If this atom is united with an active acid radical, will these electrons cause anomalous rotation? According to the present theory they might; while, according to the Drude theory it is difficult to see how they could cause anomalies, because they are little influenced by molecular structure.

Most organic salts of the rare earths are insoluble, but a soluble active compound of neodymium was finally obtained in the following manner. Neodymium nitrate was dissolved in a neutral solution of sodium tartrate and a crystalline substance separated out slowly. At the end of a week this was filtered off and was found to be soluble in a basic solution. Using sodium hydride the solution was made just basic enough to dissolve all the crystals. The solution was very dilute. The compound is probably a double tartrate of neodymium and sodium. It may be mentioned that the original nitrate was not very pure as it contained some lanthanum; however its presence was not objectionable in the present work.

The solution is strongly absorbing and it was difficult to measure the rotation inside of the absorption bands. As the general rotation for 8 cm. of the solution was only one degree and the absorption bands are relatively broad compared to those of a gas, it was impossible to use the quartz wedge method. It was finally decided to use a monochromatic illuminator with a divided field.

The usual form of monochromatic illuminator is not suitable for the purpose, because the maximum brightness that can be obtained for the divided field does not depend on the ratio of focal length to aperture of the collimating lens but only upon the dispersion and aperture of the prisms.[1] After several failures with different forms of apparatus, a powerful and rather simple instrument was constructed. The optical system is shown in Fig. 3. Two high dispersion prisms were used with auto-collimation to double the dispersion. The visible spectrum was spread out over almost thirty degrees of arc. The prisms were set at minimum deviation for the band to be examined and the spectrum was moved across the slit S_2 by rotating the mirror M. The collimating lens L_1 had a focal length of 52 cm. The lens L_2, of 28 cm. focal length, formed an image of a point behind the collimating lens on the divided field F and thus gave it uniform illumination. The image of the slit

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[1] Elias, Zeit. fur Instrumentenkunde, 31, 137, May, 1911.

S_2 fell upon the pupil of the eye placed at the diaphragm D. The divided field was viewed directly without the aid of an eye piece, thus saving a loss of light by reflection. Maximum brightness was secured by making the apparent size of the illuminated area of F only just large enough for settings to be made and by always keeping the slits S_1 and S_2 of the same width. A cell 8 cm. long and containing the active solution was placed at C.

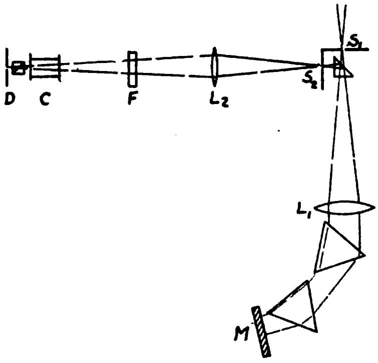

Fig. 3.

Scattered light was carefully screened out. It is impossible to get rid of all the light reflected from the collimating lens, but with the present arrangement all such light goes to form a small image of the slit, which appears as a speck in the divided field and thus causes no error.

An intensely bright field could be obtained, monochromatic to 2A°; but the absorption of the neodymium solution was so great that it was necessary to use 10 A° even with the brightest sun light. The 10 A° means the range of wave-lengths that could pass through the slit S_2 when the width of S_1 and S_2 were equal.

To prove that an apparent anomaly could not result from stray light or too great a slit width, the following test was made. A cell full of the neodymium solution was placed as an absorption cell just behind the slit of the monochromatic illuminator. An active sugar solution examined under these conditions showed no false anomaly, thus proving that there was no serious error from the two causes just named.

In the case of the original neodymium nitrate the absorption band in the yellow could be resolved into five lines: 5,716, a pair of strong lines 5,744 and 5,750, 5,784 and a weak line 5,807. There were two strong lines at 5,219 and 5,202. In the double tartrate the band in the yellow was shifted about 30 A° toward the red, and it could again be resolved into five lines: 5,733 weak, 5,770, 5,800 very strong, 5,819, 5,837. The band in the blue was shifted to 5,364 and could no longer be resolved into two components. Such changes as these are to be expected for the same reason that a change of solvent will shift and change the character of the bands. However, the bands are so characteristic of the atom that it is to be supposed that no great change takes place in the mode of vibration of the electrons.

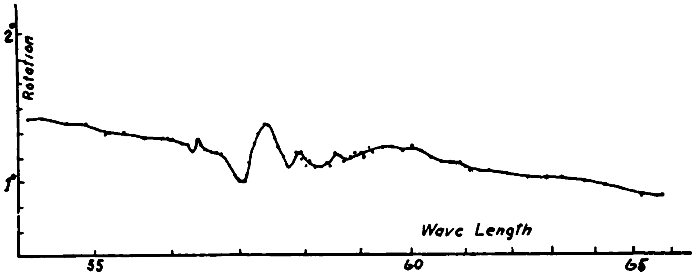

Fig. 4.

The curve shows the rotation of 8 cm. of the solution near the band 5,800.—With this depth of solution the five lines appear merged together. —The rotation is anomalous as predicted from the present theory. The large anomaly is apparently connected with the line 5,770 which is relatively weak. There are indications of anomalies in the strong absorption region covered by the three lines on the long wave-length side of the band. It is possible that larger anomalies really exist and that they have been smoothed over on account of the slit width of the illuminator. However, if all the lines tend to have anomalies of the same type, we should expect the effect to cancel out for such closely spaced lines. A small anomaly was found at 5,642, and subsequent examination showed a very faint absorption band at that wave-length. The strong band near 5,200 also showed an anomaly of the same type as the one at 5,800, but the absorption is too great for a satisfactory curve to be made.

As mentioned before, these results are in agreement with the present theory, but could hardly be explained on the Drude theory of spiral paths.

In conclusion the writer wishes to thank Professor Lehner, of the chemistry department, for his assistance in preparing an active compound of neodymium.

Summary.

Assuming that the atoms of a molecule may be electrically polarized by a light wave and that they then interact according to the ordinary laws for an electric doublet, it has been shown that the optical activity of isotropic substances may be explained as a result of such interactions.

From this view point, an asymmetric molecule will be active and a symmetrical molecule inactive. A molecule must have more than three atoms to be active.

A connection has been shown between molecular activity and atomic refractivity.

A possible explanation of the variation of rotation with temperature has been given.

According to the present view of activity any absorption band belonging to an active molecule might cause anomalous rotation.

This last statement has been tested experimentally in the most extreme case that could be obtained. A neodymium atom was introduced into an active molecule and the rotation was found to be anomalous at the bands characteristic of the atom, a result which supports the present theory.

Note.—After the theoretical part of the present paper had been completed and while the experimental work was being carried out, two excellent articles were published on the same subject by J. Stark and M. Born. Stark[1] suggested that natural activity might be explained as the result of the interaction of doublets; and later Born[1] published a mathematical treatment of the same subject. The present paper is simpler and therefore less rigorous. It may also, in a way, be looked upon as a special case of Born's more general theory; instead of an indefinite coupling the atoms have been coupled in a very definite manner. There are two important differences: (1) Born views a molecule as a system of coupled electrons, the coupling and the restoring forces being identical. In the present paper the atom is looked upon as a particle of dielectric. The atoms are coupled according to the ordinary laws for a doublet, and the restoring force on an electron is not identical with the coupling but may be influenced by it. This is supported by the fact that many absorption bands are roughly characteristic of the atom and that for many organic compounds the refractive index is an almost additive property. (2) Born neglected the retardation of the interaction between particles. This retardation might be important because the rotation depends only on a space derivative of E, *i. e.*, upon the variation of E from one side of a molecule to the other. However, the conclusions agree with those of Born, and the experimental result is a support to both theories.

In the Annalen der Physik, 48, 1, 1, September, 1915, C. W. Oseen has also presented a theoretical treatment of optical activity, very similar to that of Born.

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[1] Stark, Jahrbuch der Rad. und Elektronik., 1914.

[2] M. Born, Phys. Zeit., 16, 251, July, 1915.

THE THEORY OF IONIZATION BY COLLISION.

I. The Distribution of Velocities of the Electrons.

By K. T. Compton.

Introduction.—The fundamental equation in any theory of ionization of gases by impact is

$$n = n_0 \frac{(\alpha - \beta)\epsilon^{(\alpha-\beta)d}}{\alpha - \beta\epsilon^{(\alpha-\beta)d}}, \tag{1}$$

where n_0 is the number of negative ions which start per second from the negative electrode, n is the number per second which reach the positive electrode and α and β are the average numbers of new ions of either sign formed per centimeter path by each negative and positive ion respectively. In many cases the ionization is due almost entirely to the negative ions and equation (1) reduces to

$$n = n_0\epsilon^{\alpha d}. \tag{2}$$

These equations were first proposed by Townsend[1] and are correct on any theory of ionization due exclusively to collisions. They afford no information concerning the mechanism by which impacts may result in the formation of new ions.

Attempts to derive, from assumptions regarding the nature of ionization, expressions for α and β in terms of properties of the colliding particles have not been very successful. Townsend[2] assumed that collisions of electrons with molecules are inelastic, that ionization occurs at an impact if the velocity of the electron exceeds a critical value characteristic of the gas and that no appreciable recombination or formation of "clusters" occurs within the range of electric fields and pressures used in the experimental tests. The resulting formula for α agreed so well with the facts then known that these assumptions have been rather generally accepted as at least approximately true. However we know now that the constants in Townsend's formula differ from the correct values in most cases by more than a hundred per cent., being in some cases too large and in others too small. Moreover the equation is not successful for small fields and in the case of helium (and probably the other inert gases) the plotted curves are not even of the right general shape.

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[1] "Ionization of Gases by Collision," pp. 4, 41.

[2] Ibid., p. 23.

Campbell[1] recently pointed out an error in the derivation of Townsend's formula. The corrected equation conforms slightly better to the experimental results, but the difference is not great and the constants are still far from correct.

The most satisfactory equation for α which has been proposed was derived by Bergen Davis[1] on assumptions similar to Townsend's except that the necessary condition for ionization at an impact is that the colliding electron must possess a velocity whose component normal to the surface of the molecule (considered spherical) exceeds a critical velocity characteristic of the gas. For many gases this formula fits the facts acceptably over a wide range of pressures and gives much more accurate values of the constants than does Townsend's formula. But it also fails in the case of helium and the other inert gases. Moreover it will be shown that certain conditions exist in the ionized gas which necessitate a modification of Bergen Davis's formula.

It appeared to the writer that a study of the distribution of velocities of the ions in the gas under various conditions should be of value in forming a theory of ionization by collision. The present paper gives the more important results of a study of the velocity distribution. On this foundation expressions for α have been derived which conform more nearly to the facts than any formula yet suggested and which may be applied to the inert gases and to mixtures of gases. The derivation and discussion of these equations will be given in a later paper.

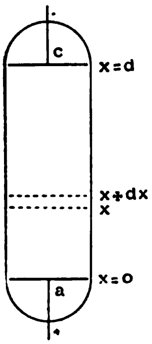

Fig. 1.

Notation.—For convenient reference the uniform notation employed is explained here. Let c be the cathode and a the anode in a discharge tube. By ultra-violet light, or otherwise, electrons are liberated from c and move toward a under the influence of the applied field. We shall let

x = distance from anode a measured in the direction of the potential drop;
d = distance between anode a and cathode c;
X = electric intensity (supposed uniform);
n_0 = number of electrons leaving c per second;
n = number of electrons passing plane x per second;
n_a = number of electrons reaching a per second;
ν = average number of collisions made by an electron in traveling one centimeter in the direction of the electric force;

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[1] Phil. Mag., 23, p. 400 (1912).

[2] PHYS. REV., 24, p. 93 (1907).

P = probability that an electron ionizes when it collides;
P_x = probability that an electron goes a distance x without being stopped by a collision;
α = number of ionizing collisions by an electron per centimeter path;
β = number of ionizing collisions by a positive ion per centimeter path;
p = pressure of the gas;
e, m = charge and mass of an electron;
$F(v)dv$ = probability that an electron has a velocity between v and $v + dv$;
$F(x)dx$ = probability that it has moved freely a distance between x and $x + dx$;
$F(V)dV$ = probability that it has a velocity due to a potential drop between V and $V + dV$;
n', ν', P', P_x' are the corresponding quantities which refer to the positive ions.

It has been recently shown,[1] as was previously suspected, that in some gases the electrons start approximately from rest after each collision while in others their kinetic energy is retained except at collisions which result in ionization. We shall call these cases of inelastic and elastic impact, respectively, and shall consider first the case of ionization by electrons only, which occurs when X/p and d are small. We shall be safe in assuming that no appreciable recombination or formation of "clusters" occurs within the range of pressures and electric fields employed in measurements of ionization by collision.[2]

A. Inelastic Impact.

(1) *Ionization Due Exclusively to Electrons.*

Consider the electrons colliding and originating in a layer dx distant x from the anode a (Fig. 1). The number of collisions per second in this layer is $n\nu dx$. Of these collisions, $Pn\nu dx$ result in the formation of new ions. Thus $(1 + P)n\nu dx$ ions start from rest in the layer dx each second. According to the kinetic theory of gases the probability that one of these will reach the anode a without further collisions is

$$P_x = \epsilon^{-\nu x}. \tag{3}$$

Thus

$$n_a F(x)dx = (1 + P)n\nu\epsilon^{-\nu x}dx$$

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[1] Franck and Hertz, Verh. d. D. Phys. Ges., 15, p. 373 (1913); 16, p. 457 (1914).

[2] Townsend, Phil. Mag., 23, p. 856 (1912).

is the number of electrons reaching a per second after having traveled freely a distance between x and $x + dx$.

But in the layer dx the total number of electrons has been increased by the number $dn = -P n \nu dx$, whence

$$n = n_a \epsilon^{-P\nu x}. \tag{4}$$

Substituting this value of n we obtain

$$F(x)dx = (1 + P)\nu\epsilon^{-(1+P)\nu x}dx. \tag{5}$$

These reach a with a velocity determined by the relations

$$\tfrac{1}{2}mv^2 = eV = eXx; \tag{6}$$

whence

$$F(V)dV = (1 + P)\,\frac{\nu}{X}\,\epsilon^{-(1+P)\frac{\nu V}{X}}\,dV, \tag{7}$$

and

$$F(v)dv = (1 + P)\,\frac{\nu m v}{eX}\,\epsilon^{-(1+P)\frac{\nu m v^2}{2eX}}\,dv. \tag{8}$$

Now it is a well-known theorem in the kinetic theory of gases that the molecules which pass a plane in a gas have components of velocity normal to the plane whose distribution is given by an equation of the form

$$F(u)du = Au\epsilon^{-Bu^2}du.$$

By comparison with equation (8) it is seen that the electrons in the discharge tube reach the anode a with velocities distributed according to Maxwell's law. The average velocity with which they arrive is

$$v = \int_0^\infty vF(v)dv = \sqrt{\frac{\pi eX}{2(1 + P)\nu m}}$$

and the most probable velocity is

$$v = \sqrt{\frac{eX}{(1 + P)\nu m}}. \tag{9}$$

We have assumed here that the discharge tube is of infinite length. If it is of length d the initial velocity distribution $f(v)dv$ at c must be considered and the probability of ionization at a collision cannot be considered independent of x. Under these conditions an argument similar to that used to derive equation (8) leads to

$$\begin{aligned} F(v)dv = {} & \left[\epsilon^{-\nu\left(d + \int_0^d P dx\right)} f(v - v')\right]_{v=v'}^{v=\infty} \\ & + \left[(1 + P)\,\frac{\nu m v}{eX}\,\epsilon^{-\frac{\nu m}{eX}\left(\frac{v^2}{2} + \int_0^v P v dv\right)}\,dv\right]_{v=0}^{v=v'}, \end{aligned} \tag{10}$$

where v' is the velocity gained in a free passage from c to a. This expression reduces to equation (8) when d and v' are infinite and P is constant. The first term arises from those electrons which pass from c to a without suffering a collision and gives a velocity distribution identical with that of the emitted electrons increased by the amount v'. The second term gives the velocity distribution of those which have undergone collision or been produced in the gas and applies only to velocities less than v'. There is thus a discontinuity in the function $F(v)$ at $v = v'$.

Fortunately equation (10) becomes practically identical with (8) when d exceeds several mean free paths l. I have therefore used equation (8), or its equivalent (7), as the basis of part of the theory of ionization and in applying experimental results as a test I have rejected the few data taken under conditions in which equation (10) differs appreciably from (8). In the remainder of this paper we shall consider only cases in which d is at least several times greater than l.

(2) *Ionization Due Both to Electrons and to Positive Ions.*

In this case we have, as before, $(1 + P)n\nu dx$ electrons starting from rest in the layer dx every second as the result of collisions by electrons in this layer. In addition $P'n'\nu'dx$ electrons are formed per second by the impacts of positive ions. Thus $[(1 + P)n\nu + P'n'\nu']dx$ electrons start from rest in dx each second. Therefore

$$n_aF(x)dx = [(1 + P)n\nu + P'n'\nu']e^{-\nu x}dx \qquad (11)$$

electrons arrive at a from dx each second.

But

$$dn = -[Pn\nu + P'n'\nu']dx = -[Pn\nu + P'(n_a - n)\nu']dx,$$

since of necessity $n_a = n + n'$ at any point in the gas. Integrating this equation we find

$$n = \frac{n_a}{(P\nu - P'\nu')}[P\nu\epsilon^{-(P\nu - P'\nu')x} - P'\nu'],$$

from which

$$n' = \frac{n_a}{(P\nu - P'\nu')}[P\nu - P\nu\epsilon^{-(P\nu - P'\nu')x}].$$

Substituting these values of n and n' in equation (11) and regrouping the terms we find that

$$F(x)dx = \frac{1}{(P\nu - P'\nu')}\{[(1 + P)P\nu^2 - P\nu P'\nu']\epsilon^{-[(1+P)\nu - P'\nu']x} - P'\nu'\nu\epsilon^{-\nu x}\}dx. \qquad (12)$$

This equation of course reduces to (5) when $P' = O$ or $\nu' = O$.

By substituting for x from equation (6) we find that the distribution of velocities is given by

$$F(v)dv = \frac{(1 + P)P\nu^2 - P\nu P'\nu'}{P\nu - P'\nu'} \frac{mv}{eX} \epsilon^{-[(1+P)\nu - P'\nu'] \frac{mv^2}{2eX}} dv$$

$$- \frac{P'\nu'\nu}{P\nu - P'\nu'} \frac{mv}{eX} \epsilon^{-\nu \frac{mv^2}{2eX}} dv, \tag{13}$$

which is the difference between two Maxwell's distributions and is not itself a Maxwell's distribution except in particular cases. The maximum value of the first term obtains when

$$v = \sqrt{\frac{eX}{m[(1 + P)\nu - P'\nu']}} \tag{14}$$

and of the second term when

$$v = \sqrt{\frac{eX}{m\nu}}. \tag{15}$$

When there is no ionization by collision the second term in (13) vanishes and the first comes to a maximum given by equation (15). In case ionization is due entirely to electrons $P'\nu' = 0$ and equation (13) reduces to equation (8) with the maximum given by equation (9). The most probable velocity is seen to be less than it would be if there were no ionization. This is due to the fact that those ions formed very near the plate a reach it with velocities below the average. Another particular case of interest occurs when $P\nu = P'\nu'$, or when the positive and negative ions are equally efficient in the production of new ions. Then both terms of (13) combine to give a Maxwell's distribution identical with that found when there is no ionization. In this case the number n of electrons passing a plane in the gas in one second is the same for all positions of the plane. Only when the distance d is very small can this case be realized experimentally; otherwise an arc is set up.

B. ELASTIC IMPACT.[1]

Ionization Due Exclusively to Electrons.

The only points in which the treatment of this case differs from that in the case of inelastic impact are in the expressions for the number of electrons starting from rest in any layer and for the probability of reaching a without being stopped. We thus have $2Pn\nu dx$ electrons starting from rest in the layer dx each second and

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[1] This treatment is only approximate, since it does not follow the new-formed electrons in their subsequent history. An exact treatment is given in the following paper.

$$Px = \epsilon^{-P\nu x}.$$

Also

$$dn = -Pn\nu dx,$$

whence

$$n = n_a\epsilon^{-P\nu x}.$$

Therefore

$$F(x)dx = 2P\nu\epsilon^{-2P\nu x}dx. \tag{16}$$

From equations (6)

$$F(V)dV = 2P\frac{\nu}{X}\epsilon^{-2P\frac{\nu V}{X}}dV \tag{17}$$

and

$$F(v)dv = 2P\nu\frac{mv}{eX}\epsilon^{-2P\nu\frac{mv^2}{2eX}}dv. \tag{18}$$

The velocity distribution again conforms to Maxwell's law and the most probable velocity is

$$v = \sqrt{\frac{eX}{2P\nu m}}. \tag{19}$$

By comparison with equation (9) it is seen that the velocities are greater in the case of elastic than in the case of inelastic impact. It will be shown that this accounts quantitatively as well as qualitatively for the relatively large ionization in helium, argon, etc.

A certain amount of caution must be exercised in interpreting ν in these equations. ν refers always to the average number of collisions in one centimeter path in the direction of the electric force. In inelastic impact there is no motion in any other direction except the negligible amount due to the effects of thermal agitation. In the case of elastic impact, however, there is in general a considerable component of velocity perpendicular to the electric force, due to the rebound at elastic collisions. Thus the ν which appears in the formulæ for elastic impact is larger than the reciprocal of the mean free path. Direct measurements of ν in various gases are being made in this laboratory.

C. IONIZATION IN A MIXTURE OF ELASTIC AND INELASTIC GASES.

Let P_e and P_i be the probabilities of ionization at an impact of an electron with molecules of the elastic and inelastic gases respectively, and let ν_e and ν_i be the average numbers of collisions with molecules of these gases in one centimeter path of an electron. The relative magnitudes of ν_e and ν_i will vary according to the proportions in which the gases are mixed.

Following the same reasoning as before we find that $n[(1+P_i)\nu_i + 2P_e\nu_e]dx$ electrons start from rest in the layer dx each second. $\epsilon^{-\nu x}$

is the probability of reaching a without an inelastic collision and $\epsilon^{-P_e\nu_e x}$ is the probability of arriving without having been stopped by an ionizing elastic collision. Thus

$$P_x = \epsilon^{-(\nu_i + P_e\nu_e)x}.$$

Also

$$dn = -\,(P_i\nu_i + P_e\nu_e)n\,dx,$$

whence

$$n = n_a\epsilon^{-(P_i\nu_i + P_e\nu_e)x}.$$

Therefore

$$F(x)dx = [(1 + P_i)\nu_i + 2P_e\nu_e]\epsilon^{-[(1+P_i)\nu_i+2P_e\nu_e]x}dx, \tag{20}$$

$$F(V)dV = \frac{(1 + P_i)\nu_i + 2P_e\nu_e}{X}\epsilon^{-\frac{(1+P_i)\nu_i+2P_e\nu_e V}{X}}dV, \tag{21}$$

$$F(v)dv = [(1 + P_i)\nu_i + 2P_e\nu_e]\frac{mv}{eX}e^{-[(1+P_i)\nu_i+2P_e\nu_e]\frac{mv^2}{2eX}}dv. \tag{22}$$

In this case also the distribution follows Maxwell's law. Equation (22) reduces to equation (8) when $P_e\nu_e = O$ and to equation (18) when $\nu_i = O$.

Expressions for $F(v)dv$ similar to equation (13) may easily be obtained to apply to cases of elastic impact by both positive and negative ions. Since there is yet no evidence of elastic impact by positive ions it has not been thought worth while to include a discussion of this possibility.

The importance of equation (21) will be suggested in a following paper in which it will be shown that the expression for α derived from equation (21) explains the extreme electrical sensitiveness of helium and argon to the slightest traces of ordinary gases and that it accounts for the departure from theory of many experimental results.

PALMER PHYSICAL LABORATORY,
PRINCETON, N. J.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

THE TRUE TEMPERATURE SCALE FOR TUNGSTEN AND ITS EMISSIVE POWERS AT INCANDESCENT TEMPERATURES.[1]

BY A. G. WORTHING.

THE experimental part of the work here reported consisted in measuring the brightness of an incandescent, hollow, cylindrical, tungsten filament perforated with small holes and mounted in a large lamp bulb. Such determinations were first made for a hole and then for the adjacent surface. The ratio of the latter brightness to the former, when corrected for the difference in temperature between the interior and the surface and for the departure from blackness of the radiation from the interior due to the presence of the small hole and any lack of symmetry in the filament temperature distribution, gives the emissive power for the substance at the surface temperature. Because the radiation from tungsten deviates from Lambert's cosine law, the emissive power has in the present work been so limited as to refer only to that radiation which leaves the surface normally.

Black-body temperatures were obtained by comparing the observed brightness, with the aid of Wien's radiation formula, with that of a black body at the palladium point. The true temperature scale consists of the relation thus found between the black-body temperature of the filament surface and that of the adjacent hole, *i. e.*, the true temperature of the filament.

A linear relation was found between emissive powers and true temperatures for the region 1200° K. to 3200° K. such that the emissive powers for the two limiting temperatures were respectively .467 and .406. The uncertainty in these values should not be much over 1 per cent. The respective corresponding differences between the red ($\lambda = .666\,\mu$) black body temperatures and the true temperatures are 33° and 375°.

Measurements were attempted on the tungsten arc much after the manner described by Langmuir. Due to large temperature gradients across the molten tungsten no satisfactory determinations could be made of the emissive power. However, it was noticed, contrary to what was concluded by Langmuir, that the brightnesses of molten and of solid tungsten at the melting point are nearly identical. Assuming the linear relation between emissive power and temperature for solid tungsten to continue up to the melting point, it follows that this occurs at 3630° K. This corresponds to a red black body temperature of 3140° K. and an emissive power of .390.

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[1] Abstract of a paper presented at the Columbus, O., meeting of the Physical Society, December 27–30, 1915.

Though the results given above are believed to be dependable within reasonable uncertainties of measurement, further verification with other filaments will be attempted previous to the publication of the completed paper.

Nela Research Laboratory,
National Lamp Works of General Electric Company,
Nela Park, Cleveland, Ohio.

The X-Ray Spectrum of Tungsten.[1]

By Arthur H. Compton.

As a source of X-rays for the examination of the spectrum of tungsten, a Coolidge X-ray tube with a tungsten anti-cathode was used. The tube was excited by means of a transformer with a rectifying device. The spectrum was analysed by means of a recording X-ray spectrometer, the intensity of the beam reflected from the crystal being determined by an ionization method with the help of a highly sensitive electrometer.

In addition to the five well-known lines of the characteristic L-radiation from metals, the spectra obtained verify the two new lines discovered by Barnes[2] in the spectrum of tungsten, and indicate the existence of five other lines. They also show that one of the lines which has been thought to be single is really a close double. This gives a total of 13 lines observed in the L-radiation from tungsten, 2 of which are, however, somewhat uncertain.

Line.	Glancing Angle.	$\lambda \times 10^{-8}$ Cm.	Barnes' Values.[1]	Remarks.
a	9° 44.5′	1.0387		
b	9° 53.1′	1.0539		Line somewhat doubtful.
c_1	10° 3.7′	1.0725	1.082	Very close.
c_2	10° 7.7′	1.0796		Double.
d	10° 25.7′	1.1107	1.113	Strong.
e	11° 36.3′	1.2349		
f	11° 49.9′	1.2587	1.258	Strong.
g	12° 0.4′	1.2771	1.277	
h	12° 11.4′	1.2962	1.296	Strong.
i	12° 22.7′	1.3160	1.312	
j	12° 44.7′	1.3543		Line uncertain.
k	14° 4.7′	1.4933	1.477	Moseley gives 1.486.
l	14° 11.1′	1.5044		

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, February 26, 1916.

[2] J. Barnes, Phil. Mag., 30, p. 368, 1915.

The wave-lengths of these lines have been determined by reflection from a cleavage plane of calcite. The angles and corresponding wave-lengths are given in the accompanying table. The probable erro rin the observed angle for the more intense lines is less than ± 0.5′, but for the fainter lines is somewhat greater.[1]

Palmer Physical Laboratory,
Princeton University,
February 12, 1916.

Note on the End Effect in the Electrostriction of Cylindrical Condensers.[2]

By Edwin C. Kemble.

The electric field in the neighborhood of the ends of the coatings of a cylindrical condenser exerts a longitudinal traction

$$P = \frac{ad\delta_2 H^2}{4}$$

by direct action on the dielectric. Here,

a = internal radius of cylinder,
d = thickness of cylinder,
H = average electric force,
δ_2 = constant depending on the change of the inductivity of the dielectric by strain.

There is also a force exerted by the electric field on the edge of the coating. Its value is

$$P^1 = \frac{adKH^2}{4}$$

where K is the inductivity of the dielectric.

By taking these forces into account we can bring the formulas for the elongation of charged cylindrical condensers derived by Sacerdate[3] by the "energy method" and by Adams[4] by the "elasticity method" into complete agreement. The discrepancies are explained both for the case of adherent and for that of non-adherent coatings.

Harvard University,
Cambridge, Mass.

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[1] Note added March 30, 1916: The wave-lengths given in this table are calculated using for the distance between the layers of atoms in calcite the value 3.0695×10^{-8} cm. As will be explained elsewhere, this grating space is rather 3.0279×10^{-8} cm., so that the wave-lengths here given should be divided by the factor 1.0135.

[2] Abstract of a paper presented at the New York meeting of the Physical Society, February 26, 1916.

[3] Journ. de Phys. [3], 8, p. 457, 1899, and [3], 10, p. 196, 1901.

[4] Phil. Mag. [6], XXII., p. 889, 1911.

ON POSSIBLE PLANETARY MAGNETIC EFFECTS.[1]

BY L. A. BAUER.

THE question of the existence of cosmical effects arising from possible planetary magnetic fields is one engaging the attention at present of various investigators, especially of astronomers. For example, it has been known for some time that there are certain irregularities in the motions of the moon too large to be accounted for apparently by any known outstanding gravitation effects. Similar irregularities are shown by the motions of Mercury, Venus, and the earth. It therefore becomes a matter of interest to ascertain, if possible, whether these irregularities in the motions of astronomical bodies can be associated, in any way, with magnetic effects. Accordingly, as a first step, a preliminary formula has been theoretically established for determining the field strength of possible magnetic fields enveloping the members of our solar system for which certain requisite constants are known. Let M_e represent the strength of the earth's magnetic field at its poles, namely, about 0.65 of a C.G.S. unit, and let ω_e, R_e, D_e, G_e represent, respectively, the earth's angular velocity of rotation, mean radius, mean density, and gravity at the surface. Give similar significance to the quantities, M_a, ω_a, R_a, D_a, G_a for any rotating astronomical body (a). Then we may have approximately: $M_a : M_e = (\omega_a R_a^2 D_a) : (\omega_e R_e^2 D_e) = (\omega_a R_a G_a) : (\omega_e R_e G_e)$. If, for example, the astronomical body (a) is the sun (S), we get a field strength (M_S) which agrees within about 10 per cent. of Hale's provisional value ($80 M_e$). For the planets Mars, Jupiter and Saturn, whose angular velocity of rotation is known, we get: $M_m = 0.2\ M_e$; $M_j = 68\ M_e$; $M_s = 24\ M_e$. It thus appears that, on the hypothesis underlying the formula, Jupiter, Saturn, and probably also Uranus and Neptune, may be enveloped by stronger magnetic fields than is the earth. An interesting numerical equality (within about 10 per cent.) was found to exist between the products (ωv), ω being the angular velocity of rotation and v the orbital velocity, in the case of the four planets (Earth, Mars, Jupiter and Saturn) for which both quantities are definitely known.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, February 26, 1916.

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THE

PHYSICAL REVIEW.

THE THEORY OF IONIZATION BY COLLISION.

II. Case of Inelastic Impact.

By K. T. Compton.

In the preceding paper[1] an investigation was made of the distribution of velocities among the ions in a discharge tube when the current through the gas is sufficiently small that the potential gradient may be considered uniform. Townsend[2] has shown that under these conditions α, the average number of new ions formed per centimeter path by the impacts of each negative ion (electron), may be expressed by an equation of the form

$$\frac{\alpha}{p} = f\left(\frac{X}{p}\right), \tag{23}$$

where X is the potential gradient and p the pressure. This relation has been supported by all experiments with gases in which the impacts of electrons with gas molecules are inelastic, *i. e.*, when the electron loses its kinetic energy at each collision.

Various assumptions regarding the mechanism of ionization by collision have led to various forms of the function f. Townsend,[3] assuming that ionization always occurs at an impact when the electron has moved through a potential drop exceeding the minimum value V_0, derived the equation

$$\alpha = \nu e^{-\frac{\nu V_0}{X}}, \tag{24}$$

where ν is the average number of collisions with gas molecules made by each electron in a centimeter path. By assuming arbitrary values of the constants ν and V_0 for each gas this equation may be made to fit the experimental results, but the values of the constants so chosen are at wide variance with the values which have been determined directly. Recognizing this, Townsend has recently[1] suggested that V_0 in his equation be considered as the average rather than the minimum ionizing potential. Under these conditions equation (24) must be considered largely as an empirical equation.

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[1] Phys. Rev., April, 1916.

[2] Ionization of Gases by Collision, p. 17.

[3] *Ibid.*, p. 22.

Much more satisfactory is the relation derived by Bergen Davis[2] on the assumption that ionization occurs at an impact whenever the electron possesses a velocity whose component normal to the surface of the molecule (considered spherical) exceeds the minimum value corresponding to V_0. This equation is

$$\alpha = \nu\left[e^{-\frac{\nu V_0}{X}} + \frac{\nu V_0}{X} Ei\left(-\frac{\nu V_0}{X}\right)\right], \tag{25}$$

where $Ei(\ )$ represents the exponential integral. It will be shown that Bergen Davis' assumptions really require a modification of equation (25). Further discussion of this equation will appear later.

The purpose of this paper is to derive a new expression for α and to test it with such experimental data as are available. It is found to agree well with experimental results as regards both the variation of α with X and the values of V_0 for various gases.

Theory.—The two factors which determine the laws governing the production of ions by impact are the distribution of velocities among the colliding electrons and the function of the velocity which represents the probability that an electron will ionize at an impact. Thus if $F(V)dV$ represents the probability that an electron has fallen through a potential drop between V and $V + dV$ and if $f(V)$ represents the probability that an electron with a velocity V ionizes when it collides, then $f(V)F(V)dV$ is the probability that an electron will possess a velocity corresponding to V and will ionize when it collides. Thus

$$P = \int_0^\infty f(V)F(V)dV$$

is the probability that any electron, chosen at random, ionizes when it collides. This quantity, when multiplied by ν, is equal to α.

$F(V)dV$ is given by equation (7) of the preceding paper.[3] As to the form of $f(V)$, we know that $f(V) = 0$ when $V \leqq V_0$ and that it must approach the value unity when V is large. The simplest form of such a function is one which fits the facts as far as we know them; we shall therefore assume that

$$f(V) = \frac{V - V_0}{V} \quad \text{when} \quad V \geqq V_0. \tag{26}$$

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[1] Electricity in Gases, p. 295.

[2] PHYS. REV., 24, p. 93, 1907.

[3] *Loc. cit.*

This relation is essentially the same as that which follows from Bergen Davis' assumption regarding the necessary condition for ionization at an impact. It may be correct, however, even if the interpretation given by Bergen Davis is not true. The bearing of certain experiments by Glasson[1] on this point will be discussed later.

If we make these substitutions in the equation for P we obtain

$$P = \int_{V_0}^{\infty} (1 + P)\frac{\nu}{X}\,\frac{V - V_0}{V}\,e^{-(1+P)\frac{\nu V_0}{X}}\,dV,$$

of which the solution is

$$P = e^{-(1+P)\frac{\nu V_0}{X}} + (1 + P)\frac{\nu V_0}{X}\,Ei\left(-(1 + P)\frac{\nu V_0}{X}\right),$$

where $Ei(\ )$ denotes the exponential integral. Thus we find

$$\alpha = P\nu = \nu\left[e^{-(1+P)\frac{\nu V_0}{X}} + (1 + P)\frac{\nu V_0}{X}\,Ei\left(-(1 + P)\frac{\nu V_0}{X}\right)\right], \quad (27)$$

which conforms to equation (23) since ν is proportional to p.

Discussion of Equation.—Equation (27) differs from the expression given by Bergen Davis by the presence of the factor $(1 + P)$ whose omission from his equation results from neglecting to consider the effect of the newly formed ions on the velocity distribution.

It is impossible to solve equation (27) for α or P explicitly. However P may be found in terms of $(1 + P)(\nu V_0/X)$ (see Làska's "Sammlung von Formeln"), from which I have computed the following table of values:

Table I.

P	$\frac{\nu V_0}{X}$	P	$\frac{\nu V_0}{X}$	P	$\frac{\nu V_0}{X}$	P	$\frac{\nu V_0}{X}$
0.00099	5.00	0.0731	1.446	0.3893	0.2880	0.725	0.0580
0.0032	3.99	0.1485	0.872	0.518	0.1645	0.804	0.0333
0.0106	2.97	0.2008	0.666	0.574	0.1271	0.854	0.0216
0.0375	1.963	0.2762	0.4705	0.656	0.0846	1.000	0.0000

In order to test equation (27) P was plotted as a function of $\nu V_0/X$, whence P, and therefore α, could be determined graphically for any given value of $\nu V_0/X$.

Comparison with Experiment.—(*a*) *Variation of α with X.*—Numerous experiments on the variation of α with X in various gases contained between parallel plate electrodes afford a means of testing equation (27). Of the available data, those for nitrogen, carbon dioxide, hydrochloric acid and water vapor are found to be in excellent agreement with equation

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[1] Phil. Mag., 22, p. 647, 1911.

(27). In helium, argon and hydrogen the collisions are entirely or partly elastic and equation (27) cannot be applied. They will be considered in a subsequent paper.

In selecting experimental data I have taken the results of the most recent, or otherwise apparently most reliable, experiments on each gas. All results which are appreciably affected by the error in such measurements pointed out by Farwell,[1] or for which there is an appreciable difference between the velocity distributions given by equations (8) and (10) of the preceding paper, have been rejected. In view of the labor of gathering and testing the available data it may be of value to include them here. The data given in Table II. have been reduced by equation (23) so as to apply to a pressure of 1 mm.

TABLE II.

$p = 1$ mm.

N_2[2]		CO_2[3]	
X	α	X	α
43.75	0.021	87.5	0.62
87.5	0.337	175	2.32
131	0.96	350	5.65
175	1.77	700	10.64
262	3.59	875	11.83
350	4.42	1050	12.68
525	6.41		
H_2O[4]		HCl[5]	
32.5	0.02	55.5	0.12
54	0.29	68.5	0.41
65	0.57	69	0.36
97.5	1.25	103	1.10
103.5	1.41	104	1.38
107.5	1.56	112.5	1.36
134	2.25	116	1.66
163.5	2.80	138	2.30
205	3.79	172	3.38
221	4.08	203.5	4.23
307	5.49	232	5.08
374	6.23	305	6.96
		357	8.00
		407	9.18
		535	11.20
		715	13.40

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[1] PHYS. REV., 5, p. 149, 1915.

[2] H. E. Hurst, Phil. Mag., 11, p. 535, 1906.

[3] *Ibid.*

[4] J. S. Townsend, Phil. Mag., p. 389, 1903.

[5] *Ibid.*

The exactness with which equation (27) conforms to these data is indicated by Fig. 2. The solid curve in each case represents the graph of equation (27), the constants ν and V_0 being chosen to give the best agreement between the curve and the data. I find that equations (24) and (25), for Townsend's and Bergen Davis' theories, agree nearly, though not quite, as well as the present theory. Possibly all are within the limits of experimental error. But when the values of the constants necessary to bring this agreement are compared with the results of direct measurements there is evident support of the present theory.

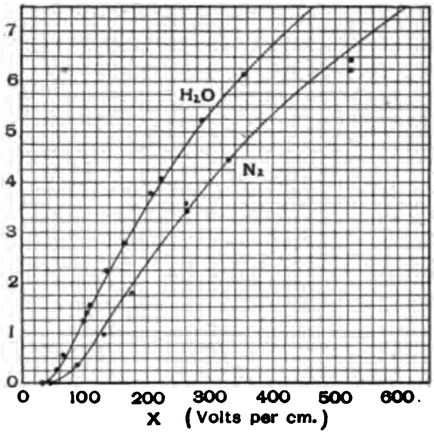

Fig. 2(a).

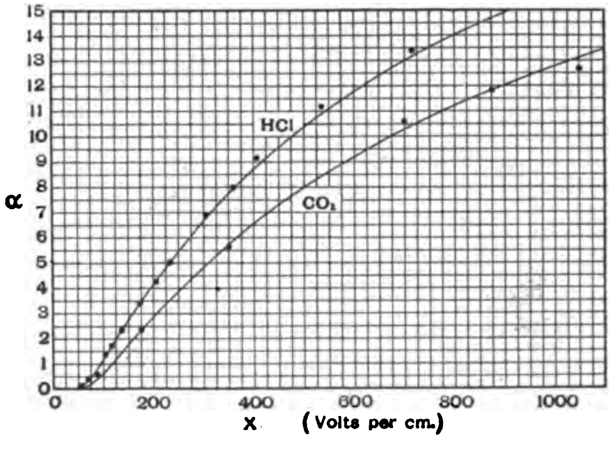

Fig. 2(b).

Values of V_0.—The minimum ionizing potential V_0 for a few gases has been measured directly by Franck and Hertz[1] and by Pawlow.[2] Their values are given in Table III. in the column marked *Exp.* In their experiments the source of electrons was a hot filament, from which the electrons escape with a definite velocity distribution. Pawlow determined this distribution and found that the fastest electrons which he could detect escaped with an initial velocity corresponding to about 2.5 volts. Now as the accelerating potential is raised ionization will first be noticed when these fastest electrons acquire the minimum ionizing velocity. Apparently, however, this initial velocity has not been included in the published values of V_0. It appears to me, therefore, that these values should each be increased by about 2.5 volts. These revised values are placed in the column marked *Exp**. In the remaining columns are given the values of V_0 which cause the curves representing the three theories to accord best with the data in Table II. I have also anticipated the results of the following paper and included the value of V_0 for the elastic gas helium, since the theoretical treatment is exactly the same as in the present paper except for the effect of elastic impact. The results for hydrogen have not been included since in this case impact appears to be partly but not wholly elastic and I have not been able to solve the equation for such a case. Townsend's and Davis' equations do not really apply to the case of elastic impact, and the value for helium calculated from these equations has been included in order to illustrate the important fact that when applied to cases of elastic impact they tend to give values of V_0 which are too small. Since for nitrogen they give values of V_0 which are too large it is not likely that the impacts in nitrogen are elastic enough to make the present treatment appreciably in error.

TABLE III.

Values of V_0.

	Exp.	Exp.*	Equation (27).	Townsend.	Davis.
N_2	7.5	10.0	10.05	27.5	12.36
CO_2			8.2	23.3	9.63
H_2O			7.47	22.4	8.98
HCl			6.05	16.5	7.30
He	20.5	22.5	22.5	12.3	5.6

Where comparison with direct experiment is possible the present theory is seen to fit the facts most acceptably. In fact, if the suggested correction of the experimental data is true, the agreement is almost exact. It is unfortunate that comparison is possible in so few cases. It is known however, that CO_2, H_2O and HCl may be ionized with comparative ease and to this extent there is support of the theory.

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[1] Verh. d. D. Phys. Ges., 16, p. 457, 1914.

[2] Roy. Soc. Proc., 90, p. 398, 1914.

Another possible test of these values of V_0 is afforded by photoelectric experiments in which the minimum ionizing potential for a gas may be calculated from the maximum wave-length of light which can ionize the gas. These wave-lengths, however, are so extremely short that no accurate data are available and it can only be said that no existing evidence of this sort casts doubt on the values of V_0 given by the theory.

Bergen Davis[1] has recently modified his equation to apply to the experimental arrangements employed by Franck and Hertz and has found good support of his equation in their experimental data. It is to be noted that their measurements were carried on under conditions in which the factor $(1 + P)$ which appears in equation (27) does not need to be considered, whence the formulæ in Bergen Davis' and the present theories become identical. Unfortunately the lack of a source of electrons of homogeneous velocities complicates the interpretation of such results.

Values of ν.—Another test of the relative merits of the theories is afforded by a comparison of the calculated values of ν with those determined experimentally. From the assumptions underlying the theories ν, for inelastic impact, should equal $\pi R^2 N$, where N is the number of molecules per unit volume and πR^2 is that part of the molecular cross section which is effective in stopping the impinging electrons. In Table IV. I have included the values of ν calculated from the three theories and also calculated on the assumption that R is the known molecular radius.[2]

Table IV.

Values of ν *for* $p = 1$ *mm.*

Gas.	$\pi R^2 N$	Equation (27).	Townsend.	Davis.
N_2	34.5	22.5	12.4	17.6
CO_2	49.5	30.0	20.0	23.2
H_2O	47.0	21.7	12.9	15.7
HCl	46.0	33.3	22.2	26.8
He	13.5	10.0	2.8	3.05

Evidently the part of the molecular cross section which is effective in stopping the electron is less than that which functions in collisions of molecules with each other. One cannot speak very definitely about this point since the meaning of a collision is not entirely clear in either case.

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[1] Phys. Rev., p. 118, 1915.

[2] Kaye and Laby's Tables.

An interesting hypothesis is this: that the explanation of the above results is to be found in the discrete structure of the molecule, for it is conceivable that a rapidly moving electron might penetrate a molecule without colliding with any of its constituent parts. If this explanation is correct, the ratio of the actual values of ν to the values of $\pi R^2 N$ in Table IV. would represent the probability that an electron could not pass through the space occupied by a molecule without colliding with some of its constituent parts.

Glasson has found[1] that, with very high velocities, the number of ionizing collisions per unit path by an electron *decreases* as the velocity increases, which is contrary to what we might expect. In line with the suggestion just made, it is likely that Glasson's results are not due to a decrease with velocity in the probability of ionization at a collision, but rather to an actual decrease in the number of collisions. In other words, we may consider ν to be a function of the velocity such that ν decreases at very high velocities. Direct evidence of this is afforded by the photographs of paths of β particles through gases. The reason appears to be that a molecule has a discrete structure and that an electron has the best chance of getting through freely when its velocity is large compared with the velocities of the constituent parts of the molecule in their orbits. Such a view removes an apparent objection to equation (26).

However this may be, no test of the ionization theories by considerations of ν can be made at this time owing to the lack of experimental measurements of ν. Such measurements are now being attempted in this laboratory.

Further support of the assumptions underlying equation (27) will appear in a following paper on the subject of elastic impact.

PALMER PHYSICAL LABORATORY,
PRINCETON, N. J.

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[1] *Loc. cit.*

THE THEORY OF IONIZATION BY COLLISION.

III. Case of Elastic Impact.

By K. T. Compton.

Introduction.—In the preceding paper[1] there was derived an equation by which α, the average number of new ions of either sign formed per centimeter path in a gas by each electron, could be accurately calculated from a knowledge of the potential gradient X, the minimum ionizing potential V_0 and the average number of collisions ν made by each electron in a centimeter path in the direction of X. This equation was shown to accord with experimental results more accurately than any equation hitherto proposed. It was based on the assumption of inelastic impact between electrons and gas molecules and therefore does not apply to such gases as helium, argon and neon in which the collisions have been shown to be elastic,[2] *i. e.*, the kinetic energy of the colliding electron is retained except at impacts which result in ionization. The problem of deriving a theoretical relation between X, α and ν applicable to these gases is much more difficult than in the case of inelastic impact and has, so far as I am aware, been attempted only by Franck and Hertz[3] who did not obtain an equation to which experimental data could be applied.

In the present paper I have succeeded in deriving, for cases of elastic impact, an equation based on the same assumptions as those underlying the theory for inelastic gases developed in the preceding papers. This equation seems to receive very satisfactory experimental support.

Functional Relation between α, X and p.—Townsend[4] has shown that a relation of the form

$$\frac{\alpha}{p} = f\left(\frac{X}{p}\right)$$

must hold in all cases of ionization of gases by inelastic impact. His argument does not necessarily apply in case the collisions are elastic. It is necessary, therefore, to see whether the same type of relation is true in the case of elastic impact.

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[1] Phys. Rev., this issue.

[2] Franck and Hertz, Verh. d. D. Phys. Ges., 15, p. 373, 1913; 15, p. 613, 1913; 16, p. 457, 1914.

[3] Verh. d. D. Phys. Ges., 16, p. 12, 1914.

[4] Ionization of Gases by Collision, p. 18.

Let ν represent, as formerly, the average number of impacts by each electron while advancing one centimeter in the direction of the electric intensity X. Let ν_1 represent the average number of impacts by each electron in a centimeter path in any direction. ν_1 is the reciprocal of the mean free path of the electron.

In inelastic gases ν equals ν_1, since there is no component of velocity perpendicular to the electric intensity except that arising from thermal agitation, which is negligible in comparison with the velocity due to the electric field. In the elastic gases, however, there is generally a sidewise component; the electron will bound and rebound many times and in all directions. Thus in the present case ν exceeds ν_1 by an amount which we shall now calculate.

During a free path l the electron experiences an acceleration equal to Xe/m in the direction of the electric field. If $\bar{v}$ is the average velocity of an electron just before it ionizes, $\bar{v}/2$ is its average velocity and hence $2l/\bar{v}$ is the mean free time. Therefore

$$s = \frac{1}{2}at^2 = \frac{1}{2}\frac{Xe}{m}\left(\frac{2l}{\bar{v}}\right)^2$$

gives the average distance moved in the direction of the electric field during one free path. The reciprocal of s is ν, whence

$$\nu = \frac{1}{s} = \frac{m\bar{v}^2}{2Xel^2} = \frac{m\bar{v}^2\nu_1^2}{2Xe}.$$

But Xe is the energy acquired by the electron in moving one centimeter in the direction of the field and $\frac{1}{2}m\bar{v}^2$ is the average energy of the electron when it ionizes. The ratio of these quantities evidently equals α, the number of ionizing collisions made per centimeter by each electron. Thus

$$\nu = \frac{\nu_1^2}{\alpha}, \quad \text{or} \quad \alpha = \frac{\nu_1^2}{\nu}. \tag{28}$$

But $\alpha = P\nu$, where P is the probability that an electron ionizes when it collides. Thus

$$P = \left(\frac{\nu_1}{\nu}\right)^2, \quad \text{or} \quad \nu = \frac{\nu_1}{\sqrt{P}}. \tag{29}$$

Since P is always less than unity, ν always exceeds ν_1. In the limit, with so large a field that ionization would occur at every impact, ν_1 would equal ν and P would equal unity, as is otherwise obvious. This shows that no numerical factor has been neglected in equation (29) owing to the use in the above argument of the average velocity rather than the exact velocity distribution.

Now ν_1 is proportional to the pressure p. Also it is obvious that if X varies as p, then ν varies as p. Thus, from equation (28), if X varies as p then α varies as p; whence

$$\frac{\alpha}{p} = \psi\left(\frac{X}{p}\right). \tag{30}$$

We see, therefore, that the type of functional relation between α, X and p is the same for elastic and inelastic gases; the form of the function ψ is of course different in the two cases.

We shall proceed to determine the form of the function ψ for an elastic gas. It will be convenient to deal with ν instead of ν_1. At the end of the argument equation (29) will enable us to replace ν by the more familiar quantity ν_1.

Determination of $\psi(X/p)$.—As in the case of inelastic impact the probability that any electron, chosen at random, ionizes at an impact with a gas molecule is

$$P = \int_0^\infty f(V)F(V)dV, \tag{31}$$

where $f(V)$ is the probability that an electron with a velocity due to a potential drop V ionizes when it collides and $F(V)dV$ is the probability that an electron possesses a velocity due to a potential drop between V and $V + dV$. For the reasons given in the preceding paper we shall take

$$f(V) = 0 \qquad \text{if} \quad V \leqq V_0,$$

$$f(V) = \frac{V - V_0}{V} \quad \text{if} \quad V \geqq V_0. \tag{32}$$

It remains to determine $F(V)dV$. We shall suppose that the mean free path l and the minimum ionizing path x_0 are small in comparison with the distance between the electrodes of the discharge tube, and in testing the equation so derived we shall reject experimental data taken under circumstances in which this restriction can be shown to lead to an appreciable error. Under these conditions we may take P and ν to be uniform throughout the discharge tube.

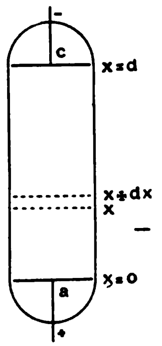

Fig. 3.

Consider the subsequent history of the electrons which start from rest in any layer dx in the gas. (Fig. 3.) Only at an ionizing collision is an electron stopped, whence two electrons start from rest as the result of each ionizing collision in dx. Thus the number of electrons starting from rest in dx per second is $2P\nu n dx$, where n is the number of electrons entering the layer dx per second. The number of new negative ions originating per second in dx is

$$dn = -P\nu n dx,$$

whence

$$n = n_a e^{-P\nu x}. \tag{33}$$

where n_a is the number per second which reach the anode a. Therefore

$$n_0 = 2P\nu n_a e^{-P\nu x}\, dx \tag{34}$$

electrons start from rest in dx each second.

If x_0 is the distance which an electron must go in the direction of the electric force in order to acquire the minimum ionizing velocity corresponding to V_0, it is evident that none of the electrons starting in dx can ionize (or be stopped) in going a distance less than x_0. Thus

$$[F(x)dx = 2P\nu e^{-P\nu x}\, dx]_{x=0}^{x=x_0} \tag{35}$$

applies within the limits noted. In terms of potential drop this is

$$\left[F(V)dV = \frac{2P\nu}{X} e^{-\frac{P\nu}{X}V}\, dV\right]_{V=0}^{V=V_0}. \tag{35}$$

Equation (35) gives the distribution of velocities of the electrons passing any plane in the gas, for velocities less than the minimum ionizing velocity. For all of these $f(V) = 0$, whence they contribute nothing toward the value of P in equation (31).

Consider now the history of those n_0 electrons after they have gone a distance x_0. We shall call $f(x)$ the probability that one of these ionizes at an impact after having gone a distance x from its starting point in dx. $f(x)$ is obviously identical with $f(V)$ and is equal to $(x - x_0)/x$.

Let n_x be the number of the original n_0 electrons which have not been stopped by ionizations in going the distance x from dx. Of these, $\nu n_x dx'$ will collide in the ensuing layer dx'. Thus $f(x)\nu n_x dx'$ of the original n_0 electrons are stopped by ionization in the layer dx' distant x from dx. In other words

$$dn_x = -f(x)\nu n_x dx',$$

whence

$$n_x = n_0 e^{-\int_{x_0}^{x} f(x)\nu dx'} = n_0 e^{-\nu\int_{x_0}^{x}\frac{x-x_0}{x}dx'}$$

$$= n_0\left(\frac{x}{x_0}\right)^{\nu x_0} e^{-\nu(x-x_0)}.$$

Thus

$$\frac{n_x}{n_0} = \left(\frac{x}{x_0}\right)^{\nu x_0} e^{-\nu(x-x_0)}$$

is the probability that an electron, originating in dx, goes a distance x, $(x \geqq x_0)$, without being stopped.

From this quantity and equation (35) we obtain

$$\left[F(x)dx = 2P\nu \left(\frac{x}{x_0}\right)^{\nu x_0} e^{\nu x_0} e^{-\nu(1+P)x} dx \right]_{x=x_0}^{x=\infty}, \tag{36}$$

or

$$\left[F(V)dV = \frac{2P\nu}{X} \left(\frac{V}{V_0}\right)^{\frac{\nu V_0}{X}} e^{\frac{\nu V_0}{X}} e^{-(1+P)\frac{\nu V}{X}} dV \right]_{V=V_0}^{V=\infty}, \tag{36}$$

for the velocity distribution, in terms of potential drop, of those electrons which are able to ionize while passing through any plane of the gas.

Finally, substituting from equations (32) and (36) in (31) we find

$$P = \int_{V_0}^{\infty} \frac{V - V_0}{V} \frac{2P\nu}{X} e^{\frac{\nu V_0}{X}} \left(\frac{V}{V_0}\right)^{\frac{\nu V_0}{X}} e^{-(1+P)\frac{\nu V}{X}} dV,$$

or, in terms of x,

$$P = \int_{x_0}^{\infty} 2P\nu e^{\nu x_0} \frac{x - x_0}{x} \left(\frac{x}{x_0}\right)^{\nu x_0} e^{-(1+P)\nu x} dx,$$

which is easily reduced to

$$\frac{x_0^{\nu x_0}}{2\nu e^{\nu x_0}} = \int_{x_0}^{\infty} x^{\nu x_0} e^{-(1+P)\nu x} dx - x_0 \int_{x_0}^{\infty} x^{\nu x_0 - 1} e^{-(1+P)\nu x} dx.$$

In order to plot this equation it will suffice to solve it only for integral values of νx_0, in which case the integrals are of the familiar type $\int x^m e^{ax} dx$. The factorial terms arising from the two integrals may be combined in such a way that the above equation reduces to

$$e^{P\nu x_0} = 2\nu x_0 \left\{ \frac{1}{[\nu x_0(1 + P)]^2} + \frac{2(\nu x_0 - 1)}{[\nu x_0(1 + P)]^3} + \frac{3(\nu x_0 - 1)(\nu x_0 - 2)}{[\nu x_0(1 + P)]^4} + \cdots + \frac{(\nu x_0 - 1)(\nu x_0 - 1)!}{[\nu x_0(1 + P)]^{\nu x_0}} + \frac{\nu x_0!}{[\nu x_0(1 + P)]^{\nu x_0 + 1}} \right\} \tag{37}$$

This is the simplest form of the equation expressing the theory of elastic impact.

The calculation of corresponding values of νx_0 and P from this equation was exceedingly laborious. I gave νx_0 various integral values between 0 and 80 and in each case determined by trial the value of P which satisfied the equation. Some of the values so obtained are given in Table V. Since we wish to deal ultimately with the mean free path l, or its reciprocal ν_1, instead of ν, the values from the above calculations have been translated into terms of ν_1 by the use of equation (29). Instead of x_0 its more useful equivalent V_0/X has been written in Table V.

TABLE V.

$\frac{\nu V_0}{X}$	P	$\frac{\nu_1 V_0}{X}$	$\frac{\nu V_0}{X}$	P	$\frac{\nu_1 V_0}{X}$
0	1.000	0.000	10	0.0478	2.186
1	0.249	0.499	15	0.0337	2.756
2	0.161	0.802	20	0.0265	3.256
3	0.1213	1.045	25	0.0222	3.725
4	0.0984	1.255	30	0.0189	4.124
5	0.0831	1.441	40	0.0144	4.793
6	0.0723	1.613	60	0.0101	6.030
8	0.0574	1.917	80	0.0075	6.940

For any given value of $\nu_1 V_0/X$ the corresponding value of P may thus be found, whence

$$\alpha = P\nu = \nu_1 \sqrt{P} \qquad (38)$$

by equation (29).

Experimental Verification of Equation (37).—Experiments on exceedingly pure helium by E. W. B. Gill and F. B. Pidduck[1] enable us to test equation (37). I have applied to their data tests similar to those mentioned in the paper on inelastic impact, whence I have rejected all values for which P and ν cannot be considered approximately uniform throughout the ionization chamber. The values have been reduced as suggested by equation (30) to correspond to a pressure of 1 mm. The agreement

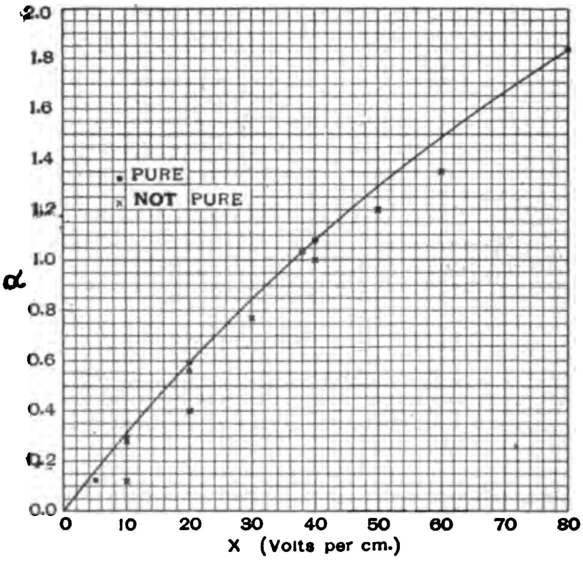

Fig. 4.
Helium.

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[1] Phil. Mag., 23, p. 837, 1912.

between these data and equation (37) is shown by Fig. 4 in which the dots represent the experimental data and the solid curve represents the equations (37) and (38) with the constants V_0 and ν_1 so chosen as to give the best agreement.

Additional support of the theory is found in the calculated values of V_0 and ν_1. In Table VI. the column marked *Exp.* gives the directly determined value of V_0 as published, the column marked *Exp.** gives the revised experimental value which appears to me most probable from a consideration of the experimental results[1] and the fourth column gives the value of V_0 determined by the present theory. Corresponding data from the preceding paper on inelastic impact are included for comparison.

Table VI.

Gas.	V_0			ν_1	
	Exp.	Exp.*	Theory.	$\pi R^2 N$	Theory.
He	20.0	22.5	22.5	13.5	10.0
A	12.0	13.5	14.5?	32.0	15.0?
N_2	7.5	10.0	10.05	34.5	22.5
CO_2			8.20	49.5	30.0
H_2O			7.47	47.0	21.7
HCl			6.05	46.0	33.3

Table VI. contains also the values of ν_1 calculated for a pressure of 1 mm. In the fifth column are given values calculated on the assumption that $\nu_1 = \pi R^2 N$, where πR^2 is the molecular cross section calculated from the kinetic theory and N is the number of molecules per unit volume. Where direct test is possible the theoretical values of V_0 appear to be accurate and the values of ν_1 appear to be consistently smaller than $\pi R^2 N$, for which a reason was previously suggested.

In the case of the elastic gas argon the ionization experiments are untrustworthy and the values calculated from them by the present theory can only be regarded as very approximate in view of the following considerations.

Effect of Impurities in the Gas.—Not only is there good agreement between theory and experiment as shown by Fig. 4, but the deviations from the theory are in a direction to be explained by traces of impurities in the gas, as is shown by the crosses which represent experiments in which the helium was known to be less pure.[2] The effect of even slight traces of ordinary gases is to give helium, in small fields, the characteristics of an inelastic gas. In other words, impurities cause the $\alpha - X$ curve to leave the origin horizontally instead of at a definite angle. It is not surprising, therefore, that the only available data for argon do not conform accurately to the present theory when we know that the experiments were performed under conditions identical with those in which the crossed data for helium were obtained.[1] The values of V_0 and ν_1 for argon in Table VI. were obtained by constructing a curve which bore the same relation to the data for argon that the curve of Fig. 4 does to the crossed data. These values are, of course, much more uncertain than those for the other gases.

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[1] See the preceding paper, *loc. cit.*

[2] E. W. B. Gill and F. B. Pidduck, Phil. Mag., 16, p. 280, 1908.

Approximate Theory of Ionization in a Mixture of Elastic and Inelastic Gases.—I have not been able to work out an exact formula to cover this case but will briefly outline a treatment which various tests have shown to be not very far from correct. The equation which we shall derive has its chief value in its ability to account for the effects of small traces of impurities in gases.

Suppose that an electron, in one centimeter path, makes an average of ν_i and ν_e collisions with molecules of inelastic and elastic gases, respectively, and let V_{oi} and V_{oe} be the minimum ionizing potentials in the two gases. By equation (21) in an earlier paper[2] we have, for the velocity distribution among the electrons approximately

$$F(V)dV = \frac{(1 + P_i)\nu_i + 2P_e\nu_e}{X} e^{-\frac{(1+P_i)\nu_i+2P_e\nu_e}{X}V} dV,$$

where P_i and P_e are the probabilities of ionization at an inelastic and an elastic impact, respectively. We have therefore, from equation (31),

$$P_i = \int_{V_{oi}}^{\infty} \frac{V - V_{oi}}{V} F(V)dV = e^{-\frac{sV_{oi}}{X}} + \frac{sV_{oi}}{X} Ei\left(-\frac{sV_{oi}}{X}\right)$$

and

$$P_e = \int_{V_{oe}}^{\infty} \frac{V - V_{oe}}{V} F(V)dV = e^{-\frac{sV_{oe}}{X}} + \frac{sV_{oe}}{X} Ei\left(-\frac{sV_{oe}}{X}\right),$$

where $s = (1 + P_i)\nu_i + 2P_e\nu_e$ and $Ei(\ )$ denotes the exponential integral.

But $\alpha = P_i\nu_i + P_e\nu_e$, whence

$$\alpha = \nu_i\left[e^{-\frac{sV_{oi}}{X}} + \frac{sV_{oi}}{X} Ei\left(-\frac{sV_{oi}}{X}\right)\right] + \nu_e\left[e^{-\frac{sV_{oe}}{X}} + \frac{sV_{oe}}{X} Ei\left(-\frac{sV_{oe}}{X}\right)\right]. \quad (39)$$

The type (elastic or inelastic) of variation of α with X depends on the relative magnitudes of the two terms in s. For example, in the case of argon or helium containing a trace of nitrogen or oxygen we can easily predict the general shape of the $\alpha - X$ curve. Since V_{oi} is considerably smaller than V_{oe}, P_e becomes negligible in comparison with P_i when X

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[1] E. W. B. Gill and F. B. Pidduck, *loc. cit.*

[2] PHYS. REV., April, 1916.

is very small. Thus for small values of X the gaseous mixture shows the characteristics of its inelastic component. When X is large, however, P_e and P_i do not differ so greatly and the mixture has more nearly the characteristics of the gas which is present in the greater quantity. Though the relative effect of the inelastic constituent is always large in comparison with the proportion of this constituent present, the relative importance of this inelastic constituent diminishes as X increases. Fig. 5 illustrates this point. The upper curve refers to a pure elastic gas, the lower curve to a pure inelastic gas and the intermediate curve represents equation (39) for the elastic gas with a small proportion of the inelastic gas.

This discussion suggests the reason for the extreme electrical sensitiveness of helium and argon to the slightest traces of impurities. I have not yet succeeded in deriving an equation to replace equation (39) following the exact treatment of elastic impact given in the earlier part of this paper.

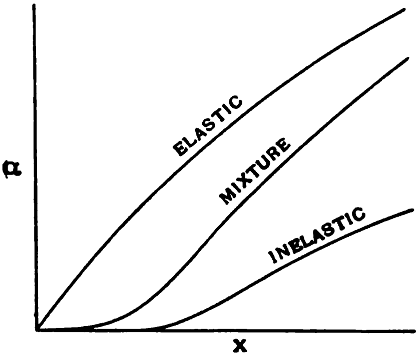

Fig. 5.

Summary.—In the first paper of this series exact expressions were obtained for the velocity distribution among the negative ions of inelastic gases and approximate expressions for this distribution in elastic gases and mixtures. These expressions applied to cases of ionization exclusively by impacts of negative ions and also to ionization by both negative and positive ions.

In the second paper an expression was derived for α, the number of new ions formed by inelastic impacts of a negative ion in a centimeter path and this expression was found to receive good experimental support.

In the present paper the functional relation between α, X and p for elastic impacts was derived and an equation for α was obtained which conformed to the best experimental data. The departure of some experimental data from this theory was shown to arise from the use of impure gases.

In all the discussion it has been assumed that velocities due to thermal agitation are negligible and that no complex ions or clusters are formed within the range of electric fields and gas pressures employed in experimental tests. The only experimental data used were those in which the effect of the finite distance between the electrodes could be neglected.

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SOME CHARACTERISTICS OF CRYSTAL DETECTORS.

By Victor A. Hunt and Laurens E. Whittemore.

THIS paper describes an investigation of the behavior of certain crystal detectors, as used in wireless telegraphy, when subjected to varying conditions of temperature, pressure and humidity.

A considerable amount of work has been done in recent years in studying the properties of various crystals and metals which have been found to be rectifiers of high frequency currents.[1] Up to the present time, however, the phenomenon of crystal rectification has been investigated chiefly by the direct application of known potential differences, both direct and alternating, to the terminals of the rectifier.

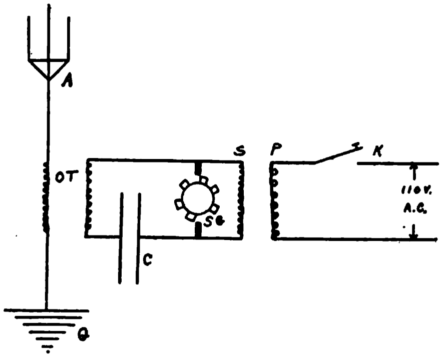

Fig. 1.

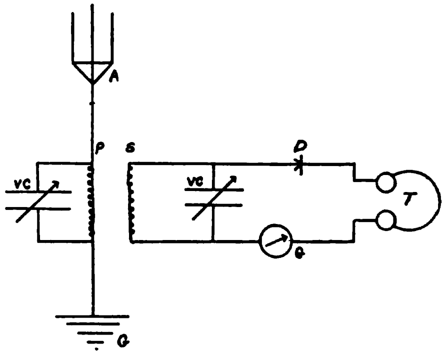

Fig. 2.

Since the most important use of crystal rectifiers at the present time is in wireless telegraphy, it was decided to undertake the investigation of the properties of four of these detectors under conditions similar to those of actual practice. Observations were made of the variations which occurred in the rectified current with changes of temperature, pressure and humidity. For the purpose we erected a sending station and a complete receiving station in different parts of the same building. Fig. 1 and Fig. 2 show simple diagrams of the two circuits. The detectors studied were galena, perikon, silicon and carborundum.

The sending station was arranged to emit a wave of about 500 meters.

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[1] The following papers give a fairly comprehensive list of the recent publications on the subject of crystal rectifiers: Goddard, Phys. Rev., 34, 423, 1912; Flowers, Phys. Rev., 3, 25, 1914; Hartsough, Phys. Rev., 4, 306, 1914.

The normal current in the closed oscillatory circuit was 4.5 amperes, and the power consumed by the transformer was 240 watts. The curve *A* in Fig. 3 shows that when this current was used, such fluctuations as would be present in the power supply would produce only small variations in the galvanometer deflection at the receiving station.

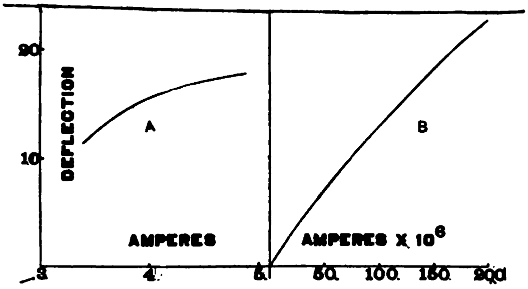

Fig. 3.

The measurements of the current in the detector were made by means of a Leeds and Northrup ballistic galvanometer of 1,069 ohms resistance and having a sensibility of 0.0026 micro-amperes per millimeter. Fig. 4

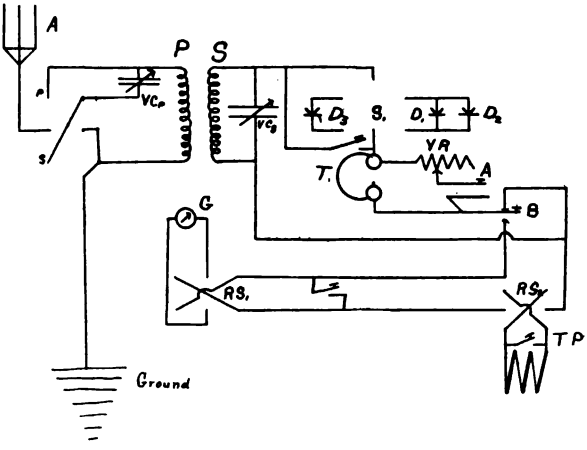

Fig. 4.

gives a complete diagram of the connection of the receiving instruments. The signals were sent out from the sending station by closing a key at the receiving station. In order to prevent any inductive effect in the galvanometer circuit from the make and break of the current in the primary of the sending transformer, the key *B* was always left open at the make and break of the sending key. The time during which the current flowed through the galvanometer was determined entirely by the time during which the key B was closed. By means of the reversing switch RS_1 the direction of the galvanometer deflections could be changed. The switch S_1 was used to connect in an auxiliary detector with which to receive messages from other stations without disturbing the detector under test.

By far the most troublesome source of error which we encountered was an "extra" E.M.F. of variable magnitude which made its appearance continually throughout the course of the investigation. It originated at the contact of the detector and would cause a seemingly unexplainable deflection of the galvanometer even when no signals were being received. To eliminate this by inserting an opposing E.M.F. from a potentiometer in the circuit proved to be inconvenient and unsatisfactory. The method of elimination which was used consisted in the use of a thermopile which was inserted directly in series with the galvanometer, and was heated by the radiation from a thirty-two candle power carbon filament lamp. To adjust the E.M.F. generated in the thermopile the position of the lamp was varied by moving it back and forth on a sliding track. Since the "extra" E.M.F. often reversed its direction it was necessary to use a reversing switch RS_2 in order that the E.M.F. of the thermopile might always oppose it. This arrangement made it possible to eliminate entirely all extraneous currents from electro-chemical or thermo-electric sources such as have been mentioned by Flowers.[1]

The following is the method of procedure used in taking a series of readings. One observer confined his attention to the galvanometer. He first eliminated whatever "extra" E.M.F. was present by opposing it with the E.M.F. generated in the thermopile. This balance was secured by closing the key B and adjusting the distance of the lamp until the galvanometer showed no deflection. The observer in charge of the galvanometer wore the telephone receivers at all times in order to detect any errors in the readings which might occur due to the interference of atmospherics or to the sending of a nearby station, and all data which contained such errors were discarded.

The other observer then closed the key which put the sending station in operation, and, with stop watch in hand, pressed the key B for a certain length of time. The first observer then noted the maximum deflection of the galvanometer. Because of its long natural period it did not have time to come to its full deflection in the three seconds during which the current was flowing through it. The curve B, in Fig. 3, shows the deflections produced by the corresponding currents flowing for a period of three seconds.

In making a test we did not attempt to secure the most sensitive adjustment of the detector, but tried above all to secure one which would remain permanent and which gave a clear, uniform tone. The deflections were adjusted to the range of the galvanometer scale by varying the coupling of the receiving transformer which was then kept the same throughout that particular experiment. In order to subject the detector to the various physical conditions desired, we had constructed a box which consisted of a glass battery cell with a suitable brass cover.

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[1] Phys. Rev., 29, 445, 1909.

To heat the detector we enclosed it in the box which was then immersed in cylinder oil of a high flash point and heated by a gas burner. In order to cool the detector we suspended it in a U-shaped Dewar flask which contained liquid aïr. The temperature obtained was measured by means of a pentane thermometer.

The pressure in the box was varied by means of a motor driven air pump and was measured with an open tube manometer.

The humidity of the air in the box was varied by varying the concentration of a sulphuric acid solution which was placed in the bottom of the cell. The air was first dried by putting in a known volume of concentrated sulphuric acid and letting it stand for some time. To increase the humidity the acid was diluted by the addition of known quantities of water. The humidity was allowed to reach a state of equilibrium in each case. This was found to require about ten minutes. The value of the vapor pressure for each concentration was obtained from the Physikalisch-Chemische Tabellen of Landolt, Bornstein and Myerhoff.

Results.

Galena.

The first physical condition which we varied was temperature, since it was natural to suppose that changes in temperature would produce the most noticeable effect. The galena detector was carefully adjusted and placed in the closed box which was then immersed in oil and heated. The heat was supplied continuously and the galvanometer readings taken at successive intervals while the temperature was rising. The galvanometer readings were then plotted against the respective temperatures. Fig. 5 shows one of these curves and is typical of those obtained with galena. In this as well as the other figures the dotted curves indicate the results obtained with other specimens or other adjustments. The rectification falls off very rapidly with the rise in temperature, approaching zero at 170° C. So long as the adjustment remained the same there was no improvement in rectification when the detector was cooled.

The decrease in rectification was not due to increased pressure at the contact, since the needle point was suspended by a very fine coiled spring, so that expansion due to increased temperature would not affect its adjustment. We were much gratified with the uniformity of the readings which we were able to secure with galena, since we had feared that, on account of the light contact needed, a permanent adjustment could not be preserved. The following data show the uniformity of the galvanometer readings under constant conditions at the detector.

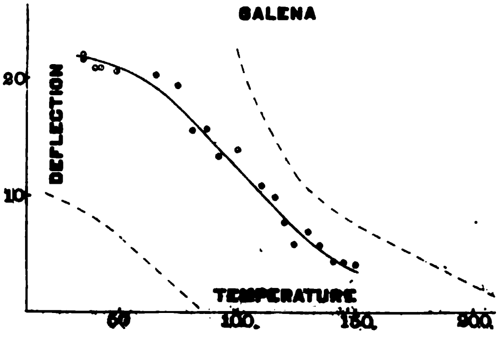

Fig. 5.

Galvanometer Deflection for Signals Three Seconds Long.
Galena Detector under Room Conditions.

15.1
14.6
14.9
15.2
15.0
15.0
14.9
15.3
14.9
15.3
14.9
14.7
15.0

One noteworthy fact in connection with the rectification by galena is that, although in most cases the current flows from crystal to point, it was found that, with some adjustments, the current would flow in the other direction.

It is a well-known fact that when a relatively large current caused either by a heavy atmospheric discharge or by the spark at a sending station nearby, passes through a galena detector, the rectification is impaired. It has occurred to us that a probable explanation of this phenomenon is the local heating of the contact by the passage of the current producing an effect similar to the heating described above.

The temperature at which rectification becomes practically negligible is not always the same but varies with the original adjustment, being higher for better adjustments.

In cooling the galena detector to a low temperature the manipulation was not so easy, for we could not obtain the decrease in temperature as uniformly as we should have liked. By lowering the detector nearer and nearer to the surface of the liquid air in the Dewar flask we were able to obtain sufficient data to determine the behavior of the galena detector in that range of temperature. Fig. 6 shows the results of this experiment. It will be noticed that at about − 50° C. there occurred a decided maximum of rectification, which was corroborated by successive tests.

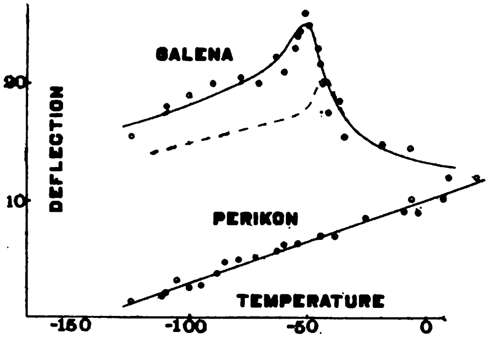

Fig. 6.

One of the greatest difficulties which we encountered in cooling galena as well as the other detectors was the precipitation of frost upon the crystals. If any of this frost melted due to heating at the contact the effect of humidity was superimposed upon that of temperature. In addition, this moisture at the contact had a decided influence on the "extra" E.M.F. causing it to be very strong, although sometimes in one direction and sometimes in the other. The glass-hardness of the surfaces of the crystals at these low temperatures made it very difficult to retain the adjustments. It will be noted that the curve obtained by cooling continues the general trend of the one obtained by heating.

No effect on the rectification by galena could be noted when the detector was completely immersed in liquid air. Good adjustments remained good and poor ones poor.

The results of varying the humidity of the air in the closed box are shown in Fig. 7. The galvanometer deflections are shown which correspond to the vapor pressures of the various concentrations of sulphuric acid solution in the bottom of the box. It will be seen that the presence of a very small quantity of moisture is sufficient to reduce greatly the rectification by the galena detector. Each of the points on Fig. 7 represents the average of at least ten successive readings taken after the vapor pressure had reached a state of equilibrium.

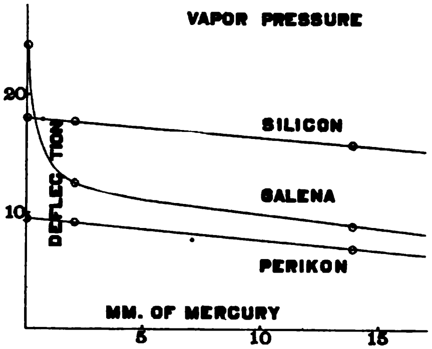

Fig. 7.
Galvanometer deflections corresponding to vapor pressures of sulphuric acid solutions.

Perikon.

The perikon detector used consisted of a contact between crystals of zincite and chalcopyrite. The effect of temperature was more marked in the case of perikon than in the case of any other detector as is shown in Fig. 8. Starting with a deflection of 3 cm. at about 55° C. for a three-second dash, the deflection passed off the scale of the galvanometer at a temperature of about 140° C. When the length of the dash was shortened to one second the deflection became 7 cm. but rose rapidly with the temperature reaching a value of 24 cm. at 210° C.

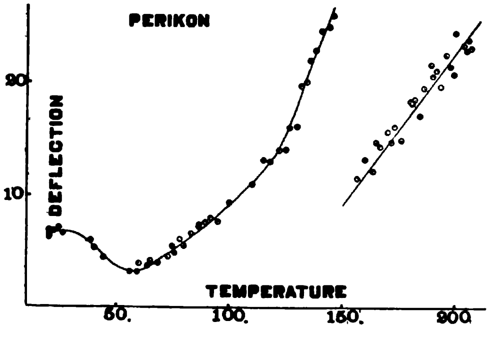

Fig. 8.

The curve obtained upon cooling the perikon detector (Fig. 6) was perfectly continuous with the heating curve, falling rapidly with the temperature. At no time was there any indication of a definite maximum or minimum of rectification within the temperature range which we used, from − 120° C. to 210° C. Immersion of the perikon detector in liquid air caused a complete loss of rectification.

When the vapor pressure of the space surrounding the perikon detector was changed, the results shown in Fig. 7 were obtained. The effect was not so marked as in the case of galena but there appeared a definite decrease in rectification with an increase in humidity.

Silicon.

We found the rectifying action of silicon to be very erratic. This corresponds with the statements made by other observers in regard to the properties of that substance. Miss Frances G. Wick, in a paper on the "Electric Properties of Silicon,"[1] makes the remark that "the position of the element in the periodic system between the metals and the non-metals may explain some of the deviations of its properties from those of the stronger metals."

O. E. Buckley[2] states that, "since different specimens, sometimes even if cut from the same piece, may vary widely in their electrical properties, it is important that measurements of different properties be made with the same specimen." His statement is confirmed by the results of our investigations as shown in Fig. 9. The conditions of heating were identical in obtaining both curves except that different specimens were used. In one case the rectification is seen to increase and in the other to decrease as the temperature rises.

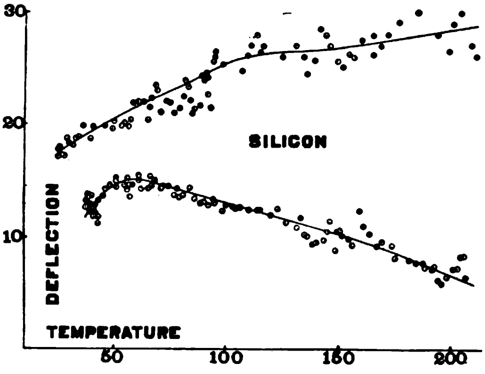

Fig. 9.

In attempting to determine the effect of subjecting silicon detectors to low temperatures it was found impossible to obtain any consistent results. The rectification would change very rapidly when there was apparently no cause whatsoever, and it was very hard to retain a given adjustment since the surface of the crystal became exceedingly hard and slippery at the low temperatures.

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[1] Phys. Rev., 25, 382, 1907.

[2] Phys. Rev., 4, 482, 1914.

When the silicon detector was completely immersed in liquid air the sound in the telephone receivers increased in the majority of cases. It is possible that in other cases the somewhat violent boiling of the liquid air destroyed the adjustment of the light needle contact.

The behavior of silicon under changes of humidity resembles very much that of the perikon detector, the rectification decreasing somewhat with the increase of vapor pressure. These results are shown in Fig. 7.

Carborundum.

In order to secure a good adjustment with carborundum we found it necessary to use considerable pressure upon the contact point, consequently imbedding the point of the steel needle below the surface of the crystal. The adjustment was very permanent and could not easily be impaired by vibration.

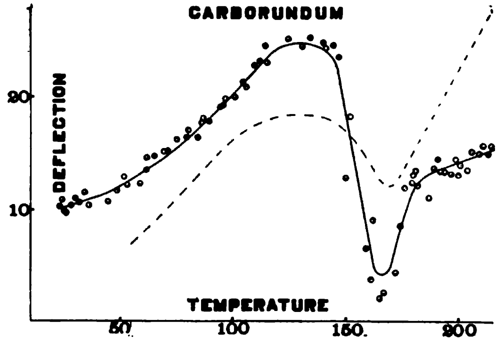

Fig. 10.

Upon heating the carborundum detector very peculiar results were obtained as may be seen in Fig. 10. Successive tests showed that the rectification attained a decided maximum at a temperature of about 130° C. and from there fell off rapidly until a minimum was reached at about 165° C. Above that temperature the rectification again increased as far as the tests were conducted. Even when the glass containing box accidentally broke and the detector was immersed in the hot oil-bath its behavior remained the same. This was probably due to the fact that the oil could not reach the actual contact between the crystal and the needle.

The cooling curve shown in Fig. 11 continued the lower portion of the heating curve. Rectification became zero at about − 85° C. It was noteworthy that upon being warmed the detector largely regained its rectifying properties, the dotted curve showing this rise. In order to prove that the decrease in rectification was not due to an imperfect contact caused by the contraction of the needle and the detector stand, we tried repeatedly to secure a new adjustment at these low temperatures but found it impossible. Likewise immersion in liquid air proved disastrous to all rectification.

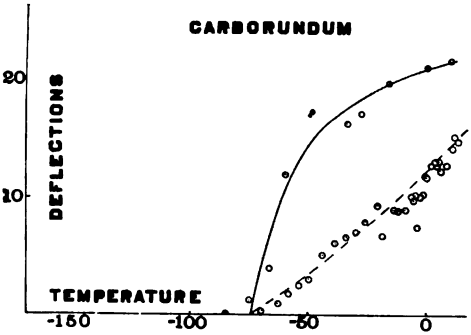

Fig. 11.

It was impossible to obtain any noticeable effects upon the rectification of carborundum by changes of humidity. It is probable that this was due to the imbedding of the contact point in the crystal so that the moisture could not affect it.

DISCUSSION.

It was found that with each of the four detectors, as far as we could observe, change in air pressure had no effect upon the rectification. We were able to vary the air pressure from 25 cm. to 135 cm. of mercury. It is perhaps not surprising that no effect was observed within this range.

One of the peculiarities noticed in our work was that certain crystals did not always rectify the oscillating current in the same direction. With one adjustment the rectified current would flow from crystal to point and with another would flow in the opposite direction. Once, indeed, during a test on the effect of change of temperature upon carborundum the rectification actually reversed although no change had occurred in the adjustment. The most common direction of rectification for silicon and for galena was found to be from the crystal to the brass point which was used. In the case of carborundum the ordinary sense of rectification was from needle to crystal. The perikon detector was the only one of the four which showed no reversal; always rectifying from chalcopyrite to zincite.

The "extra" E.M.F., to which reference has previously been made, is probably of galvanic origin. Although it might have been of thermoelectric origin, due to local heating at the contact, it could not have been the result of a thermo-junction caused by the difference in temperature between the detector and the outside circuit since both leads from the detector were of copper, causing the junctions affected by the heat to oppose each other. In addition the frequent reversal of this "extra" E.M.F. seems to preclude all possibility of its being due to a thermoelectric junction.

CONCLUSIONS.

It is possible that there are changes both in the condition of the surface on the crystal and in the body itself of the crystal during changes in the rectification. If both of these changes take place, as has been suggested by some observers, it is probable that the body effect predominates in the variations of rectification with changes in temperature which are most notable in the case of carborundum. In the case of humidity the surface effect probably has the most influence on the variation in rectification.

At certain times when the rectification was especially good, a humming sound was heard in the receivers similar to the inductive hum often heard when the detector circuit was opened. This seems to be evidence of the formation of a high resistance film on the surface of the crystal at the point of contact. Attention has previously been called to the possibility of the presence of this resistance film by Austin[1] and Goddard.[2]

There is apparently no relation between the rectification and the "extra" E.M.F. since the former remains unchanged through rapid fluctuations and often through reversals of the latter. The "extra" E.M.F. often exists even after room conditions of temperature, etc., have prevailed for some time throughout the apparatus. The cause of this "extra" E.M.F. and its relation, if any, to the rectification would be, in itself, a profitable subject for investigation.

We are indebted to Mr. Edison Pettit, of Washburn College, for suggesting this problem and to Professor F. E. Kester and the other members of the Department of Physics of the University of Kansas for their assistance and suggestions during the progress of the investigation.

BLAKE PHYSICAL LABORATORY,
UNIVERSITY OF KANSAS,
May 12, 1915.

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[1] Bulletin Bureau of Standards, Vol. 5, p. 146.

[2] PHYS. REV., 34, 423, 1912.

THE MAGNETIC SUSCEPTIBILITY OF GASES.[1] II.

By W. P. Roop.

8. The method of the gas manometer, as it may be called, has now been applied to the determination of the susceptibility of oxygen. Three features of the new determination distinguish it from previous work in the same field: observational errors have been reduced to values hitherto unattained; the observations have been made in fields of small intensity; and certain systematic errors of methods previously used have been eliminated.

Precision of measurements in this field has been very little discussed for the reason that previous work has shown wide discrepancies between successive observations taken by a given investigator, as well as between the averages obtained by different investigators. In Section 9 will be found a discussion of the work of Piccard,[2] the only previous observer who claims for his data a moderate degree of exactness.

The intensity of the field used is of importance for two reasons. In the first place, the measurement of a field of high intensity, but small extension, is a matter of considerable difficulty, and subject to undoubted errors for which allowance can not be made. In the second place, measurement of susceptibility in fields of both high and low intensity is necessary in order to obtain an experimental answer to the question: is suspectibility entirely independent of field intensity? This question will receive further consideration in Section 10.

Three systematic errors of other methods are absent in that of the gas manometer, namely: error due to the uncertainty as to the susceptibility of water; error due to imperfect determinations of pressure in the gas; and error due to solution of the gas in the liquid of the liquid manometer. These matters are discussed in detail in Section 13. In addition, the high sensitivity of the method reduces observational errors as well, since one source of the large errors of previous work has been the smallness of the quantities measured.

9. Piccard's paper unfortunately does not exhibit the data from which his mean values are calculated. It would appear, from data given on page 462 of his paper, that the probable error of an average of 15 deter-

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[1] A continuation of work described in Physical Review, 2, 497, 1913.

[2] Piccard, Archives des Sciences, 35, 476, 1913.

minations of the magnetic depression of water (Quincke's method) is 0.0028 mm. Since his determination of the susceptibility of oxygen is based on two such observations, it would seem to be subject to errors of that order of magnitude, or larger. On his page 479, the observed difference in magnetic depression of water due to presence of oxygen atmosphere is 0.033 cm. (misprinted in one place as mm.). The observational error in his mean result thus appears to lie between one and two per cent.

Observational error occurring in the use of the gas manometer is well under one per cent.

10. Systematic variation of susceptibility with field intensity has been observed by both Quincke[1] and Hennig.[2] A summary of data taken from their papers, and reproduced in the following table, shows this. The units in which susceptibility is expressed here and elsewhere in this paper, are ten millionths (10^{-7}).

Table I.

Data on Variation of κ with H.

Quincke.		Hennig.			
H	$\kappa \times 10^7$	H	$\kappa' \times 10^7$ 1 Atmos.	$\kappa \times 10^7$ 2 Atmos.	$\kappa \times 10^7$ 4 Atmos.
7,000	1.52	3,700	1.31		1.15
14,000	1.57	5,700	1.26		1.26
16,000	1.70	8,400	1.26	1.23	
		10,340	1.19	1.17	1.09

Both men, however, evidently considered their data insufficient to warrant even the suggestion that such a variation exists, for neither mentions the matter. The fact that the variation is in the opposite sense in the two cases is evidence of its spurious character. On account of the care used by Hennig, however, this should not have deterred him from calling attention to the indication given by his data. Instead of that, he excluded the two extreme values in casting up his definitive average.

Curie simply states, without giving data, that the susceptibility of oxygen was found not to vary within the limits of 100 and 1,350 gausses. These are much weaker fields, however, than those used by Quincke and Hennig, and his mean value is high, namely: 1.67. This was based on an assumed value for water of 7.9. The recent carefully determined value reached independently by Piccard and Sève is 7.2. This would convert Curie's value to 1.52.

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[1] Quincke, Annalen der Physik, 34, 401, 1888.

[2] Hennig, Annalen der Physik, 50, 485, 1893.

Piccard's value of 1.407 was obtained with a more intense field (21,000 gausses) than that used by any of the others.

Langevin's theory of magnetization of gases demands a decrease of susceptibility with increasing field intensity, but according to his estimate, this should only begin to be perceptible at fields of the order of 100,000 gausses at atmospheric temperatures. The systematic variation shown by Hennig's data, and the discrepancy between the results of Curie and of Piccard would indicate that the field intensity at which saturation begins to be appreciable is not so high as that. A comparison of Piccard's work with the new results exhibited at the end of this paper may be considered to point to the same conclusion.

11. The apparatus used in the present experiments is the same in principle as that previously described, but quite different in design. It seemed impracticable to obtain a manometer of the desired form in glass. In order to reduce the time required for the enclosed gases to settle to their equilibrium position, it was necessary to make the bore of the manometer large and uniform. It also seemed desirable that it should be circular in shape, and that the two valves should be opened and closed simultaneously. The only feasible way of doing this was by using metal as a material. Since iron was excluded for magnetic reasons, mercury could not be used for handling the gas. It was therefore determined to make use of a vacuum pump and exhaust the manometer completely at each trial. The exhaustion demanded is about 0.01 mm., and difficulties were encountered in making the somewhat complicated brass manometer tight. This, however, was finally accomplished, and the present limitation to precision lies mainly in temperature control.

The manometer consists of a brass block containing a toroidal cavity of mean radius 46.7 mm., and cross-sectional area 33.33 mm.^{2} At opposite ends of a diameter, the toroid is divided by two ground stopcocks, connected by gear wheels so as to turn together. The diameter along which the toroid is thus divided is inclined at an angle of 20° with the horizontal.

Each chamber of the manometer may be connected separately with the vacuum pump, which is of the Toepler type. By means of this pump, the gas may be transferred to the burette, and its volume measured. On the way to the pump, the gas passes through a drying tube and an absorption tube filled with solid KOH.

The figure indicates schematically the arrangement of the apparatus.

12. The operations leading to a value for susceptibility may be summarized as follows: First, the cocks *A* and *B*, leading to the two chambers of the manometer, were opened, and the entire apparatus exhausted.

Cock C was turned so as to pass the gas into the atmosphere. At the same time the burette was completely filled with mercury by raising reservoir F until mercury flowed through cock C (now in its other position) into the chamber G. On reversing C again and raising reservoir E, all air was expelled from G. C was then closed and left closed. Cocks A and B were now connected, respectively, with a supply of oxygen and of carbon dioxide. On opening the cocks A and B, the upper chamber of the manometer was thus filled with oxygen and the lower chamber with carbon dioxide. Now on closing A and B and opening the brass cocks of the manometer momentarily, the gases flowed around until the two boundaries between them lay at opposite ends of a horizontal diameter. Diffusion, of course, occurred at the same time. Difficulties thus introduced have already been treated in sections 4 and 5, and will be again discussed in section 17. After the manometer cocks were closed again, the upper chamber had received a small amount of carbon dioxide, and the lower chamber an equal amount of oxygen. Passing the gas into the burette, measurements of the volume of oxygen in each chamber were obtained.

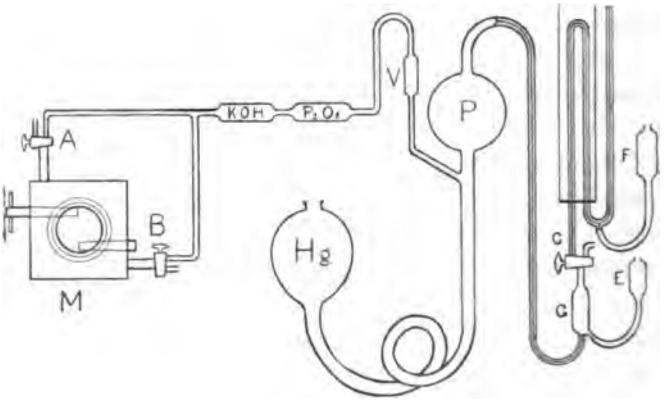

Fig. 1.

The whole process was then repeated, with the exception that the two chambers of the manometer were connected with each other under the influence of the magnetic field. The boundaries between the two gases thus reached different equilibrium positions, and the volumes measured in the burette were different from those obtained before.

If the action of the field is to decrease the volume of oxygen left in the upper chamber, the volume of oxygen found in the lower chamber will be increased. This increase and decrease should be numerically equal, and in this way we obtain a valuable check on all observations.

We will call this observed deviation in volume ΔV. Divided by the cross-sectional area of the toroid, it gives the vertical displacement of the boundary surface due to the action of the field. This we will call l instead of d, as in Section 3.

The intensity of the magnetic field was determined by means described in Section 23.

The susceptibility difference between oxygen and carbon dioxide is given by the equation

$$\kappa - \kappa_0 = \frac{4lg(\rho - \rho_0)}{H^2 - H_0^2}.$$

13. Before proceeding to the discussion of the experimental details, I will call attention to certain broad features of the method.

In the first place, it is a comparison only, and not absolute in character. The same is true of all other methods that have ever been applied to the problem. In the cases of Quincke, Curie, and Piccard, the comparison substance was water, with respect to whose susceptibility an uncertainty of absolute value 0.5 exists, unless the work of Piccard and of Sève be accepted and all previous work rejected. In Quincke's method, used also by Piccard, error of the same absolute amount must be considered to attach to the value for oxygen, since it is the difference of susceptibility of oxygen and water that is measured. On the other hand, this uncertainty is reduced in case the oxygen is used under pressure greater than that of the atmosphere. Piccard does not state the pressure used, but only indicates that allowance for pressure was made in reducing data. Curie's method introduces uncertainty, not of the same absolute magnitude as that in the case of water, but in the same proportion. Hennig's method compares susceptibilities of oxygen and air, and is based on the assumption that the susceptibility of air is due simply to its oxygen content.

The adoption of carbon dioxide as comparison susbtance leads to uncertainty of the same order of magnitude as that which exists with respect to the susceptibility of that gas. I have given elsewhere[1] a summary of work on carbon dioxide which indicates that its susceptibility is certainly less than 0.03. Since different observers disagree, even as to sign, we may, in default of better knowledge, take its value to be zero. The uncertainty thus introduced does not exceed 2 per cent. of the susceptibility for oxygen.

I hope to completely settle this difficulty by the application of an absolute method now in hand.

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[1] Physikalische Zeitschrift, 12, 55, 1911.

Another feature of the method of the gas manometer is that its indications are not affected by variations in pressure, except to the relatively small extent that specific susceptibility (susceptibility divided by density) depends on the state of compression. On the other hand, in Quincke's method, observed manometric depressions are in direct proportion to the pressure. This independence of pressure makes the method valuable for two reasons: no pressure correction is necessary; and the method is available for determining the variation of specific susceptibility with pressure, without diminution of precision at low pressures.

On the other hand, the method is rather sensitive to temperature changes, necessitating exact thermostatic control in an attempt to better the results here presented.

One type of difficulty inherent in Quincke's liquid manometer is completely avoided in the gas manometer, and that is the uncertainty as to the effect of solution of gas in the liquid. Even Piccard introduces no correction on this account, although an observation which he makes in another connection[1] would indicate that it is of considerable importance. From this difficulty Curie's work is exempt.

14. I will now proceed to discuss the following essential matters of detail in the present experiments: (*a*) Purity of gases; (*b*) Precautions in filling and operating manometer; (*c*) Diffusion in the manometer; (*d*) Temperature control; (*e*) Determination of sectional area of manometer; (*f*) Correction for curvature of manometer bore; (*g*) Field intensity measurements.

15. The gases were of the kind used for commercial purposes, stored in steel cylinders. The carbon dioxide was analyzed by absorption in the apparatus itself. In a number of trials, the largest volume of gas unabsorbed by solid KOH was observed to be 10 mm.^{3} out of a total of 5,300 mm.^{3} This quantity of air (the chief impurity) could neither cause an appreciable deviation from the density of pure carbon dioxide, nor affect the magnetic forces by its oxygen content. Therefore no correction was made for it.

The oxygen was obtained from the Linde Air Products Co., and was stated to contain about 2 per cent. of nitrogen. This figure was checked by a simple analysis. A glass bulb of 200 c.c. capacity was fitted with a stopcock at each end. It was filled with oxygen by running a steady slow stream of gas from the cylinder through it for five to ten minutes. While the gas was still running through, the lower end of the open tube was placed under the surface of a strong solution of KOH, and the upper cock was closed. The gas in the bulb was warmed with the hand, and a few bubbles driven out. When the temperature returned to that of the room, the KOH solution stood at an observed point in the bottom of the bulb. Pyrogallic acid was then poured in from above in such a way as to admit no air, at first only in small quantities. As absorption proceeded, the solution rose in the bulb, and more and more pyrogallic acid was added, until absorption was complete. The bulb was calibrated by water weighings. Two determinations yielded values of 98.03 and 97.84 for the percentage of oxygen in the gas. The mean was taken as the correct value. This somewhat crude, but very convenient analysis amply satisfies all demands of precision for the purpose in view.

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[1] Piccard, *loc. cit.*, p. 481.

16. The flow of gas from the cylinders was controlled by reducing valves, and the gas was admitted to the manometer through rubber tubes. Before each run, the rubber tube was detached from the cock and a slow steady stream directed against the opening in the end of the glass tube for two to ten minutes, depending on how recently the operation had been carried out before. The tube projecting from the cock was about 1 cm. long, and of capillary bore. It would appear that pure gas from the cylinder would thus reach the manometer, although failure to meet this condition may be partly responsible for the residual lack of uniformity in the observations.

Both gases were dried, and the oxygen was passed through KOH to insure the absence of carbon dioxide.

As soon as the rubber tube was placed over the end of the glass tube, the flow necessarily stopped, and the reducing valve maintained in the tube a steady, but small pressure. The cock (A, B) leading into the manometer was then opened, momentarily reducing the pressure, but not permanently. The reducing valve was then closed and a side cock opened for an instant to reduce the pressure in the tubes and chamber to that of the atmosphere. This, of course, was accompanied by a slight cooling of the gas in the manometer. After an allowance of time (three minutes) for the gas in the manometer to recover the temperature of the brass walls, the side cock was again opened momentarily, and the cock (A, B) leading to the manometer was then finally closed.

In spite of these elaborate precautions, the outstanding non-uniformity in observations seems to be largely due to failure to control the temperature, and hence pressure of the gas inside the manometer.

17. After the manometer had been filled, the brass cocks were opened for a certain length of time to allow the gases to settle into their equilibrium position. This time must be long enough not to interrupt the motion before the equilibrium position has been reached, and short enough to prevent diffusion beyond the limits of the uniform field while the cock still remained open.

To explain the means by which these conditions were satisfied, I will refer to the detailed drawing of the manometer. The broken circles represent the toroidal bore; *M* and *N* are the two cocks, with diagonal bore, as indicated.

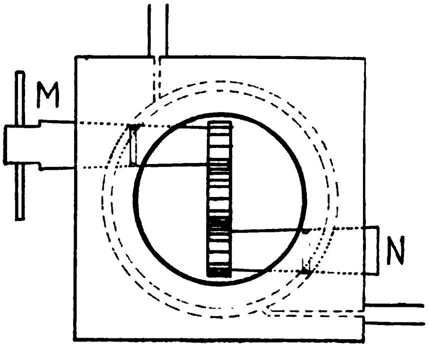

Fig. 2.

The bore of the manometer is large (diameter, 6 mm.) so as to reduce the viscous resistance to the motion of the gases. In order to reduce to a minimum the inevitable mixing during motion of the boundary, the distance through which the boundary must move is made as small as possible.

The time during which cocks *M* and *N* were left open was determined in accordance with indications as to diffusion obtained in the following way: a series of runs was carried through, with zero magnetic field, and with the cocks open for a length of time varying from 1 to 5 seconds. The results as exhibited in the table show that 1 second is certainly too little time, while 5 seconds is certainly too much, since in that time an appreciable amount of oxygen has *diffused* into the lower chamber. The four observations running from 1.6 to 3 seconds, however, show no discrepancies exceeding the observational errors. As a result of this test, 2.5 seconds was taken as the standard length of time during which the cocks were held open. Exact determination of this time being unnecessary, opening and closing was done by hand, and time estimated to fifths of seconds by ear.

TABLE II.

Table to Illustrate Diffusion.

Time, Secs.	1	1.6	2	2	3	5
Burette reading, mm.^{3} . . .	464	532	535	529	532	559

The question was also investigated, with similar results, with magnetic field on as well as off.

18. Certain details of design of the maonmeter were found essential to its proper operation.

Difficulty connected with the disposal of the gas contained within the rather large bore of the cock was settled as follows: a groove was cut around the plug in such a way as to intersect one of the open ends of the bore of the plug. In this way, one of the plugs was connected with one chamber and the other with the other, in whatever position the plugs stood. The division of the gas in the manometer into two halves was thus sharp and without any ambiguity of any kind. The design of the plug is illustrated by the accompanying photograph.

In constructing the manometer, it was necessary to turn the cavity out in two plates which were then put together, face to face. The second photograph shows a section of the manometer. The large circle in the center is the section of the cavity, and the horizontal lines extending to each side are the sections of the faces of the plates. It was found impossible in this way to make a permanently tight joint by simply soldering the two plates together, and recourse was had to an expedient which settled the difficulty satisfactorily. Just inside and outside the cavity, shallow grooves were cut in the faces of the two plates, and the sections of these may also be seen in the photograph. After the plates were soldered together, these grooves were squirted full of sealing wax. In order that the seats of the cocks might not be subject to the action of the exposed section of this sealing wax guard ring, they were somewhat enlarged and a bushing inserted. The cock plugs, which were of phosphor bronze, thus were ground into a uniform jointless brass surface, with satisfactory results.

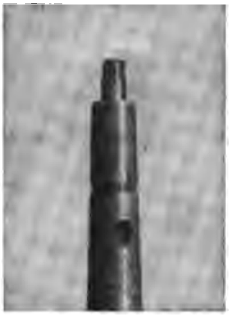

Fig. 3.

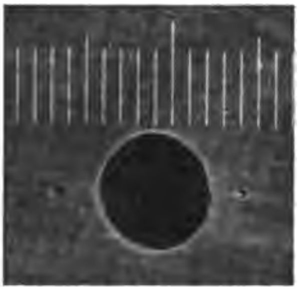

Fig. 4.

19. The temperature of the brass manometer was closely observed by means of two thermometers, graduated to twentieths of a degree, whose bulbs were placed in cavities in the brass. The two thermometers rarely agreed exactly. The burette was water jacketed to retard temperature changes. Corrections for temperature difference between manometer and burette amounted, in the maximum, to 10 per cent.; in the average, to 2 per cent. Sufficient precautions were taken, however, to reduce uncorrected errors from this source well below 1 per cent.

Different runs were effected at different temperatures ranging from 13° to 20° C. No attempt was made to correct individual runs according to Curie's law, so that deviations from the average due to this cause are included with observational errors.

20. The bore of the manometer was determined by sawing a right section of it, and enlarging photographically. The area of the photograph (See Fig. 4) was determined by means of a planimeter. The scale shown in Fig. 4 is a scale of millimeters ruled on the brass surface with a dividing engine.

21. If the manometer were of uniform, vertical bore, l, the vertical deviation of the boundary due to the action of the field, would be simply ΔV, divided by the area of cross-section. Since the bore is circular, instead of straight and vertical, we must introduce a correction. Let a_0 denote the area of the normal section, and a that of the approximate horizontal section at the point at which the boundary rests in the deviated position. Referring to the figure, we see that

$$a = \frac{a_0}{\cos\theta}. \tag{1}$$

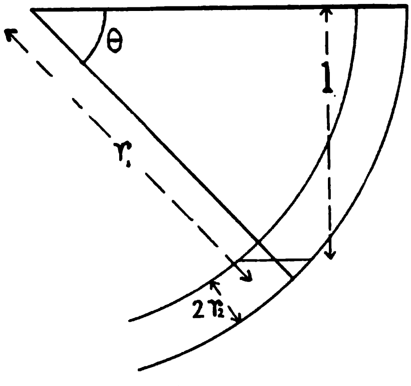

Fig. 5.

Further,

$$\frac{l}{r_1} = \sin\theta,$$

whence

$$dl = r_1 \cos\theta d\theta.$$

Thus

$$adl = a_0 r_1 d\theta,$$

and

$$\Delta V = \int_0^l adl = a_0 r_1 \theta. \tag{2}$$

To determine l, we therefore have the equation

$$l = r_1 \sin\theta = r_1 \sin\frac{\Delta V}{a_0 r_1}. \tag{3}$$

This is based on the assumption that the boundary remains horizontal, and that equation (1) is exact. The latter condition is not satisfied, and we proceed to determine the error as follows: The true value of the horizontal sectional area we will denote by $a(1 + \epsilon)$. ϵ will, of course, vary with l, but by obtaining a number of distributed values, and taking the average, we can determine a factor which can be taken out from under the integral and applied directly to the approximate result. To do this, we determine the exact area of horizontal cross-section as a function of l, r_1 and r_2.

Measuring x horizontally in the plane of the toroid, y vertically, and z normally to the plane of the toroid, we have, as the equation of the toroidal surface,

$$z^2 + (\sqrt{x^2 + y^2} - r_1)^2 = r_2^2.$$

The equation of the line of intersection with the plane

$$z = \pm l$$

is

$$z = \sqrt{r_2^2 - (\sqrt{x^2 + l^2} - r_1)^2},$$

and this is the curve bounding our horizontal section. To determine its area, we must evaluate the integral

$$\int_{r_1-r_2}^{r_1+r_2} z dx.$$

This I have not been able to do in general, but numerical evaluation at a series of values of l is simple and yields the desired result with sufficient exactness.

I find the maximum error to be 4.0 per cent., and the mean error, $\bar{\epsilon}$, 2.0 per cent., in the case where the correction is largest.

The corrected value of l is obtained from the equation

$$l = r_1 \sin \frac{\Delta V(1 - \bar{\epsilon})}{a_0 r_1}. \tag{4}$$

22. In entire agreement with Piccard, I find that more careful attention is necessary in measuring the intensity of the field than has been given it in the past. A search coil and ballistic galvanometer were employed for the purpose. The coil was made by the builders of the magnet, the Société Genevoise, but the data accompanying the coil were found not to be trustworthy. The coil was calibrated by comparison with two precision coils, wound on glass. These two coils were constructed with great care and checked against each other to insure accuracy. Glass cylinders 20 × 6 mm. were ground true and calipered. Glass discs were cemented to their sides, and on each of the spools thus formed, a single layer, consisting of 25 turns of s.s.c. No. 34 copper was wound. The ends were specially arranged to avoid introduction of unevaluated magnetic area. The two coils were found by comparison in a field of 2,000 units to show a discrepancy of less than 0.2 per cent. The value of the area of the Geneva search coil as determined by comparison with the glass coils, may safely be stated to be in error by an amount not exceeding a similar figure.

Dependence was placed on the calculated area of the glass coils.

Experiment showed this to be far more satisfactory than any scheme of calibration of the coils in a calculated field.

The galvanometer was not in any sense used as a measuring instrument. It was calibrated carefully in connection with each reading with the search coils, at the identical deflections observed with the search coils. A standard solenoid and calibrated ammeter were used for the purpose.

23. I will now proceed to consider the data obtained. I give first in full the record of a sample pair of runs.

November 7.	Series 3, Run 37.		Page 217.
Manometer temp., 15.95.	$I = 5.00$ amps.		Cocks open 2.8 sec.
Upper Chamber.—Burette Readings,	4,664 mm.^{3}		
Temp. of burette,	697		
16.55.	3,967		
Temperature correction,	8		
Unabsorbed gas,		3,959	
Lower Chamber.—Burette Readings,	5,143		
Temp. of burette,	3,050		
16.70.	2,093		
Temperature correction,	5		
Unabsorbed gas,		2,088	
Total unabsorbed gas,		6,047 mm.^{3}	

November 7.	Series 3, Run 38.		Page 218.
Manometer temp., 14.90.	$I = 0$ amps.		Cocks open 3 sec.
Upper Chamber.—Burette Readings,	5,790		
Temp. of burette,	251		
15.35.	5,539		
Temperature correction,	9		
Unabsorbed gas,		5,530	
Lower Chamber.—Burette Readings,	3,628		
Temp. of burette,	3,120		
15.55.	508		
Temperature correction,	1		
Unabsorbed gas,		507	
Total unabsorbed gas,		6,037 mm.^{3}	

ΔV:	5,530	2,088
	3,959	507
	1,571	1,581 mm.^{3}

24. In all, 140 runs like these were carried through. Of these, a great many, most of the first and second series, were rejected. Of the third series of 65 runs, a few also were rejected. No run was rejected by reason of its deviation in value for ΔV from other runs. The sole criterion was agreement as to total volume of unabsorbed gas.

There was a considerable unexplained variation in the value of this total volume of unabsorbed gas, but changes did not often occur suddenly. For this reason, readings at zero field were taken frequently, and combination of successive runs in pairs like that shown above gives consistent results. When sudden variations did occur, they were taken as evidence of leakage, or failure to absorb perfectly, and the readings rejected. Attempts were made to correct on a basis of equalization of total volume of unabsorbed gas. The probable error was found to be somewhat reduced in this way, but the mean values did not change appreciably. Results as given are uncorrected.

25. Data from which the average ΔV for $I = 5.00$ amps. was obtained were as follows:

1,543
1,507
1,579
1,527
1,607
1,631
1.504
1,551
1,593
1,594
1,571
1,581
1,527
1,507
1,527
1,528
1,572
1,614
Mean value, 1,559 ± 10.

The averages at different field intensities are given in the table.

ΔV	I	H	No. of Obs.
1,559 ± 10	5.00	7,495	18
268 ± 5	2.00	3,405	17
75 ± 4	1.10	1,833	12
30 ± 7	0.65	1,080	12

26. Applying all corrections as indicated, I find

$$\text{at } H = 7{,}495,\ \kappa = 1.459 \times 10^{-7},$$

$$\text{at } H = 3{,}405,\ \kappa = 1.459 \times 10^{-7};$$

less trustworthy are the other two determinations:

$$\text{at } H = 1{,}833,\ \kappa = 1.41 \times 10^{-7},$$

$$\text{at } H = 1{,}080,\ \kappa = 1.6 \times 10^{-7}.$$

The agreement between the first two figures is partly coincidental. I judge, however, that it is fair to state that the true value differs by less than 0.01×10^{-7} from 1.46×10^{-7}.

27. The discrepancy between this result and that of Piccard exceeds the estimates of probable error. This might be explained in four different ways. (*a*) The error may be larger than it is estimated to be. Experience shows that this is sometimes the case in work involving a number of possibilities of error. (*b*) The susceptibility of carbon dioxide may differ from zero. (*c*) Piccard's result may be in error by reason of failure to correct for dissolved oxygen, or by reason of error in the value for susceptibility of water. (*d*) Oxygen may show saturation phenomena at field intensities lower than Langevin's estimated limit.

The following table may be taken to give the best answer available at the present time to the question of magnetic saturation in oxygen.

Observer.	Field Intensity.	$\kappa \times 10^7$
Curie	100–1,350	1.52
Roop	3,400–7,500	1.46
Piccard	21,000	1.41

Further experiments in continuation of these here described are in progress. Acknowledgment is due the Whiting Fund, from which the magnet was purchased, Mr. W. R. Stamper, who constructed the manometer, and Mr. O. G. Steinitz, who constructed the standard search coils.

BERKELEY, CALIFORNIA,
December 24, 1915.

NOTE ON THE DAMPING FACTOR USED IN BALLISTIC CONSTANTS OF MOVING-COIL GALVANOMETERS.

By Paul E. Klopsteg.

ATTENTION is called in a recent paper[1] to an error arising from the use of the factor $\sqrt{\theta_1/\theta_2}$ in obtaining from the throws of a damped galvanometer the equivalent throws under conditions of no damping.[2] An exact factor, $(\theta_1/\theta_2)^{2t_1/T}$, is given, where t_1 represents the time of the throw and T the complete period of the damped coil; and a curve is drawn from which its value may be obtained if θ_1/θ_2 is known. From the concluding paragraph of the paper cited one might infer that a direct determination of $2t_1/T$ is essential to the evaluation of the exact factor; on account of the difficulty of this determination the correction curve would therefore seem desirable.

In this connection it may, however, be well to note that the solution of the differential equation of motion of a galvanometer coil, under the usual assumptions, leads to the simple and theoretically exact expression for the ballistic constant

$$K = \frac{i}{\varphi} \cdot \frac{T_0}{2\pi} \cdot \rho^{\frac{1}{\pi}\tan^{-1}\frac{\pi}{\Lambda}}, \tag{1}$$

where i/φ is the current constant of the instrument,[3] T_0 the complete period of the *undamped* motion of the coil, ρ the ratio[4] of two consecutive swings and Λ the logarithmic decrement. For small values of Λ this equation becomes

$$K = \frac{i}{\varphi} \cdot \frac{\tau}{2\pi} \cdot \sqrt{\rho}, \tag{2}$$

the degree of approximation being only 0.3 per cent. when Λ is as high as 0.184. Except for precise measurements this is sufficiently accurate. Equation (1) shows $\rho^{1/\pi \tan^{-1}(\pi/\Lambda)}$ to be the exact damping factor, which should be applied when Λ is greater than 0.184, or when higher accuracy is required in cases of smaller damping. Instead of using a curve to obtain from θ_1/θ_2 the value of this factor, it seems decidedly preferable to calculate it directly, since it depends only upon ρ, which is very easily determined. Or, to simplify matters still further, one might use the tables[1] which have been prepared for this purpose.

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[1] Phys. Rev., N.S., VI., 1915, 446: Lindley Pyle, "On the Correct Formula for the Damping Factor in Highly Damped Periodic Motion of the Coil of a D'Arsonval Galvanometer."

[2] Zeleny and Erikson Manual, 3d ed., 1912, p. 134; A. W. Smith, Phys. Rev., O.S., XXII., 1906, 250; Smith, Electrical Measurements, 1914, p. 147.

[3] In a paper not yet published the writer shows this factor, as ordinarily determined, to be the least accurate one involved in this equation. A method is also given for obtaining its value more exactly.

[4] ρ is identical with θ_1/θ_2 used above.

PHYSICAL LABORATORY,
UNIVERSITY OF MINNESOTA,
January 10, 1916.

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[1] Kohlrausch, Lehrbuch der praktischen Physik, 12th edition, 1914, p. 723; also Kohlrausch, 3d Eng. from 7th Ger. ed., 1894, p. 447.

THE DIFFRACTION RING PATTERN IN THE SHADOW OF A CIRCULAR OBJECT.

By Mason E. Hufford.

DURING a series of experiments[1] in producing diffraction patterns by use of circular openings and steel spheres, it was observed that the concentric ring system produced by interference in the shadow of a disc or sphere was clearly brought out.

Lommel[2] has deduced formulas for calculating the radii of these rings and has compared the calculated and measured values for a few rings in microscopic diffraction patterns.

Since the number of rings here shown is much larger than that mentioned or shown by others,[3] an opportunity is offered to compute the radii by Lommel's formulas for many more rings and to compare computed and measured values over a much wider range.

In Lommel's expression,

$$M^2 = C^2 + S^2,$$

which represents the intensity of light at points in the geometric shadow, the integrals C and S involve quantities,

$$y = \frac{2\pi(a + b)r^2}{\lambda ab} \quad \text{and} \quad z = \frac{2\pi rR}{\lambda b},$$

in which a is the distance of a point source of monochromatic light of wave-length λ from the diffracting object of radius r, b is the distance of the object from the screen where the shadow is observed, and R is the distance from the center of the geometric shadow to the point in the shadow where the intensity of light is observed.

In order to determine the presence of points of maximum and minimum intensity of light in the shadow, the intensity M^2 is differentiated in the usual way with respect to a factor z. The result shows that maxima and minima of intensity will occur when,

$$V_0 = 0$$

where,

$$V_0 = J_0(z) - \left(\frac{z}{y}\right)^2 J_2(z) + \left(\frac{z}{y}\right)^4 J_4(z) - \cdots$$

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[1] Phys. Rev., N. S., Vol. 3, No. 4, April, 1914.

[2] Lommel, Abhandlungen der Bayerische Academie der Wissenschaften, B. 15, 1886.

[3] Croft, Phil. Mag., S. 5, V. 38, 1894. Arkadiew, Phys. Zeit, 14, 1913.

and J_0 is the Bessel function. If y is large V_0 becomes equal to J_0 and the values of z for $J_0 = 0$ are approximately the values of z for $V_0 = 0$.

The values of y and z in $V_0 = 0$ plotted give a series of curves as shown in Fig. 1. The dotted lines are the graphs of $J_0 = 0$. The line $y = z$ marks the boundary of the geometrical shadow. All parts of the curves above this line apply to the shadow region. The parts of the curves which give minima are ruled heavy while the parts giving maxima are ruled light.

The possibility of using the values of z for which $J_0 = 0$ instead of those for which $V_0 = 0$ suggests itself.

The first ten of the values given in the following table of root values of the Bessel function were taken from tables. Beyond the tenth the values were calculated by McMahon's formula.[1] They are here given to the twenty-fifth value.

Z	Value of z by Formula.	Z	Value of z by Formula.	Z	Value of z by Formula.
Z_1	2.4048255577	Z_{10}	30.6346064684	Z_{19}	58.90712206
Z_2	5.5200781103	Z_{11}	33.7758991042	Z_{20}	62.04861462
Z_3	8.6537279129	Z_{12}	36.9171861194	Z_{21}	65.19011752
Z_4	11.7915344391	Z_{13}	40.05852068	Z_{22}	68.33162936
Z_5	14.9309177086	Z_{14}	43.19989382	Z_{23}	71.47314895
Z_6	18.0710639679	Z_{15}	46.34129753	Z_{24}	74.61450030
Z_7	21.2116366299	Z_{16}	49.49272626	Z_{25}	77.75602563
Z_8	24.3524715308	Z_{17}	52.63417501		
Z_9	27.4934791320	Z_{18}	55.76564161		

It is clear that if the value of y were infinite the values of z in $V_0 = 0$ would coincide with the values of z in $J_0 = 0$. Since the value of y is finite and rather small, the values of z corresponding to the value of y as they are taken from the graphs will differ more and more from the values of z in $J_0 = 0$ as they are taken from larger and larger graphs. It is clear that for the largest ring in any diffraction pattern, the value of z in $J_0 = 0$ will lack most of being the value of z for $V_0 = 0$. It is sufficient therefore to determine whether the value of z in $J_0 = 0$ may be substituted for the value of z in $V_0 = 0$ for the largest ring in any pattern.

This may be done if the error introduced is not greater than the experimental error.

To show how V_0 depends upon z we have:

$$\frac{dV_0}{dz} = -J_1(z)\left(\frac{z}{y}\right)^2 + J_3(z)\left(\frac{z}{y}\right)^4 - J_1(z)\left(\frac{z}{y}\right)^2 + \cdots,$$

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[1] Annals of Mathematics, V. 9, 1894–5, p. 25.

since $J_0 = 0$ and,

$$J_n' = J_{n-1} - \frac{n}{z} J_n.$$

The Bessel functions with higher indices are decreasing quantities and also within the geometric shadow the value of y is always greater than that of z. Therefore the fractions will decrease rapidly and the expression is converging.

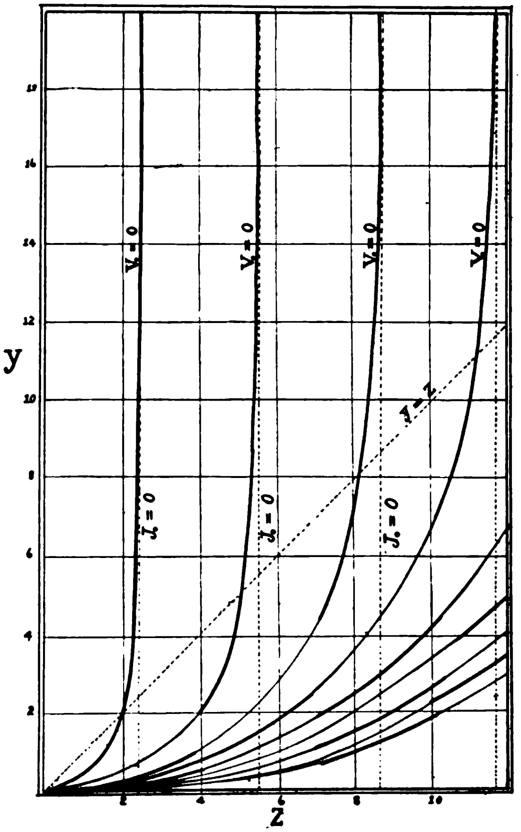

Fig. 1.

The accompanying photographs are the shadows of steel spheres, .317 cm., .6345 cm. and .9517 cm. in radius. These were obtained by apparatus similar to that described in a previous article.[1] The light from an arc was filtered through double thickness violet colored glass. From photographs of the spectrum of this filtered light it was observed that the most effective light which passed through the glass was that of wave-length, 3,880 A.U.

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[1] See Phys. Rev., N. S., April, 1914, p. 243, Fig. 2.

By means of suitable lenses this light was focused on an opening .027 cm. in diameter. The light then traversed a box 3,255 cm. in length. At a distance of 1,550 cm. from the opening the spheres were suspended by means of very fine wires. The exposures were about six hours in length.

Substituting the values, $a = 1{,}550$, and $b = 1{,}705$ in the formula,

$$y = \frac{2\pi(a + b)r^2}{\lambda ab},$$

y for the smallest sphere is 20.02 and for the next larger one, $y = 80.23$.

The photographic negatives show that in the shadow of the small sphere, as shown in Photograph A, there are six rings representing minima of light intensity. In the shadow of the next larger sphere, as shown in Photograph D, there are about twenty-five rings representing minima of intensity. Photograph C is the shadow of the largest sphere used and Photographs B and E are enlargements.

The sixth and twenty-fifth roots of the Bessel function are, 18.0710639679 and 77.7560256302 respectively. These values apply to the sixth ring in the smallest pattern, Photograph A and to the twenty-fifth ring in the larger pattern, Photograph D. Whether these and smaller values may be used for calculating the radii of the rings depends upon how V_0 changes with respect to z in the immediate vicinity of these values.

If z is taken as 17 for the smallest sphere and as 75 for the next in size, then dV_0/dz for the former is .268 and for the latter it is, .2718. If the actual values of z could be found they would probably be larger than these and therefore the rates of variation would be less.

For calculating the first of these rates, the values of J_1, J_3, J_5, etc., were taken from tables in Gray and Matthew's Treatise on Bessel Functions. For calculating the second of the rates, the corresponding values of J_1, J_3, etc., were found by formulas given in Gray and Matthews, pp. 40 and 13.

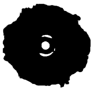
These values are given below:

$J_1(75)$	-0.085137365	$J_5(75)$	-0.078166306	$J_9(75)$	-0.0540197166
$J_2(75)$	-0.03691648	$J_6(75)$	-0.0573188674	$J_{10}(75)$	-0.0834193863
$J_3(75)$	$+0.083168445$	$J_7(75)$	$+0.0689952873$	$J_{11}(75)$	$+0.0317745466$
$J_4(75)$	$+0.046896693$	$J_8(75)$	$+0.0701979877$		

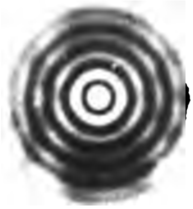
Six terms of the expansion, dV_0/dz were used in calculating the rates of variation.

Physical Review, Vol. VII., Second Series.
May, 1916.

Plate I.
To face page 548.

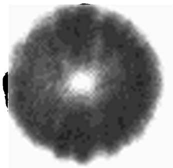


A.

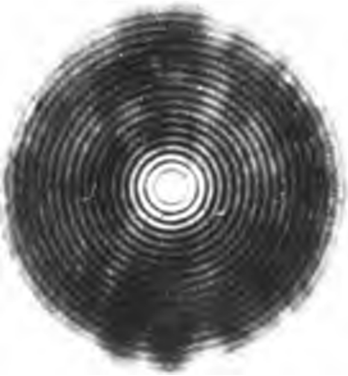


B.

C.



D.



E.

MASON E. HUFFORD.

The results of several trials showed that there was an error of about one per cent. in setting the cross hair of the measuring instrument on the transparent rings of the negatives.

Since the rather large rates of variation of V_0 with z apply only to the last rings of the patterns, it was thought sufficiently accurate to use the root values of z for calculating the radii of the diffraction rings.

The calculated and measured values are given below with the difference between them expressed as a per cent. of the calculated value.

a = 1,550 cm. b = 1,705 cm.

λ = 3,880 A.U. D = .027 cm.

Sphere No. 1. *Radius* .317 *cm.* y = 20.02.

No.	Calculated Cm.	Measured Cm.	Diff. in Per Cent.
1	.079	.080	+1.2
2	.183	.184	+0.5
3	.287	.287	+0.1
4	.391	.386	−1.4
5	.485	.483	−0.6
6	.600	.575	−4.0

Sphere No. 2. *Radius* .6345 *cm.* y = 80.23.

No.	Calculated Cm.	Measured Cm.	Diff. in Per Cent.	No.	Calculated Cm.	Measured Cm.	Diff. in Per Cent.
1	.039	.042	+7.0	13	.663	.652	−1.6
2	.091	.092	+1.5	14	.715	.704	−1.5
3	.143	.144	+0.5	15	.767	.755	−1.6
4	.195	.194	−0.3	16	.819	.802	−2.1
5	.247	.243	−1.7	17	.871	.858	−1.4
6	.299	.292	−2.4	18	.923	.910	−1.8
7	.351	.344	−2.0	19	.975	.958	−1.9
8	.403	.396	−1.8	20	1.027	1.013	−1.3
9	.455	.436	−4.2	21	1.079	1.062	−1.6
10	.507	.499	−1.8	22	1.132	1.113	−1.7
11	.559	.549	−1.7	23	1.184	1.162	−1.8
12	.611	.601	−1.6	24	1.236	1.208	−1.4

The above calculated values of the rings were found from the formula:

$$R = \frac{2\pi r}{\lambda b z}.$$

The measured values are the averages of four independent measurements of each radius. These were made by means of a comparator.

While it was possible to count twenty-five rings on the larger negative without the aid of a microscope it was found that there was so much lack of detail near the edge of the shadow that the cross hair could not be set on the outermost ring with any degree of accuracy. Twenty-four of the twenty-five rings were therefore measured.

In conclusion I wish to express my thanks to Prof. A. L. Foley, who has made many valuable suggestions throughout the progress of this work. I wish also to thank Prof. R. D. Carmichael, formerly of Indiana University, for valuable hints in the mathematical part of the work. I am indebted to Indiana University for placing the apparatus at my disposal and to the Chicago University Library for the loan of useful books.

LABORATORY OF PHYSICS,
INDIANA UNIVERSITY,
December, 1915.

THE EFFECT OF TEMPERATURE ON THE RESISTANCE, THE LIGHT-SENSITIVENESS, AND THE RATE OF RECOVERY OF CERTAIN CRYSTALS OF METALLIC SELENIUM.

By Kathryn Johnstone Dieterich.

THE purpose of this article is to set forth certain facts concerning the effect of variation of temperature on the rate of recovery after illumination, the sensitiveness to light, and the resistance of certain crystals of metallic selenium.

Apparatus.

The apparatus used is shown in Fig. 1. The crystal was placed between platinum electrodes, *E*, such that the weight, *W*, suspended from the upper electrode, maintained a constant pressure on the crystal.

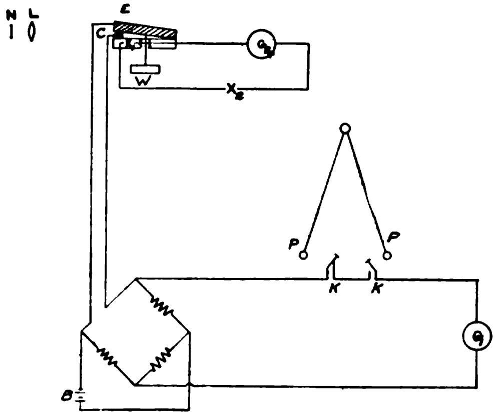


Fig. 1.

The source of illumination was a Nernst glower, *N*, at a distance of 45 cm., the intensity of illumination being kept constant by maintaining over the lamp a voltage of 115 volts from storage batteries. The crystal constituted one arm of a Wheatstone bridge circuit, over which a constant voltage of 10 volts was kept. As nearly as possible all conditions of light intensity, pressure, voltage, and time of recovery were kept constant and only temperature varied. The latter was measured by means of a copper-constantin thermocouple in series with a galvanometer. One junction, X_1, was placed beside the crystal, the other, X_2, was kept at 0° C.

In carrying out this investigation it was necessary, at each temperature, to measure the resistance of the selenium when it had reached equilibrium in the dark, in the light, and after it had been in the dark the desired short interval of time, viz., .02 sec. The first two measurements were made by the usual balanced bridge method, the last by the method of Brown and Clark.[1] This consisted in balancing the Wheatstone bridge circuit with the selenium in equilibrium in the light, and then opening the galvanometer circuit. A timing pendulum, which had been arranged to operate a metal shutter to cut off the light from the crystal and throw the galvanometer in circuit simultaneously, was then released. A deflection of the galvanometer resulted, due to the change in resistance of the crystal when the light was removed. The resistance of the crystal at the end of .02 sec. in the dark was determined, later, by substituting for the crystal a carbon resistance, which could be adjusted, until upon releasing the pendulum, the same galvanometer deflection was obtained as had been given by the crystal. The carbon resistance, since it was constant, was then measured by balancing the bridge circuit.

THEORETICAL.

When a crystal of metallic selenium has been exposed to light, and the light is suddenly removed, the conductivity of the crystal does not return instantly to the value of the equilibrium dark conductivity, but decreases rapidly at first, then more slowly, requiring several minutes or even hours to attain its final value, depending on the temperature. When a gas is exposed to a source of ionization, and the source is suddenly removed, the conductivity decreases rapidly at first, then more slowly, until equilibrium is restored. In gases the decrease in conductivity during the first short interval of time is ascribed to the recombination of the conducting electrons with the positive residues. It has been shown by Rutherford,[2] McClung,[3] McClelland[4] and more recently, Plimpton,[5] that if there are present n_1 negative ions and n_2 positive ions, the rate of recombination is directly proportional to $n_1 \times n_2$, that is

$$\frac{dN}{dt} = -\alpha n_1 \times n_2,$$

where α is a constant and is called the coefficient of recombination.

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[1] Phys. Rev., Series 1, XXXIII., p. 53, 1911.

[2] Phil. Mag., Series 5, Vol. 44, p. 422, 1897; Vol. 47, p. 109, 1899.

[3] Phil. Mag., Series 6, Vol. 3, p. 283, 1902.

[4] Phil. Mag., Series 5, Vol. 46, p. 29, 1898.

[5] Phil. Mag., Series 6, Vol. 25, p. 65, 1913.

In order to study the effect of pressure, light intensity, and electromotive force on the rate of recovery in selenium, in cells and crystals, Professor Brown[1] assumes that for the first short interval of time after the light has been removed the decrease in conductivity of a selenium cell or crystal is due to the recombination of the conducting electrons with positive residues, and that this recombination takes place according to the same law as that which holds for gases. If the number of electrons taking part in the conduction is proportional to the conductivity then the rate of recombination may be calculated from the formula:

$$\alpha_1 = -\frac{\Delta C}{C^2 \Delta t},$$

where ΔC is the decrease in the conductivity during the time, Δt, that the crystal has been in the dark, C is the equilibrium value of the conductivity in the light, and α_1 is proportional to the coefficient of recombination. In order to obtain this result, Professor Brown found it necessary to make the following assumptions:

1. When the crystal is in equilibrium in the light or in the dark the conductivity is proportional to the number of negative electrons taking part in the conduction.
2. Recombination takes place according to the same law as in gases.
3. The rate of recombination is the same in the light as in the dark.

The above conception was verified approximately by varying the conductivity over a wide range. This was done by changing the intensity of illumination. Since the rate of recombination was found to be nearly proportional to the square of the conductivity it was assumed that the recombination followed the same law as in gases.

In this paper the results of an investigation of the effect of temperature on the rate of recovery of selenium crystals will include a discussion of the effect of temperature on the coefficient of recombination, calculated on the above theory.

Rate of Recovery.

The crystals used were prepared by Dr. Brown[2] as described by him. They were of three kinds: the large hexagonal, the needle shaped acicular, and the monoclinic lamellar. In each case the current was passed through the crystal in a direction perpendicular to the direction of the long axis, and the crystal illuminated in a direction perpendicular to the direction of the current.

In general the coefficient of recombination was found to decrease with increase in temperature.

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[1] Phys. Rev., Series 2, Vol. 5, p. 395, 1915.

[2] Phys. Rev., Series 2, Vol. 5, p. 236, 1915; Vol. 4, p. 85, 1914.

TABLE I.

Temperature in °C.	Temperature in °Abs.	Conductivity in Light. Ohms^{-1} $\times 10^7$.	$\alpha \times 10^{-7}$.	$\alpha \times$ C.
−89	184	1.685	3.640	6.11
−68	205	2.057	3.280	6.75
−40	233	3.580	1.680	6.02
0	273	7.070	0.875	6.17
19	292	7.460	0.835	6.22
21	294	7.880	0.754	5.93
26	299	8.070	0.730	5.89
33	306	8.660	0.606	5.23
43.5	316.5	9.070	0.548	4.95
56	329	9.310	0.537	4.99
64	337	5.890	0.367	2.17
75	348	8.320	0.316	2.62

Table I. shows the results obtained with hexagonal crystal No. 2. There is seen to be a large fall in the value of α with increase of temperature from − 89° C. to about 0° C., after which the decrease is more gradual to 75° C. which is as high a temperature as could be reached. Upon examining the electrodes it was discovered, early in the work, that at the higher temperatures a dark deposit was formed between the crystal and the electrodes. The effect of this deposit was to depress both the values of conductivity and of α, so that the last two points on the curve, Fig. 2, showing the relation between α and temperature, are only approximately correct.

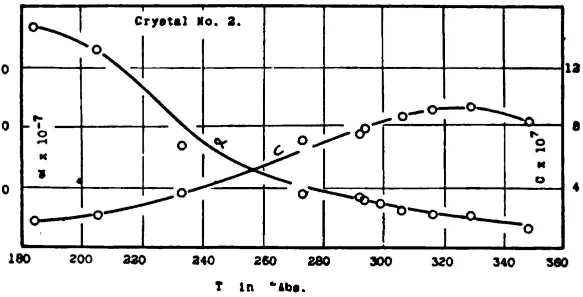

Fig. 2.

Crystal No. 13, Table II., shows in general the same result; a large decrease in α with increased temperature. In this case the temperature was not high enough to produce sufficient deposit to cause any appreciable error.

The curve, Fig. 3, shows a slight depression in the values of α at 10° C., which may or may not be a genuine effect. However, two complete series of readings gave a low value of α at just this temperature. It may be of interest to mention here that, although a crystal had been removed from the electrodes for several weeks, it could be returned and, if the previous conditions as to light intensity, pressure, and voltage were restored, the same curve of α and temperature was obtained, giving the same values of α as before.

Table II.

Crystal No. 13. E.M.F. = 6 volts. Pressure = 75 gr.

Temperature in °C.	Temperature in °Abs.	Conductivity in Light. Ohms^{-1} × 10^7.	$\alpha \times 10^{-7}$.	$\alpha \times C$.
−40	233	2.63	2.010	5.26
− 7	266	4.35	0.925	4.03
10	283	7.10	0.460	3.26
21	294	6.05	0.548	3.31
32	305	6.82	0.398	2.73
41	314	7.38	0.348	2.57

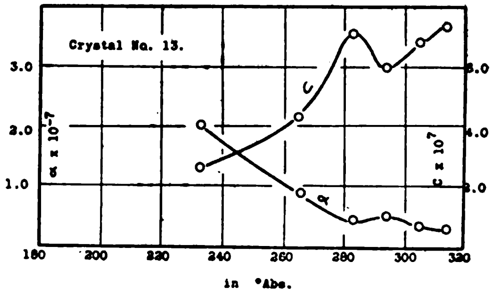

Fig. 3.

Other crystals showed essentially the same change of α with temperature. In some cases there were depressions as in No. 13; one crystal, No. 3, a lamellar type, showed quite a sharp kink at 35° C. Thus, each of the three types of crystals showed the same effect of temperature on the coefficient of recombination: a general decrease in α with increase of temperature.

H. A. Erikson,[1] in 1909, made a determination of the effect of temperature upon the coefficient of recombination in gases. He obtained a curve showing the relation between α and T which resembles very closely the curve obtained for crystal No. 2, Fig. 2. His curve is shown in Fig. 4. However, no definite significance can be attached to this striking similarity as a result of this investigation only.

[1] Phil. Mag., Series 6, Vol. 18, p. 328, 1909.

If then, the recovery of selenium crystals is due to a recombination of negative electrons with positive residues, and the rate of recombination is expressed by the formula,

$$\frac{\Delta C}{\Delta t} = -\alpha C^2,$$

the coefficient of recombination, α, varies with temperature.

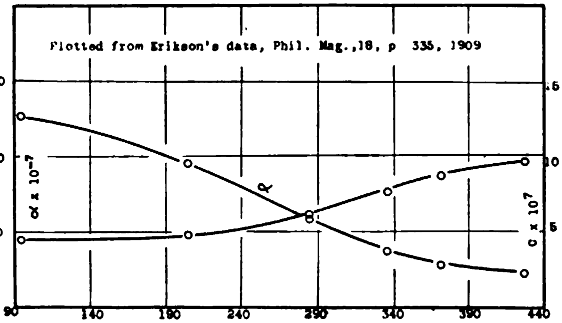

Fig. 4.

It will be noticed, in Tables I. and II., that, while the coefficient of recombination is not a constant at all temperatures, the product, $\alpha \times C$, is very nearly so. A glance at the curves, Figs. 2 and 3, will show that this relation suggests itself here also; since wherever a rise occurs in the α-curve a depression appears in the C-curve. The same general relation between the shape of the α-curve and the C-curve is apparent in Erikson's work in gases, Fig. 4. Dr. Brown[1] has found that $\alpha \times C$ is a constant for selenium crystals when pressure or voltage varied the conductivity, although α is not constant.

Since $\alpha \times C$ is a constant and is equal to $\Delta C/C \times \Delta t$, the rate of recovery during the first short interval of time is equal to some constant, β, times the conductivity in the light.

$$\Delta C/\Delta t = \alpha C^2 = \beta C;$$

where β is equal to $\alpha \times C$, and we have the simple relation that the rate of recovery of selenium crystals is proportional to the conductivity in the light. This proportionality did not appear to hold with much accuracy at the higher temperatures.

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[1] *Loc. cit.*

Resistance.

Conductivity is determined by taking the reciprocal of the resistance, so that a study of the effect of increase of temperature upon the conductivity of selenium crystals should give an adequate idea of its effect on their resistance. In crystal No. 2, Fig. 2, it will be seen that the conductivity in the light at low temperatures was small, increased with increasing temperature, and passed through a maximum in the region of 55° C. The fall of conductivity above 55° C. might have been influenced by oxidation of the brass electrodes which were used with this crystal, so that the apparent decrease was somewhat exaggerated. Crystal No. 13, Fig. 3, showed a general increase of conductivity in the light with increased temperature. There was, however, a decrease for a short interval from 10° C. to 21° C., after which the conductivity rose again as the temperature increased. Depressions of this kind were found in the *C*-curves of many of the crystals, although not always at this same temperature. One crystal, No. 4, had a sharp depression and sudden rise in the region of 0° C. Ries[1] found the same condition to hold for selenium cells, with the minimum most often around 100° C., but that he was able to shift the position of this minimum to below 0° C. by certain heat treatment. The general tendency, however, in cells and crystals is for an increase of conductivity, and a consequent decrease of resistance, to result from an increase in temperature.

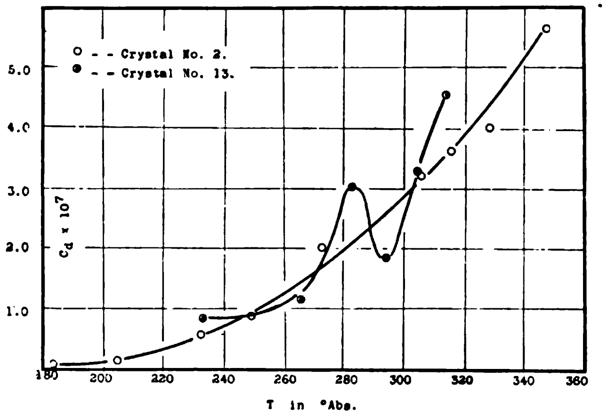

Fig. 5.

The equilibrium dark conductivity will be seen from the curves in Fig. 5 to follow much the same general tendency: an increase of conductivity with increase of temperature. The same maxima and minima which were found in the light conductivity curves are apparent here.

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[1] Die Elektrischen Eigenschaften und die Bedeutung des Selens für die Electrotechnik, 1908.

LIGHT SENSITIVENESS.

The definition of light sensitiveness may be formulated arbitrarily to best meet the needs of practical use, as in the operation of galvanometers and relays, or it may arise from purely theoretical considerations arrived at in an attempt to explain the effect of light on crystals. Several of the more common expressions for light sensitiveness determined by practical usage are,

1. The ratio of the change of resistance to the resistance in the dark. $(R_d - R_l)/R_d$.

2. The ratio of the resistance in the light to the resistance in the dark. R_l/R_d.

3. The difference between the conductivity in equilibrium in the light and in the dark. $C_l - C_d$.

In the third case it is necessary to know not only this difference but also the terms which produce it, namely, C_l and C_d, to form an adequate idea of the possibility of adapting the crystal to the work at hand. For the same reason one would need to know the intensity of the light producing this change.

However, the theory which underlies this paper makes it reasonable to define sensibility in still another way, namely, in terms of the rate of production of conducting electrons. Dr. Brown[1] has shown how this rate of production may be calculated.

$$\frac{dC}{dt} = M + q;$$

where dC/dt is the rate of production of electrons when the crystal is in equilibrium in the light; q is the rate of production in the dark and M is the rate of production by the light. dC/dt must also be equal to the rate of recombination, αC^2, since the crystal is in equilibrium. For the first few hundredths of a second after the illumination is removed

$$dC/dt = \alpha C^2 = M + q.$$

When the crystal is in equilibrium in the dark the rate of production of electrons, q, is equal to the rate of recombination, αC_d^2, and M is zero. When the crystal is in equilibrium in the light

$$\Delta C/\Delta t = M + \alpha C_d^2;$$

$$M = \Delta C/\Delta t - \alpha C_d^2$$

$$= \alpha C_l^2 - \alpha C_d^2.$$

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[1] *Loc. cit.*

TABLE III.

Crystal No. 2, in °Abs.	C_l (Ohms$^{-1}\times 10^7$.)	C_d(Ohms$^{-1}\times 10^7$.)	C_l—C_d (Ohms$^{-1}\times 10^7$.)	M
184	1.685	.106	1.58	10.2
205	2.057	.135	1.92	13.8
233	4.190	.583	3.61	26.8
273	7.070	2.150	4.82	32.47
293	7.880	2.280	5.60	34.92
299	8.070	3.030	5.04	35.37
306	8.660	3.230	5.43	39.10
316	9.070	3.620	5.45	38.00
326	9.310	4.080	5.23	37.80
348	8.320	5.670	2.65	11.60
Crystal No. 13.				
233	2.630	.588	2.042	12.21
266	4.350	1.150	3.20	16.25
283	7.100	3.030	4.07	19.08
294	6.050	1.830	4.22	18.17
305	6.820	3.280	3.54	14.29
314	7.380	4.550	2.83	11.80

Table III. and Fig. 6 show the effect of temperature upon light sensitiveness as defined by the practical working definition, $C_l - C_d$. It will be seen that the sensitiveness at first increases with increase of temperature to about the region of room temperature after which it decreases. Prof. L. P. Sieg[1] has noted that selenium plates show a decrease in sensitiveness to light, from room temperature to 100° C. when the light sensitiveness is defined as the ratio of conductivity in the light to the

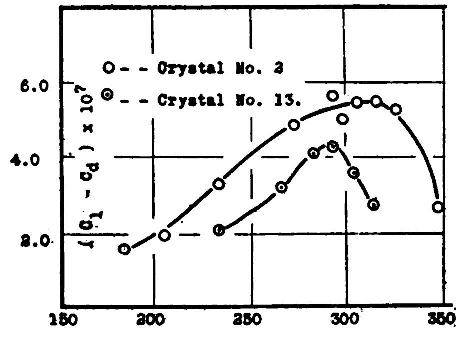

Fig. 6.

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[1] Phys. Rev., Series 2, Vol. 6, p. 213, 1915.

conductivity in the dark. Fig. 7 shows the same general relation between temperature and light sensitiveness, when the latter is measured by the rate of production, M, of electrons by the light.

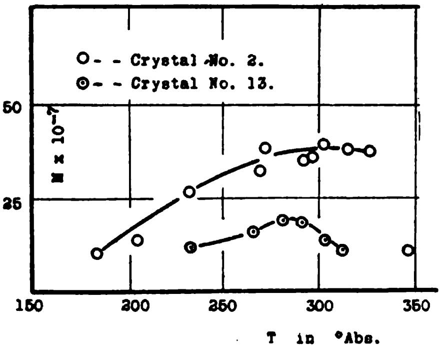

Fig. 7.

SUMMARY.

In conclusion, it may be said in regard to the effect of temperature on certain properties of selenium crystals, that,

1. The resistance in the light and in the dark is, in general, decreased by increase of temperature, with a small increase in some cases in the region 0° C. to 50° C.

2. The coefficient of recombination, α, calculated according to the formula,

$$\alpha = -\Delta C/C_1^2\Delta t,$$

is not constant at all temperatures but decreases with increase of temperature in much the same manner that the coefficient of recombination decreases in gases with increase in temperature.

3. Over a wide range of temperatures the rate of recovery for the first few hundredths of a second is proportional to the conductivity in the light.

$$\Delta C/\Delta t = -\beta C_1.$$

4. The sensitiveness to light does not vary greatly over a wide range of temperature, although a tendency toward a maximum is found in the region 30° C. to 50° C.

I wish to acknowledge my indebtedness to the staff of the Department of Physics for their interest in the problem, and especially to Dr. Brown for suggesting it, and for his encouragement during the progress of the work.

PHYSICAL LABORATORY,
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SUB-HELMHOLTZIAN VIBRATIONS OF A RUBBED STRING.

By Harry Clark.

I. Statement of the Problem and Review of Previous Investigations.

THE problem of the longitudinal vibration of a rubbed string has been extensively studied by Davis,[1] to whom it was suggested by its usefulness in the study of the effects of mechanical stress upon the magnetic properties of materials. The writer has undertaken a further study of the subject with a two-fold purpose: first, of determining more carefully the conditions necessary for the production and accurate duplication of the various wave forms with unfailing regularity; and second, of studying in detail particular conditions of rubbing hitherto uninvestigated.

A satisfactory solution of the first of these problems is due largely to the adoption of a new rubbing device whose action is extremely uniform and reliable. The second problem refers to the so-called sub-Helmholtzian vibrations, which under favorable circumstances may be produced by rubbing with slight pressure. These waves were investigated by means of an improved photographic method of study. Several features of interest are presented by them.

In addition to the study of sub-Helmholtzian waves, the experiments to be described have verified nicely some of the previous work on Helmholtzian forms, and have brought out a new criticism of the laws governing them. A complete understanding of the work will be facilitated by reviewing briefly at this point some of the studies of transverse vibration.

The problem of the bowed string cannot be successfully attacked by the use of analysis without a complete knowledge of the nature of the bowing process. Much of this knowledge is embodied in the following laws:

Young's Law.[2]—No overtone is present which would have a node at the point of excitation.

Helmholtz's Law.[3]—When a string is bowed at an aliquot point ($1/q$), the part of the string immediately under the bow moves to and fro with

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[1] Proc. of the Am. Acad. of Arts and Sci., Vol. XLI., No. 32, May, 1906.

[2] Phil. Trans. of the Roy. Soc. of London; 901, 106, 1800.

[3] Proc. Glasgow Phil. Soc., Dec. 19, 1860; reprinted in Phil. Mag., Ser. 4, 21, 393, 1861.

constant velocities whose ratio is equal to the ratio $(1/(q-1))$ of the segments into which the string is divided by the point in question.

Helmholtz's Velocity Law.—The smaller of these two velocities has the same direction as that of the bow and is equal to it.

From these facts Helmholtz concluded that the action of the bow is accounted for by something analogous to the difference between the coefficients of static and moving friction. That wave will be developed which allows the element of string under the bow to travel as much of the time as possible, in the direction of the bow's motion, without slipping.

On the basis of these laws Helmholtz solved completely all aliquot cases of the problem. His solution is given by

$$u = u_1 - u_2,$$

where u_1 and u_2 are defined by

$$u_1 = A \sum_n \frac{1}{n^2} \sin \frac{n\pi y}{l} \sin \frac{n\pi t}{T}, \tag{1}$$

$$u_2 = A \sum_m \frac{1}{q^2 n^2} \sin \frac{qm\pi y}{l} \sin \frac{qm\pi t}{T}, \tag{2}$$

u represents the displacement of a point, y, on the string at time, t. The length of the string is l, and the period of a complete vibration is $2T$, m and n are integers, A is constant for a given speed of bowing, and q has been defined above. Equation (1) is called Helmholtz's *major* solution. The value of u, determined by it, is seen to be independent of q, which defines the place of bowing. Equation (2), the *minor* solution, serves, for any particular value of q, to exclude those partials whose presence would violate the law of Young. It becomes relatively unimportant for large values of q.

Rational bowing points, which include the aliquot cases of course, may be solved if the law of Helmholtz be replaced by

Krigar-Menzel's Law.[1]—When a string is bowed at any rational point (p/q), where p and q are prime to each other, the part of the string immediately under the bow moves to and fro with constant velocities whose ratio depends only on q and is $1/(q-1)$.

These four laws constitute the established theory of rational cases of strings transversely bowed. No investigations of other cases have been published. It is to be noted that the solutions which have been obtained are considered to be independent of the nature of the bow and of the force with which it is pressed against the string. All waves obeying the above laws, whether transverse or longitudinal, are called *Helmholtzian waves.*

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[1] Inaugural Dissertation, Berlin, 1888.

Although the theories of transverse and longitudinal waves are very much alike, it is desirable to note some of the differences between the two classes of phenomena in order to see for what reasons a particular interest attaches to the study of longitudinal forms.

Sharp corners of transverse waves are quickly rounded off by the quality of "stiffness" in the string. In the case of longitudinal vibrations, however, these sharp corners are replaced by discontinuities of density. Such discontinuities are not difficult of conception, and there is no evidence to show that they exercise any distorting influence on the waves. On the theoretical side therefore the longitudinal is the simpler type. Since the frequency of longitudinal vibrations involves only the nature of the material in the string and the length of the vibrating portion, it remains very nearly constant. The operation of longitudinal "bowing" may by mechanical means be made continuous, so that a given state of vibration may be maintained indefinitely.

The very complete investigation by Davis of longitudinal Helmholtzian waves is briefly reviewed below.

He used for the "string" a steel wire 4 mm. in diameter and somewhat less than 7 meters in length, with ends firmly clamped to massive iron blocks. A knife edge, pressed lightly against the wire at the middle, divided it into two equal segments, one of which was reserved for observation. In the other was placed the "bow," consisting of a motor-driven wheel, of which the face was covered with well-resined chamois skin.

The methods of observation were two. In the first, a very small glass ball, perhaps 1 mm. in diameter, was cemented to the wire, brilliantly illuminated from a point source, and observed through a microscope. From the limits of its motion the amplitude of vibration was obtained. Such amplitudes, measured at many points along the wire, and plotted as though the motions were transverse, gave the envelope of the wave form. In the second method, a short piece of a needle was attached to the wire, and the point was allowed to trace a record of its motion upon a smoked glass drawn transversely underneath. Although the amplitudes employed were very small, only a few tenths of a millimeter, very perfect tracings were obtained.

By the experiments it was shown that all of the laws of transverse vibration apply without change to the longitudinal forms. A large number of cases were investigated, and the verification is quite complete.

Among other contributions of Davis is the *integral surface*, a graphical construction designed to facilitate the study of both longitudinal and transverse Helmholtzian waves. It gives the motion of all points on the string if the motion of one point be completely known. Since any curve may be represented approximately by a broken line of straight segments, the usefulness of the integral surface may be extended to types of waves other than Helmholtzian. The original paper[1] should be consulted for a complete description of the method.

Davis discovered that when, under certain circumstances not well understood, the pressure of the rubbing wheel against the wire is decreased, a point is reached below which the waves, though quite stable, cease to be Helmholtzian. Their amplitudes (observed by his first method only) were found to be too small to fit Helmholtzian theory. In some instances there were maintained waves with amplitudes scarcely more than six-tenths as great as those of high-pressure waves. To distinguish waves of this type they are called *sub-Helmholtzian.*

II. Apparatus and Methods of Use.

The string used in these experiments was a steel wire .101 cm. in diameter, of the quality commonly known as "music wire." It was carefully straightened and nickel-plated. To secure consistent results it was found that the use of a simple vibrating segment is preferable, although the difficulties of observation are thereby increased.

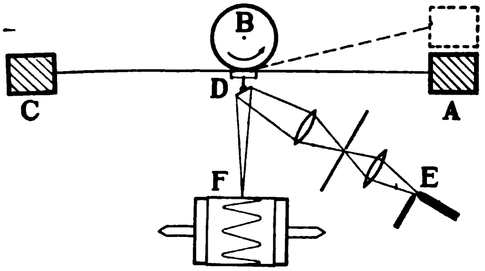

Fig. 1.

Thus arranged the vibrating portion of the wire is ten meters long. This gives a frequency of 254 vibrations per second corresponding to a pitch very nearly that of C_3. The low frequency of vibration allows the recording mechanism to work under more favorable conditions.

The wire was stretched horizontally, as shown in Fig. 1. One end is held firmly in the jaws of a vise, *A*, weighing about 6 kg., which can be moved by a feed-screw horizontally in a direction perpendicular to that of the wire. By means of this adjustment the wire can be pressed against the rubbing wheel, *B*, with any desired pressure. At the other end the wire, after passing between the jaws of a second vise, *C*, and over a pulley, supports a load which serves to hold the wire under tension. This second vise, weighing also 6 kg., is suspended by a two-point support, so that although by its inertia it furnishes an effective end for the vibrating segment, it does not interfere with the slight longitudinal motion of the wire necessary to maintain a constant tension under varying conditions of temperature and adjustment. Although the tension has no direct effect on longitudinal waves, its exact value must be known for the computation of rubbing pressures from the settings of the feed screw.

Many different rubbing wheels were tried, some of which will be considered in detail later. They rotate about a vertical axis on centers which are mounted on the slide-rest of a small lathe-bed. This mounting makes possible the necessary adjustments of the wheels with respect to the wire. Power for rotation is furnished by a 1/8 H.P. induction motor, of which the speed under favorable conditions has been found to remain very nearly constant.

A transmission unit, consisting of a pair of counter shafts with cones of pulleys, reduces properly the motor speed, so that by shifting the belts the rubbing wheel can be given a peripheral speed having any one of thirty-three values between 6 and 50 cm. per second. Greater variety of speeds is obtained by changing the pulley of the motor shaft. An essential part of the transmission is a small fly-wheel mounted on centers and nicely balanced. This is placed close to the rubbing wheel and influences it through the single belt which furnishes power to both. All pulleys are of rolled brass, carefully turned and balanced. Endless braided linen belts, such as are used on dental engines, transmit the motions, and the design of the transmission prevents slipping and automatically regulates the tension of these belts. The constant positive motion of the rubbing wheel thus made possible has contributed much to the success of the experiments.

To examine the motion of a point on the wire a photographic method is used. A small plane mirror suitably mounted to allow free rotation about a fixed, vertical axis communicates with the wire and indicates by its angular position the wire's longitudinal displacement. The device is shown at *D* in Fig. 1. Light from a fixed "point-source," *E*, having passed through a suitable system of lenses, is reflected by this mirror and comes to a focus at a point on the surface of a photographic film wrapped around the cylinder, *F*. If the exposure be made while the cylinder rotates with uniform speed a record of the displacements of the wire plotted against *time* is obtained. A detailed description of the recording mechanism follows.

The small mirror is attached by beeswax to the flat side of a steel staff shown at *A* in Fig. 2. The staff is carefully fitted to rotate freely but without shake in cylindrical jeweled bearings with end-stones. A radial arm, *B*, fits into a conical hole bored through the thick middle part of the staff. Two of these staffs were used, to each of which were fitted radial arms of various lengths. The larger, used for waves of greater amplitude, measures 7 mm. in length and weighs 20 mg., while the smaller is but 4 mm. long and weighs 9 mg.[1]

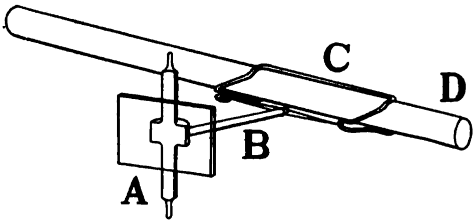

Fig. 2.

The motions of the wire are communicated to the arm *B* through the spring clamp shown at *C*, and a fine silk thread. Such clamps are bent from natural hard music wire (no. 0000, diameter .215 mm.), are about 6 mm. long and weigh from 4 to 5 mg. each. Although they grip the vibrating string, *D*, with sufficient force to prevent slipping, their design permits them to loose their hold in case of accident before harm comes to the staff or jewels. Midway between the horns of the clamp the end of the staff's radial arm is held rigidly by fine silk thread.

This mechanism differs radically from those commonly used for similar purposes in that motions are produced in the staff positively and without the use of a "restoring force," usually derived from a spring. Springs, on account of their inertia, free period, and uncertain reactions, have proved to be a source of much trouble. The natural frequency of the new device is so great that no evidence of its existence has been found. The photographs of Helmholtzian waves agree with theory to a degree of approximation represented by 150 terms of a harmonic series; yet there appears no evidence of natural period so great as the smallest of these. Therefore the frequency must exceed 37,500 vibrations per second.

The mounting of the jeweled bearings proved to be a difficult problem. At first they were held rigidly in a fixed position, but the slight lateral motions of the wire, resulting from unavoidable irregularities in the rubbing surface, caused the staff to bind in its bearings. The resulting frictional losses interfered seriously with sub-Helmholtzian waves of slight stability. Worse than this was the danger to the staff from greater motions of the wire of an accidental nature. That the staff can endure rough usage is demonstrated by the fact that in responding to Helmholtzian waves its angular acceleration at the "corners" often exceeds 7,000,000 radians per second per second.

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[1] For the admirable workmanship on these pieces the writer is indebted to Mr. W. B. Hughes, in the employ of the B. C. Ames Co. of Waltham, Mass.

The mounting finally adopted is shown in Fig. 4. The bearings with the larger staff in place are shown at *A*. Together with necessary means of adjustment they are attached to the end of a steel rod supported near the middle in a universal joint, *B*, and balanced by a counterweight, *C*. This arrangement permits the staff to follow the wire in all of its transverse movements.

The bearings at *A* can be moved parallel to the wire through any desired distance by a turn of the micrometer head, *E*. Their movement produces a rotation of the staff equal to that caused by a longitudinal displacement of the wire through the same distance. In this way the film may be calibrated for magnification of the vibrations. The calibration, which requires a slight correction, is represented by the parallel straight lines in many of the photographs, for example, numbers *1*, *2* and *3* in Fig. 5. Magnifications of from 80 to 140 diameters were commonly used.

The entire unit shown in Fig. 4 rests upon a shelf which communicates with nothing except the brick wall to which it is attached. Since even this precaution did not suffice to prevent response to vibrations of the building, nearly all of the photographs were made at night.

A small self-feeding arc-lamp furnishes the necessary light. In front of it is mounted a screen of colored celluloid, upon which an image of the arc is formed by a lens. In order that holes of various sizes may be brought to the brightest part of the image, the screen is movable, an arrangement which provides "point-sources" of various sizes, circular in shape, evenly illuminated, and with sharp edges.

The cylindrical drum at *F*, Fig. 1, is shown again at *A* in Fig. 3, which is a vertical section of the photographic unit. The drum is of wood and measures about 15 cm. in diameter. It rotates on centers at speeds ranging from zero to 4,800 R.P.M. The corresponding velocities of the film vary from zero to 38 meters per second. Power is supplied by a small direct-current motor, direct-connected to the drum through a spring clutch.

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[1] In many oscillographic experiments it is a difficult problem to get sufficient illumination. Granted a plane mirror, it is easy to show that the problem is solvable. Let it be assumed that the size of the mirror and the frequency and angular amplitude of its motion have been arbitrarily determined; also that in the photographic record is desired a certain "definition," or limiting ratio between the width of the line and its length per cycle. By adjustment of distances the amplitude in the record can be varied without changing the definition and the amount of "exposure" of the film varies inversely as the square of this amplitude.

The cylinder is enclosed in a light-tight box, *B*, provided with a simple hand-operated shutter, *C*, in front and a "loading sleeve," *D*, of black cloth behind. A method of attaching the specially adapted films has been devised, whereby they can be changed easily in a few seconds, yet are held with sufficient firmness to prevent tearing, even at the highest speeds. On account of the simplicity of the shutter it is difficult to restrict the duration of exposure to a single revolution of the film, and as a result many of the records show more than one curve.

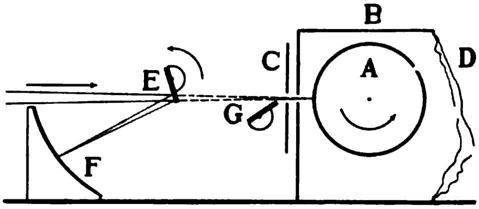

Fig. 3.

In front of the box and parallel to the axis of the cylinder, a shaft carrying a long narrow mirror, *E*, and a fly-wheel is mounted on centers and connected to the motor by a rubber belt. This mirror, by intercepting the beam of light on its way to the film and returning it to a curved screen, *F*, of white cardboard, makes possible visual observation of waves. The mirror is attached eccentrically to the shaft, which in turn is placed just above the path of the light. The shaft has an angular speed of rotation about one third as great as that of the film. The mirror in its rotation intercepts the light for visual use during an interval corresponding to one revolution of the film, leaving it free for photographic use during the two following consecutive revolutions. By proper government of speed successive visual images may be made to coincide and to produce a persistent stationary curve which need not be interrupted during exposure of the film. Transient phenomena are thus easily studied. It should be noted, however, that no attempt is made to superpose images on the film.

This entire unit is mounted on leveling screws, which rest on the table of a heavy adjustable iron stand. It is not connected to any of the other pieces of apparatus. The precaution is necessary to prevent transmission of the slight vibrations caused by the rotation of the drum.

If observations are to be accurate, the axis of rotation of the film must be strictly parallel to the motion of the vibrating spot of light. To secure this relation a straight-edge, *G*, of white celluloid is carried on a shaft mounted in bearings, which are attached to box, *B*, in front of and

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May, 1916.

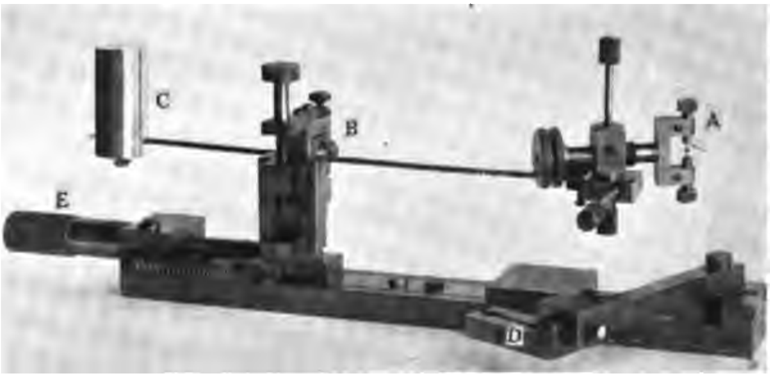

FIG. 4.

HARRY CLARK.

slightly below the level of the shutter. The parts are so adjusted that the edge in all of its possible angular positions remains parallel to the axis of the drum. When raised by hand the celluloid intercepts a small portion of the oscillating beam of light, and with adjustments well made the resulting illumination of the edge should be uniform.

III. Qualitative Description of Observed Phenomena.

It is proposed to devote this section to a qualitative presentation of longitudinal wave phenomena as observed with the aid of the apparatus just described, and to a brief discussion of some of the factors which enter into the successful production of these waves. In what follows it is to be assumed that both the rubbing wheel and the observing apparatus are located at the middle point of the wire.

In the earlier part of the work solids only were employed as friction agents, among them the various resins used on stringed instruments; also beeswax, and several mixtures. The surfaces of the wheels on which these agents were used were of various leathers, and of all thicknesses from two tenths of a millimeter to two millimeters. They varied greatly in quality. One was extremely hard, having been turned from discs of leather steeped in molten resin and tightly clamped together while hot. Another was extremely soft, consisting of a ring of thin chamois skin stretched over a wheel faced with thick gum-rubber. None of these devices, however, gave consistent results. Occasionally good sub-Helmholtzian waves were produced, but they could seldom be maintained long, and could not be duplicated at will. Although Helmholtzian forms could be produced, they often exhibited peculiarities of which something will be said later.

The influence of temperature was noticeable. Somewhat better results could be obtained at high temperatures than at low ones. When, in a cold room, a flame was applied to the wheel operating lightly with resin, the waves were appreciably modified. A considerable sharpening usually occurred, though this at times gave way to distortions.

The desired end was attained by the use of alcohol on a silk-covered wheel. This wheel, which was of brass, measured 42.2 cm. in diameter, and in its face was cut a shallow groove which served to guide the wire. Different thicknesses of silk were tried, and the best results were obtained by using perhaps ten layers of thinnest taffeta ribbon with a total thickness of less than a millimeter. The irregularity in the surface, caused by the free end of the ribbon, soon disappeared with use; the surface became hard and polished, and the accuracy of wave-forms increased. It may be of interest to note that this wheel, when nearly dry, gave the waves characteristic of resin.

Light pressure of the wire against the wheel, well saturated with alcohol, built up a wave which produced after a few minutes an audible tone of great purity. Continuing to grow, it assumed a permanent sub-Helmholtzian form, characterized by rounded "corners" and smooth symmetrical curves, and could be maintained indefinitely. Little could be learned about minimum building-up pressures. At low pressures the rate of building up was at first very small, and the necessary time varied, under fixed conditions, from twenty seconds to as many minutes.

As the pressure was increased, the wave gradually assumed the Helmholtzian form, the resulting tone closely resembling that of the violoncello. The corners of these waves, as shown by photographs, are surprisingly sharp. From the photographs it was estimated that the change of velocity is completed in less than .00001 second. When the pressure was then decreased, sub-Helmholtzian forms reappeared, which, by careful manipulation of pressure, could be carried down to very small amplitudes without loss of stability. In this way sub-Helmholtzian waves could be maintained, the amplitudes of which were not more than one-tenth or two-tenths as great as those built up directly from a state of rest under minimum pressure.

Some of the normal stable forms observed at the middle point are shown in Fig. 5, nos. *1*, *2* and *3*. These records, which are about one-half as large as the original photographs, were made at the same bowing speed and with equal magnifications. Nos. 2 and 3 are sine curves. This is true of all records of normal sub-Helmholtzian waves the amplitudes of which are not much greater than one-half that of the corresponding Helmholtzian. In Fig. 5 the time axis, in each case, extends from *left* to *right*, and the positive direction of the displacement axis, corresponding to the motion of the bow, is from *top* to *bottom* of the page.

It will be recalled that pressure was varied by a transverse movement of the vise at the end of the wire. The operation seemed to cause changes in the vibration system within, for it was accompanied by a marked increase in the dissipation of energy. To get the smallest amplitudes the pressure had to be decreased by small steps. It was moreover necessary to make each adjustment quickly and to allow time for the wave to recuperate before repeating the process.

All of these phenomena, described for a single bowing speed, were produced also at others. The forms of vibration suffered no change thereby, although the amplitudes of corresponding waves seemed to vary directly as the speed. At extremely low speeds it was found difficult to produce any waves whatsoever.

In addition to the experiments described above, a few trials were

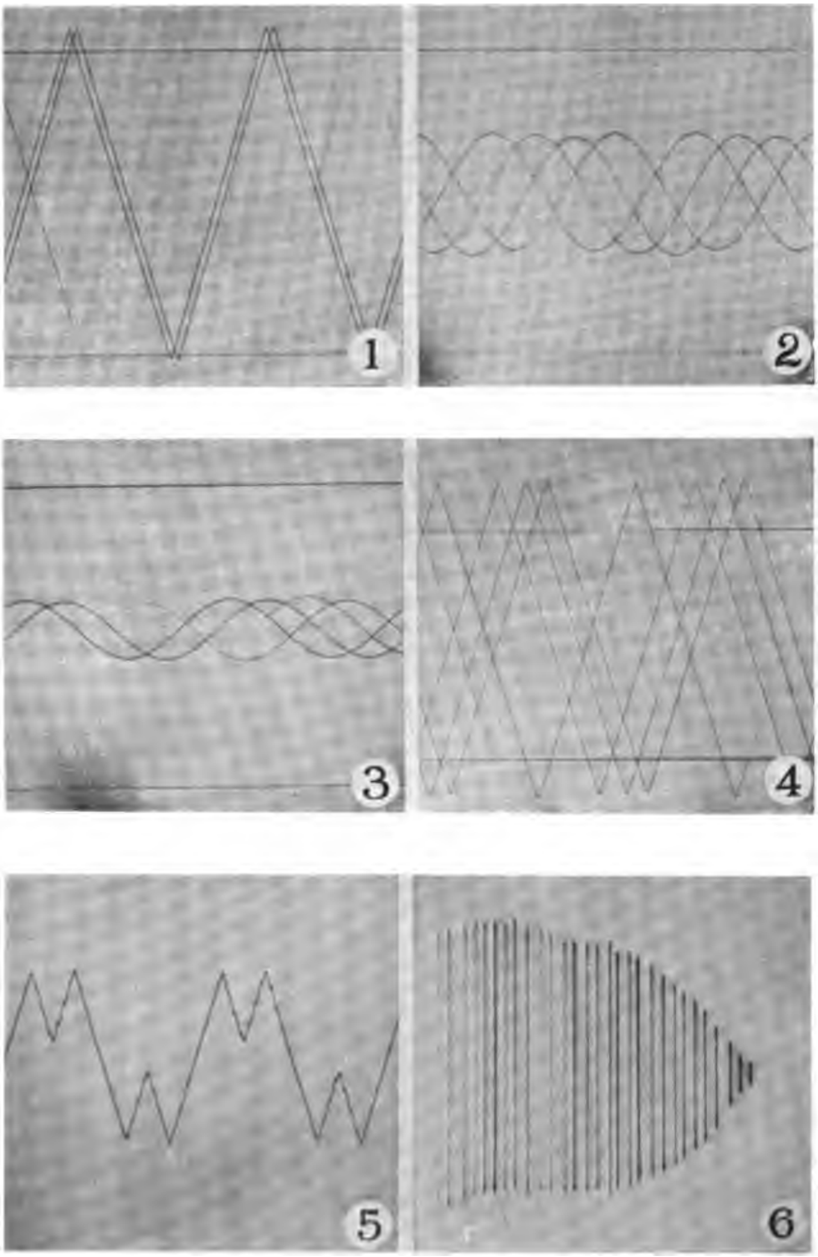

FIG. 5.

HARRY CLARK.

made in which the wire was excited by drawing the finger wet with alcohol along it. This stimulus produced rather sharp waves with sides curved in a characteristic fashion. When the finger was quickly withdrawn from the vibrating wire, the amplitude fell in about four seconds to one-half its original value. From this the rate of energy loss may be estimated, and it is found to be so small that even during rapid changes of wave form the wave equation is not violated.

It has already been seen that alcohol provides a far greater certainty of action than resin. It offers besides many other advantages. The wheel suitable to its use may be made almost truly circular, and its surface conditions, which are constant, may be readily duplicated. Unlike resin, alcohol operates with no observable temperature coefficient. Furthermore there are no incidental noises such as resin gives at the rubbed point. Finally, it is easier to handle. It is cleaner and requires less attention than resin.

Most of the conditions necessary for successful work have been pointed out in the description of apparatus. The question of tension in the wire remains. Although tension does not enter directly into simple longitudinal wave theory, it controls to a considerable extent the production of those transverse waves of slight amplitudes which have at times given trouble. Such transverse waves, it was found, could be prevented by the use of a rather high tension whose value was such as to give to the wire free transverse periods farthest removed from the period of the longitudinal wave. This tension was determined by experiment, and once learned did not need to be changed. Twenty-five kilograms gave good results.

IV. Quantitative Study of Sub-Helmholtzian Vibrations.

In this section will be presented the results of the experimental study of sub-Helmholtzian waves with special reference to the relations which connect speed, pressure, amplitude, and wave-form. For convenience the photographic records were studied by the following indirect method of procedure. The films were placed in a stereopticon, which projected their curves upon a screen of coördinate paper. When the screen had been adjusted to the proper distance and position, the curves were carefully traced with a pencil. In order to reduce them to the same degree of magnification, and also for other special purposes, all of the ordinates of the tracings had to be multiplied by the proper constant number and replotted. In much that follows it will be necessary to consider changes of the ordinates of curves whose abscissas remain unchanged. To avoid confusion, it will be found convenient to accept the following

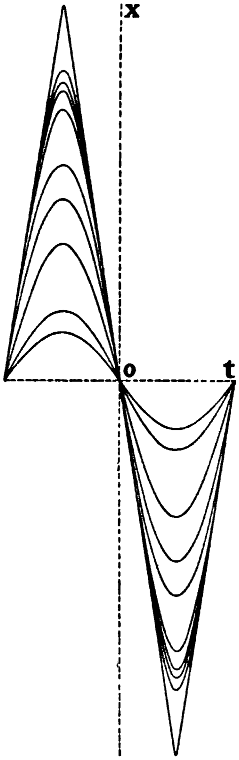

Fig. 6.

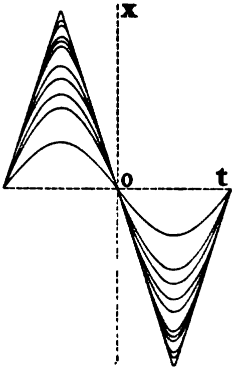

Fig. 7.

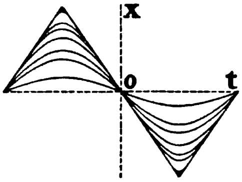

Fig. 8.

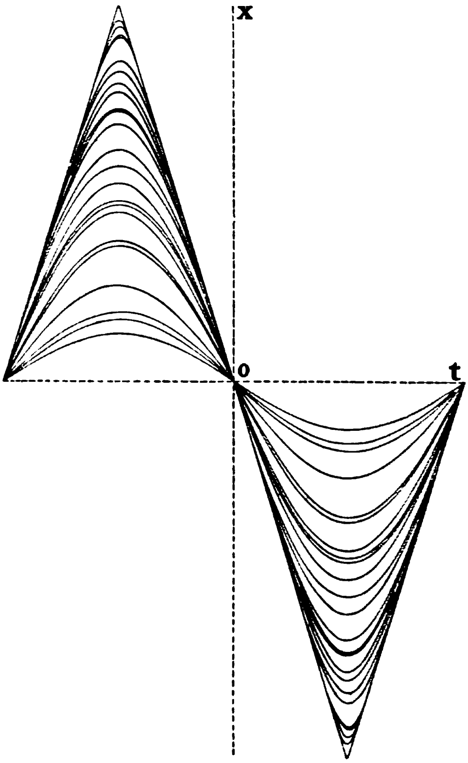

Fig. 9.

Definition.—Two curves are said to be *similar* if one of them may be produced by multiplying all of the ordinates of the other by a constant number. Figs. 6, 7 and 8 are reductions of curves treated as described above. Each of these figures represents the effect of different pressures at a single speed. The speeds, in cm. per sec., and corresponding amplitudes, in centimeters, used in each case are given in the following table.

Figure.	Bow Speed.	Helm. Amplitude.
6	27.85	.0274
7	13.30	.0131
8	6.33	.00625

By careful measurements the limiting Helmholtzian amplitudes were determined and it was found, in verification of the law of Helmholtz, that their values are given by the ratio of *speed* to *four times the frequency*. Comparative studies of sub-Helmholtzian forms could not so readily be made, but some useful suggestions were obtained by a variation of the treatment of tracings, made clear by the following illustration: The ordinates of all of the curves of Fig. 7 were multiplied by a constant number whose value was so chosen that the resulting Helmholtzian curve in particular would be identical with that of Fig. 6. The curves of Fig. 8 were similarly treated with another proper constant. The three nests of curves were then superposed. All of the abscissas of the composite nest thus obtained were then multiplied by a constant, simply for convenience in illustration, with the result shown in Fig. 9.

A consideration of the figures above referred to, and of the methods by which they were produced, brings out several interesting facts. It is evident that *similar* curves may be produced at various speed by the application of suitable pressures. The waves are seen to be in a restricted sense symmetrical, since each positive displacement is followed, half a period later, by a negative one of equal magnitude. Symmetry about vertical axes is also indicated. Finally, it is evident that the amplitudes, not only of Helmholtzian waves, but also of the members of *any* family of *similar* waves, are directly proportional to the bowing speeds.

Since it had thus been shown that a normal sub-Helmholtzian wave is completely determined by its amplitude and bowing speed, certain simplifications of the work were possible. A slight variation of the photographic method, therefore, was introduced. In the original method all records were made on rapidly rotating films. The variation consisted in allowing the film to remain motionless to receive a very short exposure. The record, under these circumstances, is a narrow straight line, from the length of which the amplitude of vibration can be determined. The device offered the advantages of saving both time and material, for many exposures could be made upon a single film. Fig. 5, no. *6*, which illustrates the result, shows decrease of amplitude with decreasing pressure, the bow speed having been kept constant. Under favorable conditions it was found possible to determine amplitude by direct measurement of the illuminated region of the screen without the aid of photography.

About six hundred determinations were made, some visually, others photographically, and the work covered a great variety of speeds and pressures. A single "run" consisted in determining the amplitudes corresponding at a single speed to a great many different pressures, and the results were embodied in a single plotted curve.

The manner of plotting requires some explanation. All of the amplitudes were modified by the same constant factor, the value of which was so determined that the modified Helmholtzian amplitude was equal to unity. All such curves representing the same speed should be identical. Most of the observed differences may be accounted for on the supposition that an inexact zero setting of the wire against the wheel introduced into all of the pressure readings errors constant for a given curve, but differing from curve to curve. Owing to the nature of the rubbing surface, and to the fact that the wire lies against it in a groove, it is difficult to determine the exact adjustment for zero pressure. An error of .01 cm. at this point introduces a pressure error of approximately one gram. The above explanation is therefore reasonable, inasmuch as the errors to be accounted for did not usually exceed one or two grams.

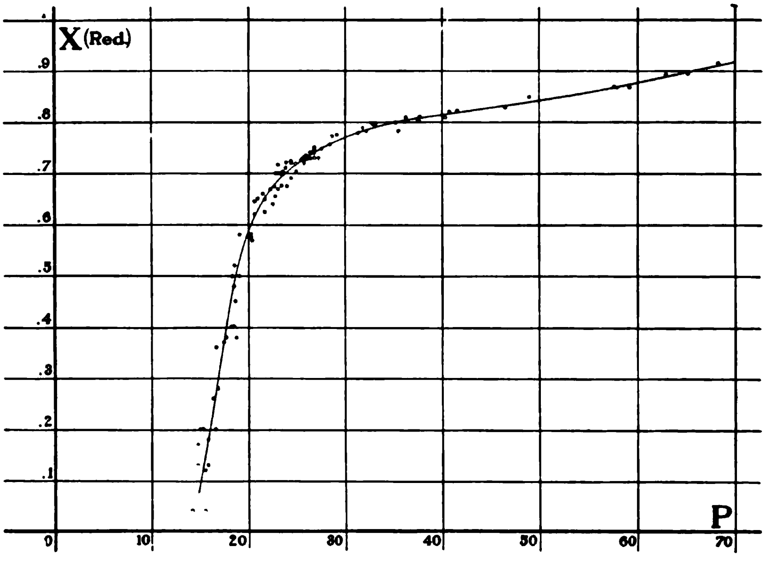

Fig. 10.

The method of averaging these amplitude-pressure curves may be briefly mentioned. They were superposed and if necessary displaced along the pressure axis to give a best compromise curve. This new curve was then replotted with a proper displacement to represent the mean zero position.. Such a curve, representing *all* of the determinations at one speed, is shown in Fig. 10. The distribution of points here shown gives an estimate of the consistency of the data. Several of these compromise curves taken at widely different speeds are shown in Fig. 11, in which the pressures are given in grams. The following table gives the speed and corresponding Helmholtzian amplitude for each.

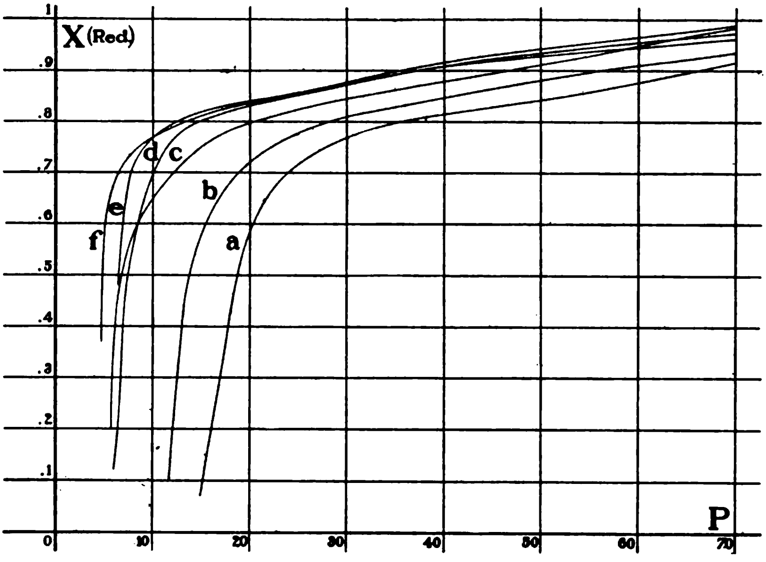

Fig. 11.

Curve.	Bow Speed.	Helm. Amplitude.
a	42.2	.0415
b	27.85	.0274
c	20.17	.0198
d	13.30	.0131
e	9.50	.00935
f	6.33	.00625

The curves are rendered comparable by the scale to which all of their ordinates have been reduced. Similar waves are represented in the figure by points having equal ordinates. At higher pressures, not here plotted, all the curves rise to within one or two per cent. of the limiting Helmholtzian. The speeds for *b*, *d* and *f* are the same as those illustrated in Figs. 6, 7 and 8.

Although the curves of Fig. 11 are very irregular, a few inferences may safely be drawn from them. They show that for the production of similar curves the necessary pressure increases with the speed. At speeds *a*, *b* and *c* the one is almost directly proportional to the other. Curves *d*, *e* and *f* on the other hand suggest a convergence of pressure upon a finite value as speed decreases indefinitely, and this apparent limiting value is far too great to be accounted for by errors in adjustment. In all of these curves, moreover, there is a certain similarity of shape which suggests similar laws of pressure and speed for all of the various wave-forms.

V. MINOR PHENOMENA PECULIAR TO HELMHOLTZIAN VIBRATIONS.

In connection with the experiments upon Helmholtzian waves it was found that any attempt to increase the sharpness of the corners by greatly increased pressure was usually accompanied by the production of ripples in the photographic curve. The phenomenon is shown in Fig. 5, no. *4*. Although these ripples are not in themselves particularly interesting, a consideration of them is rendered necessary by the possible inference that they do not exist in the wire's vibration, but instead are due to an imperfection of the observing apparatus. Their occurrence immediately after a sudden change in the direction of the string's motion points to a natural period of the oscillating mirror-system brought out by the very great acceleration which the system is at that instant undergoing.

It is not difficult to show, however, that this inference is untrue, and that the ripples constitute real phenomena of vibration in the string. In the first place their period could not be changed either by using staffs of different weights and dimensions, or by attaching considerable masses to the radial arms; and again, extremely sharp corners without ripples were sometimes observed. When resin was used ripples were very readily produced and were evident at times when the corners were far from sharp. On a few occasions they were seen superposed upon sinusoidal waves, which afterwards disappeared without destruction of the ripples. When very marked their pitch was easily recognized by the ear both before and after removal of the staff.

Their occasional occurrence in the straight parts of the Helmholtzian curve attests the innocence of the mirror, as does also the following consideration. A disturbance at any point of the wire is propagated in both directions and suffers reflection at the ends of the vibrating segment. The exact times of recurrence of these waves at its origin may be determined by an integral-surface construction. A photograph taken at $\frac{3}{4}$, the bow being at $\frac{1}{4}$, not only exhibited ripples at the corners, but also showed their recurrence later at exactly the points indicated by theory. Ripples are therefore real vibrations of the string, caused doubtless by the bow, whose soft covering may have a highly damped natural period.

Besides ripples there were observed other unexpected phenomena peculiar to Helmholtzian waves. The following theoretical treatment will make clear their meaning.

It has been stated[1] that the wave-form is completely determined by the laws of Young, Helmholtz, and Krigar-Menzel. That this is not strictly true is made reasonable at least by a consideration of the following special case. With the bow at the middle point, let it be assumed that the whole string is divided into $(2n + 1)$ equal segments, n being a positive integer. The bow, being at the middle point of the middle segment may set up in this segment a complete Helmholtzian wave similar to the fundamental one and with amplitude and period equal to $[1/(2n + 1)]$ of those of the fundamental. If the same type of vibrations exist, with proper phase-difference, in all of the other segments, the wave equation, end conditions, and Young's law are satisfied, and the motion of the bowed part obeys perfectly the laws of Helmholtz. In other words these laws do not exclude the possibility of producing certain *Helmholtzian harmonics* as well as the fundamental Helmholtzian.

Since these laws show that the fundamental type of vibration is determined in all aliquot and rational cases by the position p/q of the bowed point, this fractional form may for present purposes be utilized to indicate the type of vibration even when applied to harmonics. For example, all of the harmonics described in the preceding paragraph may be said to be of the $(\frac{1}{2})$ type.

The impossibility of producing certain Helmholtzian harmonics is evident from two illustrations: first—the third harmonic, with the bow at $(1/3)$, violates Young's law; second—the second harmonic, with the bow at $(\frac{1}{4})$, violates the spirit if not the letter of Helmholtz's velocity law, because the type is so changed that the bowed point does not travel *with* the bow during so great a part of the entire period as the fundamental form would allow. The following statement defines completely all *possible* cases save those at $(\frac{1}{2})$. If the string be rubbed at any aliquot or rational point (p/q), where p and q are prime to each other, the production of the nth Helmholtzian harmonic is consistent with the laws of Young and Helmholtz if n and q are prime to each other. Its type of vibration is given by $(np/q - m)$ where m is a positive integer so chosen that

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[1] Davis; see note 1.

$$0 < \left(\frac{np}{q} - m\right) < 1.$$

It is to be noted that although the wave equation and end conditions permit Helmholtzian harmonics to be superposed, the laws of the bow are thereby violated.

Special interest attaches to the ($\frac{1}{2}$) case, in which the motion of the middle point is so slightly restricted by the wave-equation and end conditions that it may be determined arbitrarily over an entire half-period. An infinity of complex Helmholtzian solutions exists therefore, and all necessary restrictions upon the forms of these solutions are defined by the following construction: Lay off on the time axis an interval of one half-period (of either fundamental or any odd harmonic), and connect its ends by any broken line which represents x as a single-valued function of t, and the segments of which are straight and have either one of two slopes corresponding to $\pm v$, where v is the bowing speed. When these forms are interpreted by means of an integral surface it is found that the displacement is zero along all sides of the square, so that all parts of the string pass through the position of equilibrium at the same instant. It has been pointed out by Rayleigh[1] that this is a necessary property of all natural free vibrations of strings. Since the periods of a string are not altered by the operation of rubbing,[2] the resulting vibration may be considered free and therefore, in one respect, the complex forms are *natural*.

The conditions for the production of all of these harmonics and other complex types are not as yet understood. Good rubbing wheels operating with alcohol commonly produced the simple "gable roof," which has always been considered the normal form. In contrast to this, however, hard resin wheels gave the more complex forms almost exclusively. These forms were apparently of infinite variety, and the vibration changed at times from one to another of them, slowly and without apparent cause. Complex forms could sometimes be produced with alcohol. To produce them it was necessary to remove the wheel along the wire to a distance of, say, three centimeters from the middle point. Once started, however, they usually persisted while the wheel was slowly drawn back to its normal position. History, then, as well as present conditions, determines type. In Fig. 5, no. 5, which shows a complex type produced with alcohol, the ripple is to be accounted for by the great bowing pressure required.

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[1] Theory of Sound; Vol. 1, p. 180, 1894.

[2] F. Lippich; Mittheil. d. Deut. Math. Ges. in Prag., 1, 118, 1892.

VI. SUMMARY.

The results of the experiments described in this paper may be summarized as follows:

1. The ordinary Helmholtzian form at (½) has been photographically recorded with a degree of accuracy hitherto unapproached.

2. Helmholtzian forms have been discovered which demonstrate the inadequacy of the laws, as now stated, to determine completely the form of vibration. The limitations placed by present-known laws on the number and forms of these new vibrations have been determined.

3. The study of sub-Helmholtzian vibrations has not been restricted to the forms heretofore recognized, but has been extended to include those of very much smaller relative amplitudes.

4. It has been shown that under certain well determined conditions the tone produced by the rubbed string is of exceptional purity.

5. By improved methods, standard conditions have been determined sufficient for the production and duplication of normal waves, both Helmholtzian and sub-Helmholtzian.

A NOTE ON THE SPECTRUM OF RÖNTGEN RAYS.

BY O. J. ZOBEL.

IN an application of the Fourier analysis to radiation pulses which might result in giving the proper Röntgen spectrum, Sommerfeld[1] compares the relative merits of two types of pulses; the "one-sided" (electric force always of same sign during each pulse) and the "two-sided" (time integral of electric force equals zero for each pulse). Since in each case the electric vector is to be made up of a large number of these pulses occurring at random and without correlation as to sign during the time interval of expansion, the energy in any range of the spectrum per unit time will be the sum of the separate energies in the range arising from the pulses which occur during that time. The relative distribution of energy in the spectrum will then be the same as that resulting from a single typical pulse.

As a special case for the "one-sided" pulse Sommerfeld used

$$f(t) = \pm 1 \text{ for } 0 < t < \tau,$$
$$f(t) = 0 \text{ for } t < 0 \text{ and } t > \tau,$$

giving the spectral intensity

$$J_n = \left(\frac{2}{\pi n}\right)^2 \sin^2\left(\frac{n\tau}{2}\right),$$

where n is frequency.

For the "two-sided" pulse

$$f(t) = \pm 1 \text{ for } 0 < t < \tau,$$
$$f(t) = \mp 1 \text{ for } \tau < t < 2\tau,$$
$$f(t) = \quad 0 \text{ for } t < 0 \text{ and } t > 2\tau,$$

there resulted

$$J_n = \left(\frac{4}{\pi n}\right)^2 \sin^4\left(\frac{n\tau}{2}\right).$$

From these formulæ Sommerfeld drew the following conclusion on page 725, line 17. "The 'one-sided' pulse has its maximum intensity at the frequency $n = 0$, the 'two-sided' has here the intensity zero and its maximum at $n = \pi/\tau$ approximately." Further on; "We see then that only the 'two-sided' pulse gives a spectrum, whose principal energy (for sufficiently small τ) lies in the extreme ultraviolet, and which therefore can serve to give the Röntgen spectrum. The 'one-sided' pulse shows a very improbable spectrum, since its principal energy lies in the most extreme infra-red." Extending his reasoning to any type of pulse he concludes on page 726, line 17, "Every 'one-sided' pulse, therefore, has its spectral maximum in the longest wave-lengths. On the other hand the intensity of the completely 'two-sided' pulse is here always equal to zero." And in line 24, "From this it appears that only a 'two-sided' pulse can produce an actual Röntgen spectrum."

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[1] A. Sommerfeld, Ann. der Phys., 46, pp. 721–748. "Über das Spectrum der Röntgenstrahlung."

Now the elimination of the "one-sided" pulse as the source of the Röntgen spectrum does not seem to me to be warranted on the above basis of reasoning as it can be shown that the conclusions drawn in regard to this type of pulse are unjustifiable. The discrepancy arises from the fact that the spectral intensity J when expressed in terms of frequency has its maxima at points which are different from the maxima points when J is expressed in terms of wave-length. For since

$$J_n dn = J_\lambda d\lambda$$

and

$$dn = \frac{2\pi C}{\lambda^2} d\lambda,$$

where C is the velocity of light,

$$J_\lambda = \frac{2\pi C}{\lambda^2} J_n.$$

Using the same special types of pulses employed by Sommerfeld we obtain for the "one-sided" pulse

$$J_\lambda = \frac{2}{\pi^3 c} \sin^2 \frac{\pi c \tau}{\lambda}$$

where J_λ has its broadest maximum at

$$\lambda_m = 2c\tau = c \cdot \text{(twice the duration of the pulse)}.$$

(See curve 1.)

For the "two-sided" pulse

$$J_\lambda = \frac{8}{\pi^3 c} \sin^4 \left(\frac{\pi c \tau}{\lambda}\right),$$

with the broadest maximum at

$$\lambda_m = 2c\tau = c \cdot \text{(duration of the pulse)}.$$

(See curve 2.)

This shows that both types of pulses give the broadest maximum in the spectrum at exactly the same wave-lengths, and that the other maxima occur at the same wave-lengths also. For the long wave-lengths the intensities vary as $(\lambda_m/\lambda)^2$ and $(\lambda_m/\lambda)^4$, which means but comparatively small energy in these regions for either type of pulse, the "two-sided" giving the least.

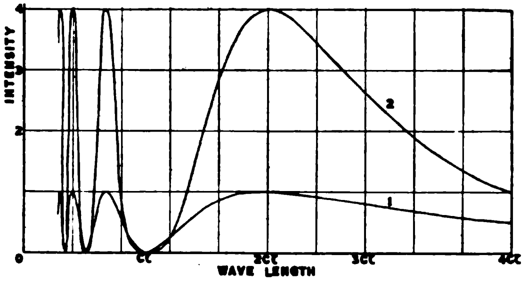

Fig. 1.

Thus the "one-sided" pulse does not have its broadest maximum (and other maxima) in the very long wave-lengths any more than does the "two-sided" pulse. Sommerfeld's objection to the former pulse has therefore lost its validity.

It is to be noted also that types of pulses other than these special ones used by Sommerfeld do not necessarily give more than a single maximum in the resulting spectrum.

Physical Laboratory,
The University of Minnesota,
February 10, 1916.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

A NEW MERCURY VAPOR PUMP.[1]

By Horatio B. Williams.

THERE appeared last year in the Annalen der Physik a paper by Gaede in which was described a new high vacuum pump whose action depends on the difference in the rates of effusion of mercury vapor and gases through a very narrow slit. The writer constructed several of these pumps and a study of their behavior has led to the construction of a very simple pump which is much more rapid in action than the Gaede type and seems to be equally effective in producing extremely low pressures.

The vapor passes through a constriction connected to the vessel to be exhausted. Within the connecting side tube is a small Hopkins condenser (see accompanying sketch), whose cooling effect aided by that of the external air condenses any vapor which enters before it has penetrated far. Diffusion takes place through the annular space between the condenser and outer side tube. This space is 2 mm. wide and has an area of about one sq. cm. A larger Hopkins condenser above condenses the main stream of vapor. The auxiliary pump is connected to it near the top.

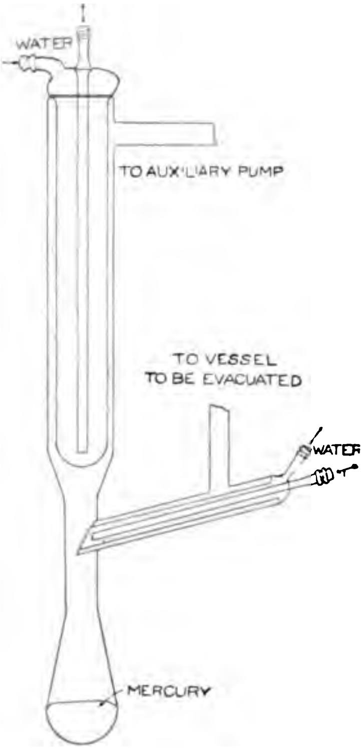

Fig. 1.

By arranging to have the diffusion take place at a constriction in the main vapor flow tube, advantage is taken of the Bernouilli effect. It is thus made possible to have the vapor pass the diffusion point rapidly without reaching a lateral pressure so high as to cause it to pass far into the side tube. A lip projecting from the side tube into the main flow tube hinders the direct passage of vapor into the side tube. The eddies in the vapor stream which this obstruction produces probably hasten diffusion. As the gas which diffuses into the stream of vapor is rapidly carried to the upper condensation chamber, the usually slow process of diffusion becomes very rapid. The auxiliary pump should produce a vacuum of something like .1 mm. of mercury; but the pump will operate with a fore vacuum of .25 mm. Its action is then much slower. Two pumps in series operate rapidly with a fore vacuum of .25 mm. The mercury may be heated with a small open flame, or preferably with a small electric oven. Effective action does not depend so critically upon precise regulation of temperature as in the Gaede type. A thermometer built in is unnecessary.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, February 26, 1916.

Preliminary experiments indicate that the exhaustion is even more rapid than with the molecular pump. A single pump with a May-Nelson oscillating ring pump as auxiliary exhausted a two-liter bulb with large aluminium electrodes from atmospheric pressure to a point where the alternative spark gap measured 18 cm. in 7 minutes. Two vapor pumps connected in series and sealed directly to a MacLeod gauge reduced the pressure from approximately .25 mm. to 4×10^{-5} mm. in 4 minutes. A short time later the pressure was too low to be measured.

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A RESONANCE METHOD FOR MEASURING THE PHASE DIFFERENCE OF CONDENSERS.[1]

BY H. L. DODGE.

IT is a well-known fact that when an alternating electromotive force is impressed upon a condenser the resulting current does not lead the electromotive force by the theoretical ninety degrees. This is particularly noticeable in the case of paper condensers of the telephone type so often used in laboratories for experiments in connection with the study of the alternating current circuit. When an attempt is made to interpret such experiments by graphic methods it is sometimes found that the power-factor of the condensers is so great that it must be taken into account. The angle by which the current lags behind the theoretical ninety degree lead is known as the phase difference of the condenser.[2] It is possible to determine this angle from data obtained in the experimental study of current resonance.

A coil of inductance L, resistance r, and impedance z is arranged to be connected in parallel with a number of paper condensers of nominally equal capacity. An alternating electromotive force being applied the values of the total current, the coil current, and the condenser current are read as the condensers are introduced one by one. The coil current will remain constant, the condenser current will increase uniformly, while the total current will decrease, pass through a minimum value and then increase.

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[1] Abstract of a paper presented at the Columbus meeting of the Physical Society, December 28–30, 1915.

[2] F. W. Grover, Bureau of Standards, Bull., Vol. 7, 495, 1911; Vol. 3, 371, 1907. These papers contain numerous references to other works upon this subject.

If one chooses several sets of these values and constructs the vector diagram of currents he will find that if the phase difference of the condensers is to be zero the minimum total current must be exactly equal to the projection of the coil current upon the voltage axis. As a matter of fact the minimum total current will be found to be larger by an amount which varies with the magnitude of the phase difference.

To obtain the phase difference proceed as follows: Draw the voltage axis. From the constants of the coil determine the angle by which the coil current lags behind the voltage and plot to scale the vector representing the coil current. With the origin as a center and a radius equal to the minimum value of the total current describe an arc reaching a short distance each side of the axis of reference. From the extremity of the coil current vector draw a line tangent to this arc. The angle which this line makes with the reference axis is the power-factor angle of the condensers. The difference between this angle and ninety degrees is the phase difference of the condensers. It is also expressed by the formula

$$\text{Phase difference} = \cos^{-1}\frac{r}{z} - \cos^{-1}\frac{I_L}{I_T},$$

in which r and z are constants of the coil, I_L the coil current and I_T the minimum value of the total current. This expression is easily formulated from the vector diagram.

A sine-wave electromotive force has been assumed. If there are higher harmonics their effect will be to increase the apparent value of the phase difference. In the ordinary laboratory experiment this is not a serious matter unless the harmonics are very prominent. The effect of the harmonics may be reduced by the introduction of inductance in series with the parallel circuit.

This method is not proposed as one possessing any great accuracy. It is, however, of some theoretical interest and has proven to be of great practical value in a laboratory where students in studying the alternating current circuit use the commercial paper condenser.

Physical Laboratory,
State University of Iowa.

A SYNCHRONO-PHOSPHOROSCOPE.[1]

BY EDWARD L. NICHOLS AND H. L. HOWES.

IN the study of the phosphorescence of substances the glow of which is of brief duration, particularly where it is desired to investigate the changes of intensity or color due to heating or cooling, it is convenient to be able to deal with substances of considerable size. For this purpose the authors have devised and constructed the apparatus described in this paper.

The object to be examined is viewed through the open sectors of a revolving disk, *WW*, which is mounted on the shaft of a synchronous alternating current motor, *A.C.* The number of open, and of closed sectors equals the number of poles of the alternator and the excitation of the object being examined is by means of a spark or series of sparks *E*, or of a suitable vacuum tube, actuated by means of a step-up transformer *T.T.* on the same alternating circuit. By proper adjustment of the disk, excitation occurs during the intervals when the phosphorescent surface is behind the closed sectors and the surface is visible for successive intermediate intervals. On a 60-cycle circuit with four sectors these intervals are of 1/240 of a second each and may begin one thousandth of a second or less after the excitation ceases. By breaking the sparking circuit the subsequent decay of materials of long duration may be observed without stopping the disk.

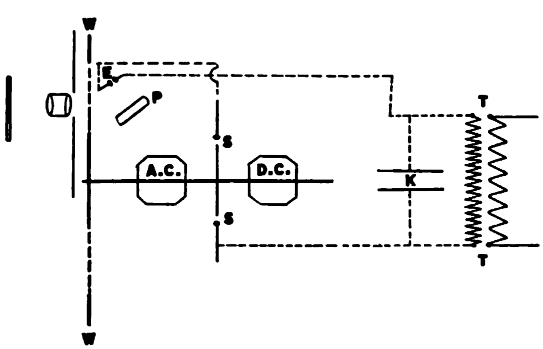

Fig. 1.

By means of a direct current motor, *D.C.*, on the same shaft, which ordinarily serves to bring the synchronous motor to speed, the disk may be driven at other speeds, the excitation in these cases being timed by means of a four point star wheel *SS* on the shaft so as to occur regularly during the passage of the closed sectors.

By the use of this device the authors have found it possible to take photographs in color by the Lumière process showing the change of color of various phosphorescent sulphides during decay and the effect of low temperatures on the color and intensity of these substances.[1] They have also made detailed comparisons of the spectra of the uranyl salts during and after the close of excitation and have determined the rate of decay of the phosphorescence of these salts.[2]

CORNELL UNIVERSITY,
April 15, 1916.

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[1] Abstract of a paper presented at the Washington Meeting of the Physical Society, April 20–21, 1915.

[2] See Transactions of Am. Philos. Soc., 1916.

[3] See Proceedings of Nat. Acad. of Sci., 1916.

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THE

PHYSICAL REVIEW.

THE INTERFERENCES OF THE DIRECT SPECTRA OF TWO GRATINGS.

By Carl Barus.[1]

1. *Introduction. Method.*—The chief purpose of the present paper is the search for phenomena similar to those recently discussed,[2] but in which the two spectra brought to interference are not inverted relatively to each other. Incidentally the strong interferences may have a value on their own account. It has been shown that the totality of the phenomena with spectra reversed on a transverse or a longitudinal axis are quite complicated and a series of companion researches in which similar results are aimed at, in the absence of inversion, is thus very desirable.

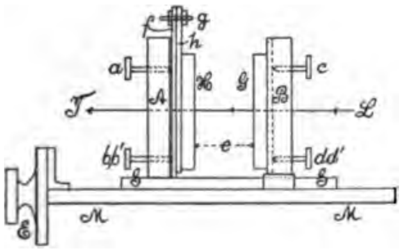

Fig. 1.

The apparatus (Fig. 1) is the same (nearly) as that shown in my last paper,[3] *MM* being the base of the Fraunhofer micrometer, *SS* the slide, *E* the micrometer screw. The brass capsules *A* and *B* are securely mounted on the slide *S*, free from the base *M*, and on the base *M* free from the slide *S*, respectively. Each capsule is provided with three adjustment screws relative to horizontal and vertical axes, *a*, *b*, *b'*, and *c*, *d*, *d'*, together with strong rearward acting springs, by which the gratings *G* and *H* at a distance *e* apart, may each be rotated slightly around a vertical or a horizontal axis (plane-dot-slot mechanism). The two gratings *G* and *H* must be identical, or very nearly so, as to the number of lines per inch and with their ruled faces toward each other.

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[1] Abridged from a forthcoming Report to the Carnegie Institution of Washingotn, D. C.

[2] C. Barus, Am. Journ. Science, XL, pp. 486–498, 1915.

[3] C. Barus; Physical Rev., VII, pp. 79–86, 1916.

These faces, as well as the ruled lines, are to be in parallel. To secure the latter adjustment a bolt, *g*, normal to the face of the grating *H*, serves as an axis and an available tangent screw and spring (not shown) is at hand for fine adjustment. This device is of great importance in bringing the longitudinal axes of the two spectra due to *G* and *H*, into coincidence and a fine wire must be drawn across the slit of the collimator to serve as a guiding line through the spectrum. Any lack of parallelism in slit and rulings rotates the fringes.

The beam of light, *L*, either white or homogeneous as the experiment may require, is furnished by a collimator (not shown), which with the telescope at *T* (in plan, in Fig. 1*a*, at *T* or *D*) are the usual parts of a spectroscope. The collimator with slit is always necessary for adjustment. It may then be removed if the phenomenon is to be studied in the absence of the slit. The telescope is frequently replaced to advantage by a lens.

The adjustments in case of white light are simple and the interferences usually very pronounced, large and striking. Brilliant spectra, chanelled with vertical narrow black lines, are easily obtained when the longitudinal axes are placed accurately in coincidence by rotating the plate *h* carrying the grating *H*, on the plate *f*, around the axis *g*. If the gratings are quite identical the sodium lines will also be in coincidence. Otherwise the two doublets D_1D_2 and $D_1'D_2'$ of the two spectra (nearly identical in all their parts and in the same direction) are placed in coincidence by rotating either grating around a vertical axis. Thereupon the strong fringes will usually appear for all distances, *e*, less than 2 cm. These fringes are nearly equidistant, vertical and they intersect the whole spectrum transversely. They are not complicated with other fringes, as in the experiments of my last paper (l. c.). They increase in size till a single shadow fills the field of view, in proportion as the distance *e* is made smaller and smaller to the limit of complete contact. During manipulations great care must be taken to keep the angle of incidence, *i*, rigorously constant; *i. e.*, to avoid rotating both gratings together, or the apparatus as a whole, as this displaces the sodium doublets relative to each other and seriously modifies the equations.

2. *White Light. Colored fringes.*—The two sodium doublets seen in the arc spectrum are usually equally brilliant and but one set of strong fringes is present in the field of the telescope.

If both gratings are rotated, changing the angle of incidence from 0^0 to i^0, the fringes disappear from the principal focal plane, but reappear strongly in another focal plane (ocular forward or rearward). In such a case the *D* lines are no longer superposed. To be specific let *i* and *i'*,

θ and θ', be the angles of incidence and diffraction at the two gratings in question, the angle between their ruled faces being $i - i'$. Let D and D' be the two grating constants, and nearly equal. Then for a given color, λ, in relation to the individual normals of the two gratings it may be shown that

$$\frac{i - i'}{D - D'} = \frac{\lambda}{D^2(1 - \cos\theta)}, \text{ nearly.} \tag{1}$$

In case of the Wallace grating below $D = 1.75 \times 10^{-4}$, $\lambda = 58.93 \times 10^{-6}$, $i - i' = 10^4 \times 3.29\ (D - D')$. Thus if the enclosed angle $i - i'$ between the plates is one degree, or .0175 radian, $D - D' = 5.3 \times 10^{-7}$, about 3/10 per cent. of D, and equivalent to about 43 lines to the inch. With adequate facilities for measuring i this method may be useful for comparing gratings, not too different, in terms of a normal or standard, practically, since the finite equations may also be expanded. In a similar way the slight adjustments of the longitudinal axes of the two spectra may be made by rotating one grating around a horizontal axis, but this correction is less easily specified. Finally one should bear in mind that with film gratings there is liable to be an angle $i - i'$ between the adjusted plates.

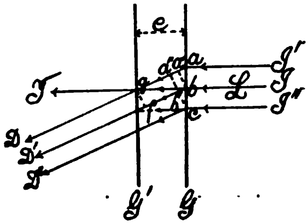

Fig. 1a.

The range of displacement of grating within which the fringes may be used with an ordinary small telescope, extends from contact of the two gratings, to a distance of $e = 2$ to 3 cm. beyond.

In Fig. 1*a*, which is a plan of the essential planes of the apparatus, G, G' being the ruled faces of the gratings in parallel, I, I', I'', three impinging rays of white light diffracted into D, D', the points a, b, c, a', a'', b are in the same phase, so that the path difference of the rays from b at g and f is easily computed. If the single ray I is diffracted into D and D', or I and I' into D, I and I'' into D', etc., the micrometer value of a fringe for a color λ under normal incidence, is

$$\delta e = \frac{\lambda}{1 - \cos\theta}. \tag{2}$$

For two colors λ and λ'

$$n\lambda = e(1 - \cos\theta) = eM;\ (n + n')\lambda' = e(1 - \cos\theta') = eM'$$

if n is the number of fringes between λ and λ'. Thus

$$n' = e\,\frac{M'\lambda - M\lambda'}{\lambda\lambda'} \tag{3}$$

or the number of fringes increases as e is greater.

Equation (2) does not, as a rule, reproduce the phenomenon very well. Since the grating space D of the two gratings is rarely quite the same, the air plate enclosed, in case of apparent coincidence, of the sodium lines, is slightly wedge shaped, as in Fig. 2. Hence the two diffractions take place at incidences 0^0 and α^0, respectively, and the corresponding angles of diffraction will be θ and θ'. If we consider the two corresponding rays I and I'', diffracted at the first and second face, respectively, and coinciding at c in the latter, the points ab and a' (ba' normal to ac) are

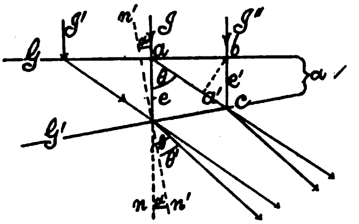

Fig. 2.

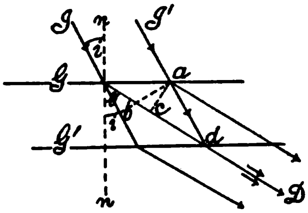

Fig. 2a.

in the same phase and we may compute the phase difference at the coincident points at c. The distance bc is

$$e' = e\frac{\cos\alpha\cos\theta}{\cos(\theta-\alpha)}$$

whence eventually

$$\delta e = \frac{\lambda\cos(\theta-\alpha)}{\cos\theta\cos\alpha\cdot(1-\cos\theta)}, \tag{4}$$

which changes into equation (3) when $\alpha = 0$ and $n = 1$. Fortunately this correction is, as a rule, small. In case of the Wallace gratings ($D = 1.75 \times 10^{-4}$ cm. for instance), if $\lambda = 58.93 \times 10^{-6}$, $\theta = 19^0\ 40^1$ or $\delta e = 1.01 \times 10^{-5}$; whereas if $\alpha = 5^0$, $\delta e = 1.04 \times 10^{-8}$; etc.

If the incidence is at an angle i and the plates are parallel, Fig. 2a, the inquiry leads in the same way to an equation of more serious import. If the gratings G and G' are at a distance e apart and the incident rays are I and I', the points a, b, c, are in the same phase. Hence the two rays leaving d and diffracted along D correspond to

$$\delta e = \frac{\lambda\cos i}{1-\cos(\theta-i)}. \tag{5}$$

Fig. 2c shows the variation of fringes with the angle of incidence i (equation (5)).

Hence if the angle of incidence is changed from -5^0 to $+5^0$, δe increases to nearly 3 times its first value. This therefore accounts for the large discrepancies of δe found in the successive data below. To secure increased sensitiveness and to make the apparatus less sensitive to slight changes of i, this angle should be about 25^0, in which case δe is about three wave-lengths per fringe. But normal incidence is frequently more convenient.

Finally in Fig. 2*b*, if the angle of incidence is i and the two faces G and G' make an angle α with each other and are initially at a distance e apart, changing successively to e' and e'', the points a, b, c, being in the same phase, the two rays D and D' leaving at d, at an angle $\theta - i$, will conform to

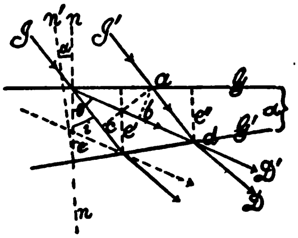

Fig. 2*b*.

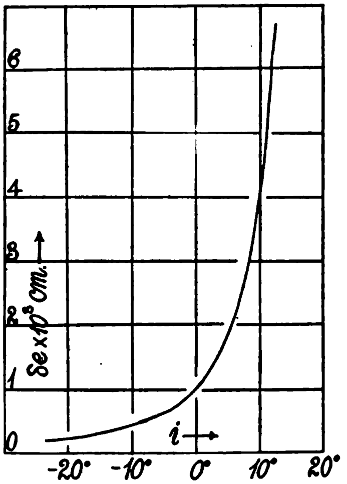

Fig. 2*c*.

$$\delta e = \frac{\lambda \cos i \cos (\theta - \alpha)}{\cos \alpha \cos \theta \,(1 - \cos (\theta - i))}. \tag{6}$$

This equation reproduces the preceding equation (5) if $\alpha = 0$ and the original equation (2) if $\alpha = i = 0$. It shows that a discrepancy or angle between the plates is of minor importance. Hence the change of this angle may be used to bring the sodium lines in coincidence when the gratings differ slightly in their grating constants D. On the other hand, changes of incidence i are of extreme importance.

Experiments made with the film gratings showed that equation (2) not only fits very badly, but that δe per fringe appears as a fluctuating quantity. Fringes were distinctly seen however within 3 cm. of displacement, by an ordinary telescope. I omit the data.

The reason for the lack of accord is given in equation (5) and (6) and Fig. 2*c*. To show that the irregularity of the above results is to be sought in the accidental variations of the angle of incidence i at both gratings, the rough experiments in Table I. suffice.

TABLE I.

Effect of Changing i.

Ives's grating,[1] i negative, ocular inward,[2] $i = \pm 0°$; i positive, ocular outward. $10^3 \delta e = .77, .78, .74; 1.18, 1.12; 1.69, 1.66$ cm.

Thus, as equation (6) implies, small variations of i produce relatively large variations of δe and if i passes continuously through zero, from negative to positive incidence, δe increases continually and may easily be more than doubled. If the phenomenon is in focal planes in front of the principal plane (ocular in), δe is small and vice versa. Moreover this enormous discrepancy is quite as marked for thin glass (2 millimeters) as for thick glass plates (8 millimeters). Again the rather stiff screw of the micrometer, which twisted the whole apparatus slightly, was sufficient to introduce irregularity.

An additional result was obtained by placing a plate of glass between the two gratings G and G'. The effect was an unexpected *enlargement* of fringes, increasing with the thickness of the glass plate (.6 cm. or more). The reason for this is given by equation (3) in paragraph 2, for the number of fringes n' between two colors λ and λ',

$$n' = \frac{e(M'\lambda - M\lambda')}{\lambda\lambda'},$$

where $M = 1 - \cos\theta$, $M' = 1 - \cos\theta'$. Since n' is a number, the glass plate can only be effective in changing θ and θ'. As both are diminished by refraction, the cosines are increased and $1 - \cos\theta$, $1 - \cos\theta'$ are both decreased. Hence n' is decreased or the number of fringes is decreased and their distance apart is thus larger.

It is obvious that when the sodium lines are not superposed, the fringes cannot lie at infinity, but are found in a special focal plane depending on the character of coincidence; *i. e.*, whether the rays are convergent or divergent. Finally a slight rotation of the slit around the axis of the collimator rotates the fringes in the opposite direction to the sodium lines and it is rather surprising, at first, that as much rotation of slit (10° or 20°) is permissible, without fatally blurring the image. The slightest rotation of one grating relatively to the other destroys the fringes.

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[1] About 15,000 lines to inch. Normal $\delta e \times 10^3 = .94$ cm.

[2] Focal plane changing.

3. *Homogeneous Light. Wide Slit. Transverse Axes Coincident.*—If there is no color difference, fringes of the same kind will nevertheless be seen in the telescope, on widening the slit indefinitely. Path difference is here due to differences of obliquity in the interfering rays. As in the preceding case, accurate adjustments of the longitudinal and transverse axes (in case of sodium, D_1 and D_1' or D_2 and D_2' coincide, horizontally and vertically) of the homogeneous color field are essential, if strong fringes are to appear in the principal focus. These fringes are as a rule well marked and widening the slit merely increases the width of the channelled, homogeneous field of view. If, owing to slight differences of grating space, the sodium lines are not quite superposed automatically, they may be corrected by rotating either grating, or else the apparatus as a whole, until the fringes are strongest. The fringes may be made to vanish under inverse conditions. The experiments (to be omitted here) showed their close relation to the preceding colored set, so far as motion of the micrometer is concerned. The fringes decrease in size as e increases and exhibit the same irregularity of δe values, due to the same causes (equation 6). Moreover δe was here usually found to be below the normally computed value, supposing the angle i to be negligible. In fact, Fig. 3 shows the optical center of the collimator C, so that Ca and Ca' are the axes of parallel pencils diffracted by the gratings G and G' at the angles θ for Ca and θ' for Ca'. The rays are subsequently condensed at F the focus of the telescope, L being the principal plane of the objective. The general path difference is thus for negative i, given by equation (5) which distributes the fringes from right to left with variation of i.

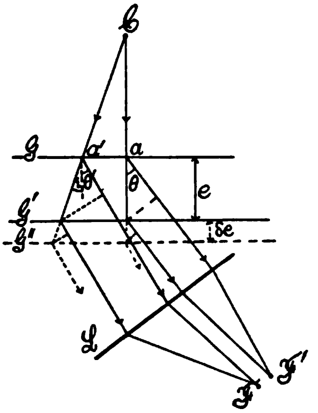

Fig. 3.

If the grating G' is displaced, δe, parallel to itself, the path difference will again be increased by λ whence

$$\delta e = \frac{\lambda \cos i}{1 - \cos(\theta' + i)}.$$

Since i is small, this equation will not differ appreciably from equation (2), with which it coincides for the central fringes.

If the sodium lines are not superposed, these fringes may still be seen; but they are not in the principal focal plane and the new focal plane changes continually as the fringes grow in size. Experiments similar to the above bearing on the reason for the discrepancies (equation (6)) were tried with the thin Wallace gratings. As before the effect of i passing from negative (through zero) to positive values, is enormous, δe increasing nearly threefold for a change of i estimated as within 20°.

4. *Homogeneous Light. Fine Slit. Transverse Axes Not Coincident.*—To obtain this group of interferences, the two sodium lines from a very fine slit are thrown slightly out of coincidence; *i. e.*, by not more than the D_1D_2 distance. In the principal focal plane, therefore, these doublets are seen sharply; while if the ocular is drawn sufficiently forward or rearward, an interesting class of fringes soon appears which resemble Fresnel's fringes for two virtual slits. These fringes may be seen, however, on both sides of the focal plane and increase in size with the distance of the plane of observation (focus or ocular) in front or behind the principal focal plane. In Fig. 4 the two gratings G and G' are struck by parallel pencils from the collimator at different angles of incidence (0° and i°). The two diffracted pencils of parallel rays are caught by the objective L of the telescope and condensed at the principal foci F and F', appearing as two bright yellow lines. In front and behind the plane FF', therefore, are two regions of interference I and I', throughout which the Fresnellian phenomenon may be seen in any plane parallel to FF', observed by the ocular. When the electric arc is used with a very fine slit, these sodium fringes often appear at the same time as the colored fringes, and though they are usually of different sizes, their lateral displacement with a change of distance apart of the gratings, δe, is the same. The fringes in question appear alone when the sodium burner is used. They may (at times) be observed with the naked eye, with or without a lens and they then fail to appear in the telescope, unless the objective is strengthened by an additional lens. They are always vertical, but finer in proportion as the D_1D_2 and $D_1'D_2'$ doublets are moved further apart. They become infinite in size, but still strong, when the doublets all but coincide, showing a tendency to become sinuous or possibly horizontal. Rotation of either grating G around an axis normal to itself and relative to the other, produces greatly enhanced rotation of the fringes, as in all the above cases, but they soon become blurred.

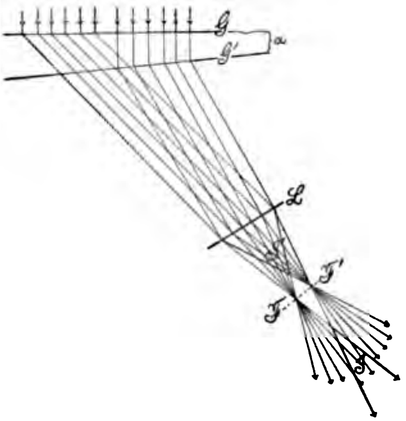

Fig. 4.

Only in the case when the horizontal axes of the field coincide (parallel rulings, etc.) do they appear strong. When the angle of incidence (or non-coincidence) is increased for both gratings, the size of the fringes increases; but when the e distance is increased by the micrometer, the fringes are apparently constant as to size. However, after displacement of 4 millimeters they are liable to become irregular and stringy, though still moving. A fine slit is not essential, particularly when e is small. They vanish gradually when the slit is too wide. If a telescope with a strong objective is used, these fringes may be seen, retaining their constant size long after those of the next paragraph vanish. Examples of data are superfluous here. With the Wallace gratings, these fringes were best produced by the aid of the sodium lines in the ordinary electric arc, simultaneously with the colored fringes and for the case of a very fine slit. They were apparent both with an ocular drawn out or drawn in. In the former case several successive groups were observed. Beginning with the sharp sodium lines in principal focus (D_2 and D_2' coincident), a slight displacement of the ocular outward showed the first group, this being a grid of very fine striations. Further displacement outward produced a second set, equally clear but larger. A third displacement of the ocular outward showed the third set and these now coincided with and moved at the same rate as the colored fringes in the same field. Other groups could not be found. No doubt for these four successive steps the interference grids of D_1 and D_1', D_2 and D_2' are coincident and superposed, until they finally find their place in the colored phenomenon.

5. *Homogeneous Light. Slit and Collimator Removed.*—Fringes similar to those seen with the wide slit above, may be observed to better advantage by removing the slit altogether. The sodium flame is then visible as a whole; and if the adjustments are perfected, it intersected with strong vertical black lines, visible to the naked eye or through a lens or a suitably strengthened telescope. They decrease rapidly with increase in e, but vanish to the eye before the preceding set (§ 4). The sodium lines need not be in adjustment, but the longitudinal axes of the field must be, as usual. If the collimator only partly fills the field of view, these diffuse light fringes and the preceding set may occur together. Both rotate markedly for slight rotation of either grating in its own plane. There seems to be a double periodicity in the yellow field, but it is too vague to be discerned. When magnified with a lens, they admit of a play of e within about .6 cm. from contact. If the sodium lines are not coincident, the focal plane continually changes with e. Otherwise it remains fixed. Again if either of the gratings is displaced and if they

are parallel, the focal plane will not change; but if G and G' are not parallel, the focal plane differs from the principal plane and now moves with the grating.

6. *Inferences.*—The above experiments showed that the equation underlying all the interferences observed is the same. These may result from different causes; but their variation in consequence of the motion of the grating, δe, is due to one and the same cause. This is best seen by producing them simultaneously in pairs. As a means of finding an accurate comparison of the number of lines per cm. on any grating, in comparison with those on the given grating, the method used in § 2 deserves consideration.

If the fringes are to be used for practical purposes, great care must be taken to keep the angle of incidence of the impinging light constant. This was not done in the present experiments, where the purpose is merely an identification of the phenomena. Moreover a micrometer with the screw running easily is essential, as otherwise the frame is liable to show appreciable twist (change of incidence) during displacement of the fringes.

The fringes are not of the sensitive type; but on the other hand they admit of a large range of displacement and are therefore adapted to special purposes.

With regard to their bearing on the behavior of reversed spectra, for the interpretation of which the present experiments were undertaken, it is obvious that the interferences with homogeneous light and a wide slit (§ 3), or in the absence of a slit (§ 5), are of analogous origin in both cases. It makes no difference, therefore, whether one of the spectra is reversed or not, except perhaps that in the former case (inversion), the coincidence of longitudinal and transverse axes is a more insistent condition. The colored fringes of § 2 obviously cannot be produced with reversed spectra. There remain the fringes with the fine slit and homogeneous light (§ 4). In other words the occurrence of a sort of generalized Fresnellian interference within the telescope, modified by causes which lie outside of it. Thus D_1 and D_1' or D_2 and D_2' may be placed sufficiently close together, to produce a region of interference before and behind the principal plane in which the sodium lines are in focus. If the D_1D_1' lines are $x = .01$ cm. apart and the fringes seen likewise at $e = .01$ cm. apart, their position measured from the principal plane will be at

$$R = \frac{xe}{\lambda} = \frac{5}{3}\text{cm.},$$

or less than 2 cm. The ocular would then have to be displaced forward or rearward by this amount. But there are two sodium doublets, each pair of which is to interfere. Suppose that D_2 and D_1' are in coincidence so that the scheme is $D_1 : D_2D_1' : D_2'$ as in Fig. 5, where *o* is the principal plane of the objective and D_1 to D_2' the principal focal plane. We should then have the separate regions of interference I and I' and the combined regions I'' and I'''. When the breadth of the latter is the whole number of fringes, the two patterns clearly merge into a single pattern. The experiments show several of these stages, terminating outermost in the focal plane of the colored fringes under the given conditions. Since the fringes lie on hyperbolic loci, the problem itself is beyond the present purposes; but it appears that the colored fringes will not occur until the correspondence of D_1D_2 lines is shared by the whole of the two continuous spectra.

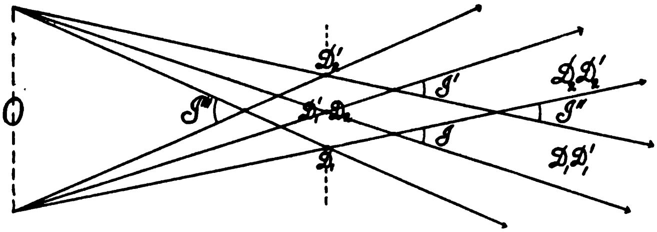

Fig. 5.

The final question at issue is the bearing of the present Fresnel phenomenon on the reversed spectra. If in Fig. 6 S and S' represent the traces of two reversed spectra in the principal focal plane, superposed throughout their extent (*i. e.*, in longitudinal coincidence), the rays *a*, *a'*, through the line of symmetry, are at once in a condition to interfere with a given difference of phase; but so are all the symmetrically placed pairs of colors *c*, *c'*, *b*, *b'* of the two spectra (the distances *c*, *c'*, *b*, *b'*, being arbitrary), provided the corresponding rays meet. As they do not

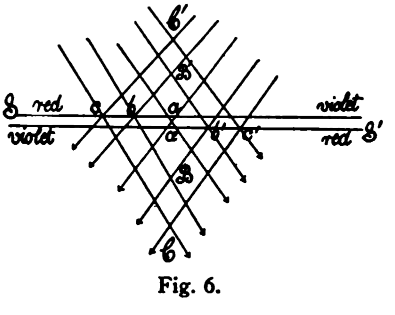

Fig. 6.

meet in the principal focal plane, they can only interfere outside of it, *b* and *b'* at *B*, *c* and *c'* at *C*, etc. Similar conditions must hold at *B'* and *C'*, inside the principal focal plane. The linear interference is thus successively transferred to different pairs of wave-lengths. The phenomena of this paper can not therefore be detected in case of reversed spectra, because in the principal focus different wave-lengths or colors are everywhere superposed, except in the narrow strip of spectrum at *aa'*, which experiment shows to be about 1/3 the width of the sodium doublet in apparent size. The latter, however, is a question of diffraction. Beyond the principal focus, the corresponding conditions, in turn, hold for the rays at *B*, *C*, etc., *B'*, *C'*, etc. Hence they can not be any Fresnellian interferences (§ 4), for there are not two but only one virtual slit, as it were, and interferences are laid off in depths, along the normal *C'C*. The phenomenon may, in fact, be detected along this line for 2 or 3 meters.

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EXPERIMENTS ON THE EMISSION QUANTA OF CHARACTERISTIC X-RAYS.

By David L. Webster.

Introduction.

It has been assumed by many authors that Planck's quantum hypothesis when applied to the energy of cathode rays, would give the highest frequency of X-rays they could produce. The first accurate experimental proof of this law was given by Duane and Hunt[1] for the case of the general radiation from a tungsten target, and was extended to a cathode ray potential of 110 kv. by Hull.[2]

Off hand one would expect the law to apply to characteristic rays also. The possibility that it does not is indicated by the fact that Whiddington[3] found that the total radiation from a metal exposed to X-rays begins to increase rapidly with increasing energy of the cathode electrons when their energy is nearly 40 per cent. above that which the quantum law was later found to demand for the production of the α lines, the strongest of the K series. A similar conclusion on X-rays from a metal exposed directly to cathode rays is indicated by some later work of Beatty.[4] As the point where this occurs is not very definite and the interpretation of the results is uncertain, it seemed desirable to test each characteristic line alone with a spectrometer. A brief report of the principal results of such a test is contained in a paper which I presented to the American Physical Society in December, 1915, and published in the Proceedings of the National Academy in March, 1916. The purpose of the present paper is to report the results more fully and discuss in more detail their theoretical significance.

Apparatus.

For this work I used a rhodium target in a tube which Dr. W. D. Coolidge, of the General Electric Company, very kindly had made up with one of his hot-wire cathodes. The spectrometer worked on the same principle as Bragg's, though the electrical system was somewhat different. The electroscope, instead of being under the axis, was in a brass chamber, placed against the back end of the ionization chamber, whose high potential electrode was a stiff wire passing through an amber plug with a guard ring to the support of the electroscope leaf. The leaf was observed through a microscope mounted on the same arm that carried the ionization chamber. Its sensitiveness was about 10 divisions per volt at 60 volts, the potential ordinarily used; the leakage was about 1 division in 6 minutes, and the drop with a fairly strong reflection was 20 or 30 divisions per second. With reflections of less intensity its readings would repeat themselves with an accuracy of about 1 per cent.

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[1] Amer. Phys. Soc., April, 1915; Phys. Rev., Aug., 1915.

[2] Amer. Phys. Soc., Nov., 1915; Phys. Rev., Jan., 1916.

[3] Roy. Soc. Proc. Ser. A85, July 5, 1911.

[4] Roy. Soc., Proc. Ser. A 87, Dec. 13, 1912.

The crystal face was a very good (1, 0, 0) face of calcite, whose irregularities were so small that they barely showed at all in photographs of the general radiation reflected by the crystal when its position was fixed and the slit was wide enough to include its whole face. The use of such a good surface made it easy to obtain comparisons of intensities in a definite narrow region of the spectrum by using the whole radiation reflected at a fixed position of the crystal with a fixed slit.

To guard against changes of the zero of angle due to heating or cooling of the support of the target, the tube was run continuously throughout each series of readings and the current was made to vary inversely with the potential. The intensities of rays given below are all reduced to a basis of 1 milliampere in the tube.

The current was furnished by 20,160 Planté cells, capable of giving 42 to 44 kv., or within 400 volts above any lower potential for which they were connected. The adjustment within this 400 volts was made by a variable water resistance. The drop in the water and battery together was kept under 1,000 volts, so that small changes in the current could produce only very small changes in the potential on the tube. This potential was measured by an electrostatic voltmeter designed by Dr. Chaffee, consisting of a small dumbbell on a bifilar suspension at ground potential, placed between two larger balls at the high potential and making an angle of 20° to 30° with their line of centers. The system was enclosed in a box lined with tinfoil and the deflection of the dumbbell was read with a telescope and scale. This voltmeter was calibrated up to 18 kv. by a manganin resistance of 894,200 ohms. Above that point the calibration was done by measuring the voltages of two parts of the battery, first separately, then in series, and then separately again. The errors in this apparatus were probably not over 1 per cent. Neighboring potentials could be compared, in any one series of measurements, to about 0.1 per cent. The current through the tube was measured with a milliammeter through which a point in the heating circuit of the cathode was connected to the grounded end of the water resistance, the anode being connected directly to the positive pole of the battery.

The apparatus was so arranged that I could watch the milliammeter and voltmeter, and adjust the potential with the water resistance if necessary, from the start to within a few seconds of the finish of each electroscope run. Thus very little error could arise from the slow decline of voltage of the battery.

The Quantum Potentials of the Characteristic Lines.

If the potential is gradually raised while the spectrometer is set for rays of a frequency not characteristic of the target, no rays will be detected until the potential reaches the value given by the relation $eV = h\nu$ applied to that frequency. At this point the general radiation of that frequency will appear, and its intensity will then increase continuously with the potential. It follows that if the spectrometer is set for a characteristic frequency, it is reasonable to expect, in addition to the characteristic rays, some general radiation obeying Planck's law as at other frequencies. Whether the characteristic rays also obey it is the point to be tested here.

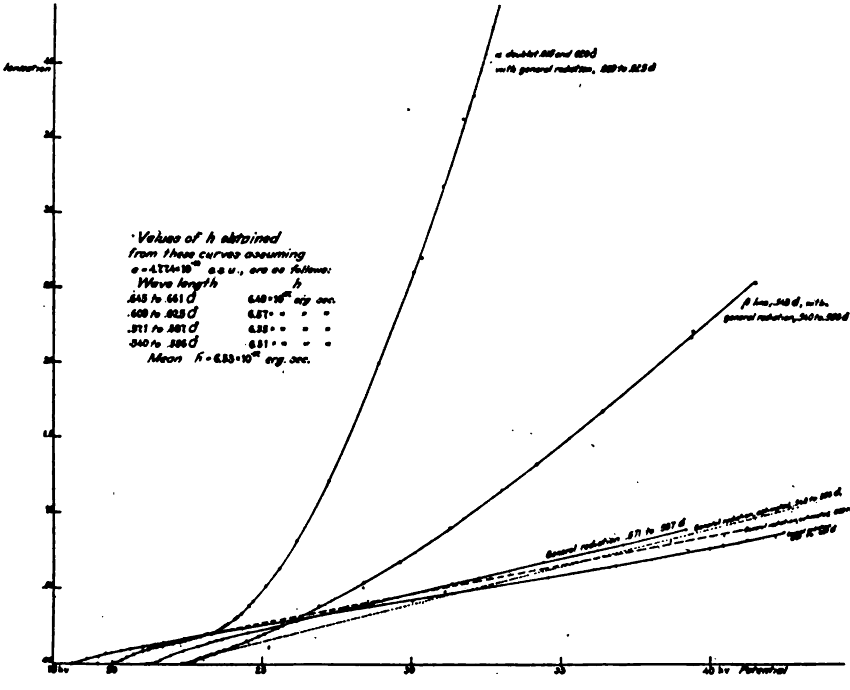


Fig. 1.

The results of such a test are shown by the graphs in Fig. 1. From these it is evident that the predictions about the general radiation are fulfilled; but the *characteristic rays of the α doublet appear, in addition to the general radiation, only above a definite potential about 16 per cent. higher than the quantum value. Above this point, instead of increasing nearly linearly, like the general radiation of that frequency, the characteristic rays increase at a continually accelerated rate.* Thus they begin to be a prominent part of the total radiation only at higher potentials. This explains why Whiddington and Beatty did not detect their presence in the total radiations from other metals at potentials less than 30 to 40 per cent. above the quantum value. The β graph indicates similar laws for that line, but not with any certainty when plotted on the scale of Fig. 1.

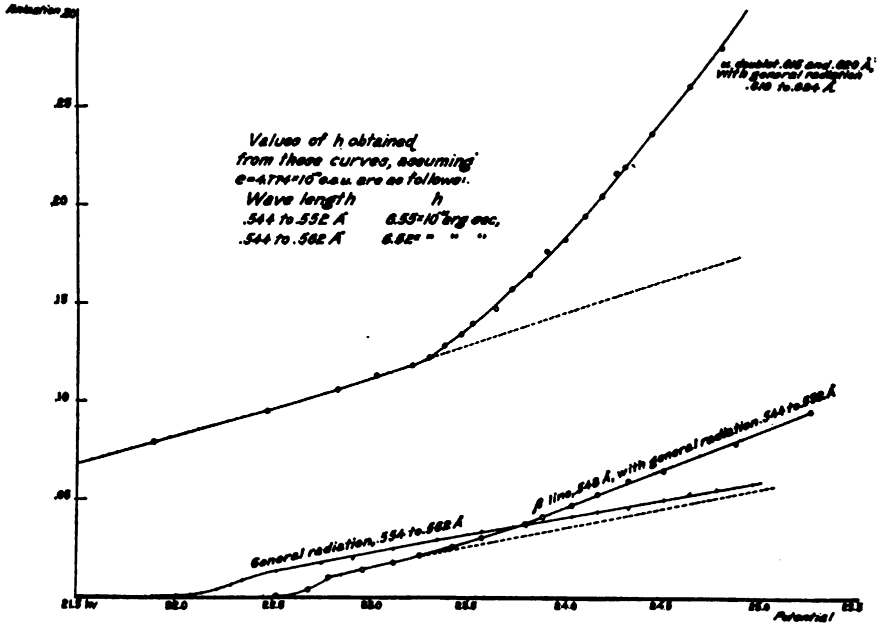


Fig. 2.

The question now arises, how definite is the point at which the α doublet appears? That is, does the curve shown here have a real discontinuity of curvature or of both slope and curvature, or of neither? One might presume neither, from the rounding of the bottoms of the general radiation curves, but it is easy to show that this is due chiefly, if not entirely, to the width of the slit and the consequent lack of homogeneity of the rays. The slit in this case was wide enough to include

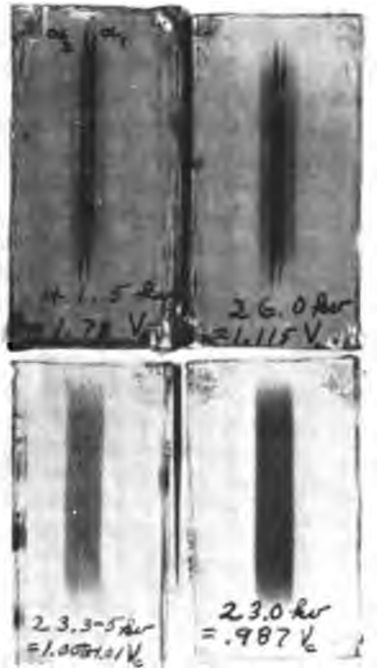


Fig. 3.

These photographs, and those of Figs. 4 and 5, have been intensified by pasting the gelatine sides of the negatives on a piece of white cardboard and photographing this combination onto a second plate. The contrast is therefore twice as good as that given by the ordinary printing process.

DAVID L. WEBSTER.

the whole α doublet and a small range of general radiation on each side. This arrangement was adopted as a precaution against shifts of the focus point with potential or temperature. The boundaries of the region used are slightly indefinite, but certainly include no rays beyond the limits given with the graphs.

To obtain more evidence on the sharpness of the corner in this curve, I have taken seven series of measurements on the α doublet under good conditions, and four on the β line. One series of each kind, with a similar series for a part of the general spectrum near β, is reproduced on a larger scale in Fig. 2. If the corners in these curves are really discontinuities in either slope or curvature, it is of course impossible to prove it definitely, and all that can ever be said is that the deviations from curves with such discontinuities are less than a certain amount. With the battery and other apparatus now available such a statement can be made of discontinuities of both curvature and slope in the α curve, with limits of error of 1 per cent. in intensity and 0.1 per cent. in potential. The β line seems to have similar discontinuities, and apparently *the critical potential is the same for both lines.* As a matter of fact there are theoretical reasons, discussed below, for doubting whether the discontinuities of slope are real, and in some of the series of measurements there seem to be traces of upward curvature below the critical point, but the change of slope is certainly very sudden. The potential at which the maximum curvature of the α graph occurs is 23.3 kv. ± 1 per cent.

To prove more definitely that this is the point at which the lines first become appreciable, I have taken a series of photographs of the α doublet at different potentials, as shown in Fig. 3. These were taken with the crystal fixed and the slit open wide enough to include about .024A of the spectrum. The exposures differed in such a way as to make the background of general radiation show stronger in the low voltage pictures, although really it is less intense; but *in spite of this the lines are stronger at 41.5 kv. than at 26.0, barely visible at 23.3–5, and totally invisible at 23.0.* The steadiness of the apparatus is shown by the boundaries of the general radiation, which are as definite as the width of the source allows them to be. A motion of almost any part of the apparatus would blur them, and a motion of the crystal alone would even blur them without blurring the lines. To test the lines at a potential even nearer the critical one, I took the pictures shown in Fig. 4. For these pictures I first took a series of intensity-potential measurements near the critical point, to determine accurately its position on the voltmeter scale, then exposed at a potential 0.997 times the critical value, or only 0.08 kv. below it, then immediately repeated the intensity-potential series to be sure everything was satisfactory, and then took the high voltage picture to be sure the lines were in the center of the field. The fact that no lines are visible at the lower voltage seems to prove that *the critical potential found by the graphs really does mark the first appearance of any appreciable quantity of characteristic rays.*

COMPARISON OF LINES ABOVE THE CRITICAL POTENTIAL.

This point being settled, the question arises whether the characteristic lines, which all appear at the same potential, show any similarity of behavior above it. The photographs in Figs. 3 and 4 show no obvious difference in the behavior of α_1 and α_2, and even those of Fig. 5 show no certainty of change in their ratio. Such a test is necessarily very inaccurate, especially as the presence of different amounts of general radiation impairs the accuracy of both the eye and the plate. Nevertheless, when one remembers that the exposure at 39kv. was only 8 minutes and at 23.4 kv. 16 hours, or nearly 1,000 minutes, and that the current varied inversely as the potential, it appears that the α_2 line is about 200 times as strong in the former picture as the line and the general radiation back of it in the latter. The lines alone, according to the ionization measurements, must be well over 1,000 times as strong in the former as in the latter. Thus even such a rough test as this acquires some value.

The relation between the α doublet as a whole and the β line is obtainable from the graphs in Fig. 1, where we may extrapolate for the amount of general radiation with each line as shown in the dotted curves. The points where these cross the other general radiation curves may be obtained approximately from the fact that such a crossing signifies a maximum of intensity in the spectrum about half way between the wave-lengths concerned, and that this maximum shifts steadily to the shorter wave-lengths with increasing potential. The curves thus obtained have the right slopes at the critical potential and are confirmed within the limits of error by plotting the spectra at different potentials, as shown below. Subtracting the estimated general radiations from the total intensities, one obtains the curves given in Fig. 6. The ratio α/β is indeterminate at the critical potential and very inaccurate near it, but soon becomes constant. In fact, *if the ordinates of the α curve are divided by this constant and plotted, they all lie on the β curve, almost within the width of the line used to represent it.* The absolute value of this ratio is of little importance, since these "intensities" being ionization currents in an arbitrary amount of ethyl bromide, are on slightly different scales, but the constancy of this ratio seems very significant.

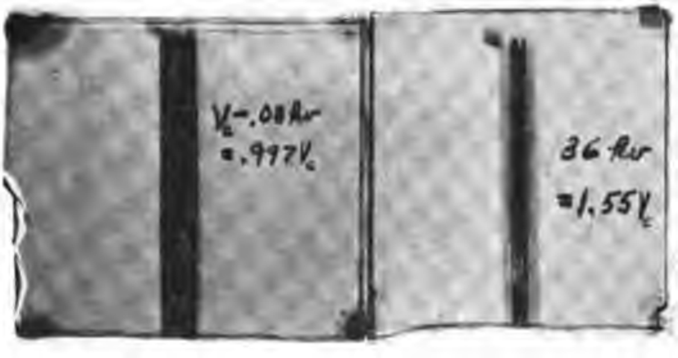


Fig. 4.

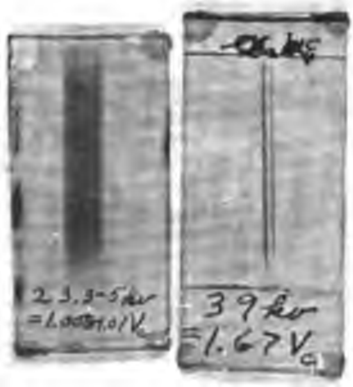


Fig. 5.

Exposure at 39 kv., 8 minutes; at 23.3-5 kv., 16 hours.

DAVID L. WEBSTER.

To test this point by an independent method, I have plotted the spectra shown in Fig. 7. This test is really not so conclusive as the other, because of the long time necessary for determining one of these spectra, and also because of the uncertainty, with the narrow slit used in such measurements, of locating and measuring the exact maximum of each line. The ratios in which the lines increase between 31.8 and 40.0 kv. are nevertheless nearly equal, and the slight inequality is in the direction opposite to that appearing in the other method. In addition to confirming the previous results these curves show that the γ line also increases in the same ratio as the others. For data on the α_1 line we can take the height of the lower hump in the α curve. The ratio of its intensities seems to be between 2.8 and 3.3, agreeing with the others as well as one can expect of such an uncertain measurement. Hence we may state as a second principle resulting from these experiments that *above the critical potential the lines all increase in the same ratio for any given increase in potential.*

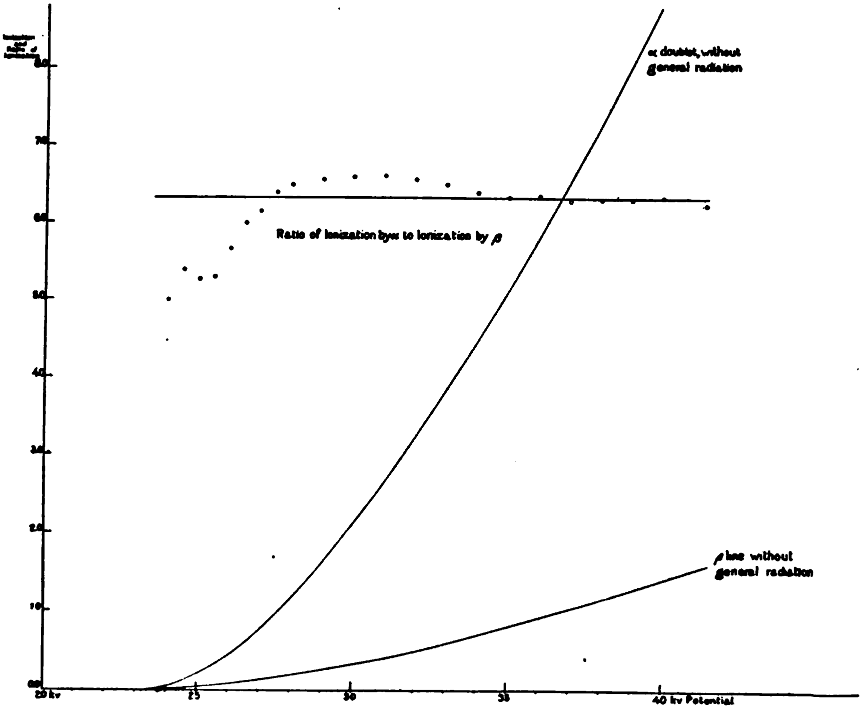
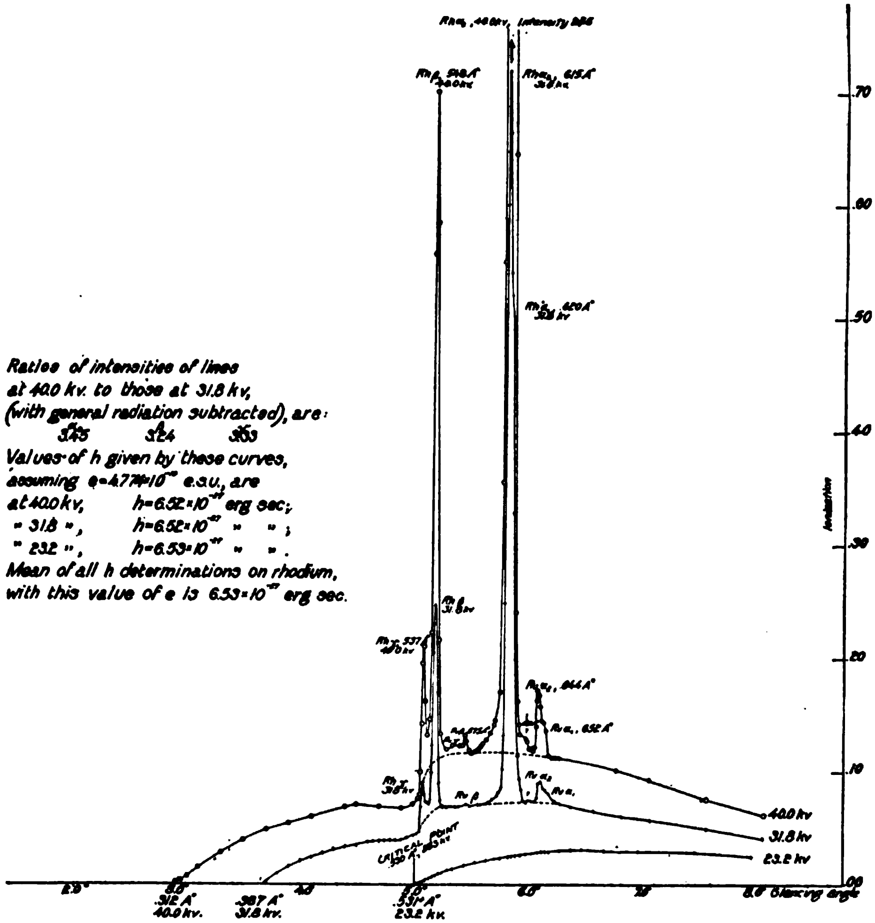


Fig. 6.

THE CRITICAL WAVE-LENGTH.

These spectra also give some information on other points, such as the way a small impurity of ruthenium in the target produces its lines as if the rhodium were not there, exactly as the zinc showed its presence in a brass target used by Moseley in his well-known series of wave-length measurements.

Another interesting point shown by these spectra is the series of values of h obtained from their short-wave limits. The agreement between these values is much better than was expected, and they also agree surprisingly well with the mean of the values from the intensity-potential curves.



Ratios of intensities of lines at 40.0 kv. to those at 31.8 kv, (with general radiation subtracted), are:
3.45 3.24 3.53
Values of h given by these curves, assuming $e = 4.774 \times 10^{-10}$ e.s.u., are
at 40.0 kv, $h = 6.52 \times 10^{-27}$ erg sec;
" 31.8 ", $h = 6.52 \times 10^{-27}$ " ";
" 23.2 ", $h = 6.53 \times 10^{-27}$ " ".
Mean of all h determinations on rhodium, with this value of e is 6.53×10^{-27} erg sec.

Fig. 7.

For all the wave-lengths and h values given here, Bragg's grating space for calcite was recomputed on the basis: $e = 4.774 \times 10^{-10}$ esu. (Millikan), atomic weight of silver = 107.88, electrochemical equivalent of silver = .00111827 gm. per co., molecular weight of calcite = 100.09, $c = 2.9986 \times 10^{10}$ cm. per sec. (Last four from Kaye and Laby's tables.) These data raise the grating spacing from 3.025Å to 3.03, and give for the mean value of h, 6.53×10^{-27} erg. sec. This lies above the values given by Duane and Hunt[1] and Planck,[2] and below those given by Millikan and Hull.[3]

The most important information given by these spectra is the location of the wave-length whose quantum potential is the critical value for the production of the α and β lines. For this wave-length, .530Å, is between 1.0 and 1.5 per cent. short of the γ line, and is in just the part of the spectrum where the absorptive power of the rhodium increases suddenly with decreasing wave-length. Some evidence of this increase of absorption is given here by the drop of intensity of the general radiation. For this drop is undoubtedly due to the slight roughening of the surface of the target by the cathode rays, combined with the fact that the rays leave it at a very small angle to the surface and therefore the humps must screen off the more easily absorbed rays coming from the hollows. In fact Hull[4] has shown that in the cases of tungsten and molybdenum the spectrum without such screening shows neither a rise nor a drop near the γ line, but with a thin screen of the same material as the target the spectrum is deformed in exactly this way. As a matter of fact I find that the point where this occurs seems to be at the γ line itself, rather than the critical wave-length found by potentials, but as I shall point out below, the latter is probably too short. I have also obtained several photographs which show that this change does occur at the γ line; but these are not suitable for reproduction, and an accident to the apparatus has rendered further work impossible for the present.[5]

We may now state the results of the present experiment in the form: *The K series rays from a rhodium target are emitted in appreciable quantities only if the potential is great enough to excite general radiation of a wave-*

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[1] Amer. Phys. Soc., April., 1915; PHYS. REV., Aug., 1915. Recomputed on the basis used here, their value is 6.50.

[2] "Heat Radiation," Translation by Masius, p. 172, gives 6.415×10^{-27} erg sec. for h and 4.67×10^{-10} esu. for e.

[3] Amer. Phys. Soc., Dec., 1915. Their values are 6.57 and 6.59 respectively.

[4] L.c., note 7.

[5] Note added in proofs. Since this was written I have got the apparatus working again, and have obtained a better series of photographs of the γ line and its neighborhood, with the tube turned at different angles to the direction of the rays used. These also show a drop of intensity of the general radiation on passing the γ line when the angle of the rays from the target is small, but none when it is as large as 10°. This confirms the above statement of the origin of the drop and the location of the discontinuity in the absorption spectrum at the γ line itself.

length slightly less than that of the γ *line, at which the sudden change of absorptive power occurs.* Since the K series lines in all elements are very similar, the law in this form probably holds for them all. *Above this potential the K series lines all increase in the same ratio for any given increase of potential; their rates of increase are continually accelerated, while that of any short region in the general spectrum is at first retarded and then nearly constant. Taking e as* 4.774×10^{-10} *esu, Planck's h appears to be* 6.53×10^{-27} *erg. sec.*

THE BASIS FOR INTERPRETATION OF THESE RESULTS.

The significance of these results depends of course on what are the fundamental laws of X-ray phenomena. If these are the laws of electromagnetics as applied to wireless telegraph radiation, we can draw one set of conclusions; if they are laws which would permit the existence of such an atom as Bohr's, our conclusions must be entirely different.

As Hull has pointed out,[1] the critical potential found in these experiments is very consistent with what might be predicted from Kossel's model of the atom,[2] which contains revolving rings of electrons rather similar to those of Bohr, and determines the frequencies of radiations by the quantum relation applied to the energy of an electron dropping from one ring to another. Such an atom, nevertheless, is known to present many difficulties, such as that of accounting for the stability of the rings in the presence of one another, and even more in the presence of other atoms. Another objection, often overlooked, is that such an atom, even though it does contain the quantum relation, does not satisfy the condition that *an oscillator must be able to possess any number of quanta at one time* without affecting its frequency. Since this is one of the essential conditions of Planck's development of his law[3] the failure to satisfy it deprives such an atom of one of the chief advantages of the introduction of the quantum.

Moreover there is the difficulty of accounting for the failure of the rings to lose energy rapidly by radiation, especially now that we have the evidence of X-ray reflection as well as optical phenomena in favor of the electromagnetic wave theory. The argument against the wave theory based on quantum relations does not seem valid, since I have proved in a previous paper[4] that these relations can be reconciled with a form of the classical electromagnetic theory containing a few additions suggested by Parson's magneton theory of the atom,[1] and Planck's method of deriving his radiation law. Consequently the laws of the classical electromagnetics will be taken as a basis for the present discussion. I shall not attempt to give a complete picture of the X-ray mechanism, or even state whether the emitting oscillators are in the nucleus of the atom or outside it, but shall merely attempt to show what conclusions about this mechanism can be drawn from the experiments described above.

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[1] Amer. Phys. Soc., Dec., 1915.

[2] Verh. d. D. Phys. Ges., Nov. 30, 1914; see also Wagner, Ann. der Phys., 1915, and Cabrera, Nature, Oct. 7, 1915.

[3] L. c., p. 153.

[4] Amer. Acad. Proc., Jan., 1915.

The Relation between the General and Characteristic Rays.

Proceeding along this line, one point may be deduced from the fact that there is a definite high-frequency limit to an X-ray spectrum at a constant potential. By Fourier's series or by direct combination of reflected waves from individual layers of atoms in the crystal, it may readily be shown[2] that this fact is inconsistent with the pulse theory of X-rays and signifies that *X-rays of the general spectrum are emitted in periodic trains by oscillators of definite frequencies, even though the spectrum is continuous.* Of course there is bound to be a certain amount of irregular, or pulse, radiation, which is impossible to calculate *a priori*, but which is thus proved to be extremely small.

We can even go a step further and say that the fact that the end of the spectrum can be located within 1 per cent. proves that there must be at least 100 waves in the train[3] to give the necessary interference. Probably there are many more.

The existence of oscillators of frequencies distributed throughout the spectrum is indicated also by the evidence of absorption. For the gradual decrease of absorption with increasing frequency, interrupted by a sudden increase near each characteristic series, is not explainable on a basis of oscillators of any one or small number of frequencies, least of all can it be attributed to the characteristic ones.[4] Consequently it is not surprising that the evidence of the emission spectra also indicates the existence of all frequencies in the target, although not necessarily all in one atom at one time.

[1] A. L. Parson, "A Magneton Theory of Atomic Structure," Smithsonian Miscellaneous Collections, Vol. 23, No. 11, 1915.

[2] D. L. Webster, Amer. Phys. Soc., April, 1915; Phys. Rev., July, 1915.

[3] From a theory of reflection which I published in the Physical Review in March, 1915, it might appear as if we could find an upper limit to the length of these trains from the depth of penetration of the rays in the crystal. However, other considerations, noted under the heading, "Errata and Addenda," Physical Review, June, 1916, have shown that the main hypothesis of that theory must be incorrect, and therefore that the effects it was designed to explain must be due to other causes than the shortness of wave trains assumed there.

[4] See *e. g.*, Lorentz, Theory of Electrons, p. 154.

In view of this agreement, it is surprising that where the absorption suddenly increases, near the γ line, there is no simultaneous increase of emissivity.[1] This may be explained by an hypothesis which we shall find extremely useful, namely that *the K characteristic rays produced by the action of cathode rays are due to the same mechanism as those produced by fluorescence.*

The natural assumption that they are produced as true fluorescence by absorption of the higher frequency rays entering the target is precluded by the experiments of Beatty[2] on total radiation. However, when an oscillator does produce the K fluorescence, it must obtain the energy by absorption from higher frequency X-rays and store it in some non-radiating system such as the magneton theory[3] provides, until it has acquired a whole quantum. Then it must remit at least a part of this quantum in a complex vibration whose component frequencies are those of the K lines. The quantum relation would indicate that if its absorbing frequency is not much above that of Kγ it probably emits most of its energy in this way. Now if this same mechanism acquires a quantum from a cathode particle, the hypothesis given above indicates that it emits most of this energy likewise as K radiation. Hence the K series oscillators will have little effect on the general radiation near the K lines, and this general radiation must be due to oscillators associated with the L, M, etc., series. Under these circumstances we should expect a lack of any sudden rise of emissivity at the critical frequency of the absorption spectrum, just as it is observed.

THE EXPLANATION OF THE INTENSITY RELATIONS OF THE LINES.

The same hypothesis on the origin of the K rays explains at once why a quantum large enough for one of the higher frequency oscillators is required for the production of any characteristic rays. Likewise it explains the contrast between the upward curvature of their intensity-potential curves and the linearity which I have found to hold for those of the general radiation above this critical frequency as well as below it. For as the potential is raised it increases not only the number of oscillators of a given frequency which a cathode particle can reach and excite, but also the range of frequency of the oscillators for which it has a quantum. Consequently, since each of these factors increases more or less linearly, their product increases as something like the square of either. Thus the intensity-potential curve for the characteristic rays, due to the excitation of such oscillators, should be exactly what is observed.

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[1] Cf. Hull. l. c., note 7.

[2] L. c., note 4.

[3] L. c., notes 13 and 14.

Since energy is required for the increased emission of high-speed electrons accompanying the fluorescent K radiation, it might be supposed necessary to allow for this energy in these comparisons. The most conclusive argument against this supposition is perhaps the work of Barkla and Shearer,[1] who have found that these electrons have the same energy as those associated with L radiation. Consequently each electron requires a whole quantum, leaving nothing for secondary X-rays from the atom emitting it. This indicates that the oscillator can emit either an electron or a quantum of X-rays, but not both at the same time. The present discussion is confined to those emitting X-rays only.

If the absorptive power has a finite discontinuity at the critical frequency, as it seems to, it is easy to prove that the present hypothesis leads to finite discontinuities in the curvatures of the characteristic intensity-potential graphs, but none in their slopes. Thus the apparent difference between the wave-length where the absorption changes and that whose quantum potential is the potential of maximum curvature of these graphs is explained as a result of the inability of the apparatus to detect the extremely small amounts of characteristic radiation in the presence of the general at potentials very near the actual critical value. *Probably some characteristic rays are produced at any potential above the quantum value for the γ frequency, though for the first one or two hundred volts the amount must be very small indeed.*

Another experimental fact to be explained is the constancy of the ratio of intensities of any two lines, a state of affairs remarkable for its difference from that of lines of the same series in light. Of course no atom can hold an infinite number of oscillators at one time, as it would have to to produce the whole general spectrum alone. Moreover, the absence of selective absorption of the characteristic rays by the element producing them shows that when the oscillator finishes emitting a set of these rays its frequency must rise again into the region of stronger absorption. Presumably it is a matter of chance what higher frequency it assumes, since any predetermined frequency for an atom would mean a difference between it and other atoms of the same element with different predetermined frequencies. Thus we must conclude that *the mechanism emitting characteristic rays is not only the same in the case of direct production as in fluorescence, but also the same for fluorescence from all higher frequencies and direct production with all sufficient potentials.* Thus it is evident that we should expect the complex characteristic vibration to be the same and the ratios of its component lines to be the same, in all cases, just as they are found to be.

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[1] Phil. Mag., Dec., 1915.

SOME EVIDENCE ON THE MECHANISM OF ABSORPTION.

These data and conclusions are far from being enough for a complete picture of the X-ray oscillator. A mystery which should not be overlooked arises from the fact that if an oscillator has stored up even a large fraction of a quantum from absorbed X-rays it still cannot be made to radiate by a cathode particle having less than a whole quantum, even though the sum of its energy and that of the cathode particle might be almost twice that amount. This fact indeed seems fatal to any model of the oscillator in which the energy absorbed from X-rays is stored as vibratory motion of any particle with which a cathode electron might collide. According to the magneton theory, the stored energy is in the form of a slight increase in the velocity of circulation of the current in the magneton, produced gradually by a mechanism which damps the oscillations on it. There is no obvious way in which a colliding particle could influence this store of energy appreciably, though it might produce a large displacement of the ring and the charge upon it. If this theory is correct, the phenomenon under consideration would indicate that the collision must be elastic unless the energy of these displacements by themselves amounts to a quantum, in which case it may be radiated, rather than returned to the colliding electron.

This assumption is very dubious, being based on but one phenomenon and not as yet subject to test by others. But even if the magneton theory turns out to be incorrect, the facts seem to show clearly an independence of the cathode ray and stored X-ray energies that must have some such counterpart in the modifications they produce in the system.

SUMMARY.

The chief experimental facts proved here are:

1. Rays of the K series of rhodium are produced only at potentials above that required to excite general radiation of a frequency equal to or slightly greater than that of the γ line.

2. Above this potential all the K lines increase in the same ratio for any given increase of potential; their rates of increase are continually accelerated, while that of any short interval in the general spectrum is at first retarded and then nearly constant.

3. The value of Planck's h appears to be 6.53×10^{-27} erg sec. taking e as 4.774×10^{-10} esu.

In the discussion of these results it is assumed that X-rays must obey the laws of the classical electromagnetics and that the quantum relations must be due to an ability of the oscillator to store absorbed energy in some non-radiating system such as the circulation postulated in Parson's magneton theory. On this basis it is shown that:

1. The short-wave limit of the spectrum at a constant potential indicates that even the general radiation consists of wave trains and not of pulses, and therefore that X-rays are emitted by oscillators of definite frequencies distributed throughout the spectrum; this conclusion is strengthened by evidence from the absorption spectrum.

2. All the behavior of the K-rays described above is explainable on an assumption suggested by the general emission and absorption spectra, namely that the same mechanism which emits fluorescent characteristic rays also emits those which arise directly from the impact of cathode rays without true fluorescence.

3. The quantum law for the end of the spectrum indicates that the energy of cathode particle cannot combine with that already stored to produce a quantum; this appears inconsistent with any type of atom in which the energy is stored as vibratory motion, but not necessarily with Parson's theory of the atom.

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NOTE ON THE END EFFECT IN THE ELECTROSTRICTION OF CYLINDRICAL CONDENSERS.

By Edwin C. Kemble.

THERE are two methods of computing the deformations of bodies due to the action of electrostatic fields. One method substitutes the expressions for the ponderomotive body and surface forces derived in the general theory of electrostriction into the equations of elastic equilibrium and attempts to solve those equations subject to boundary conditions appropriate to the body under consideration. This will be called the "elasticity" method. The second mode of attack makes no use of the general theory but refers each particular problem back to fundamental principles. It assumes that the electrical and elastic state of the body can be defined with sufficient exactness by means of a finite number of coördinates. (For example, in the problem of the elongation of a charged cylindrical condenser, the coördinates chosen are the difference of potential between the coatings, V, and a longitudinal traction, q, to which the cylinder is supposed subjected.) Otherwise the assumptions of the two methods are the same. The second is essentially an energy method and will be referred to as such.

Hitherto the application of these two methods to the problem of the elongation of a charged cylindrical condenser has led to contradictory formulas. Sacerdote,[1] using the energy method, derived a formula for the elongation of a condenser with adherent coatings in 1899 and one for the elongation of a condenser with nonadherent coatings in 1901. The first of these formulas did not seem to be in accord with the experimental results of More[2] and of Shearer.[3] These experimenters found no elongation that could not be accounted for as the result of unavoidable heating, although the sensitiveness of their apparatus seemed sufficient to detect elongations of the order of magnitude of that predicted by the Sacerdote formula. This apparent disagreement between theory and experiment led Adams[4] to attack the theoretical problem by the elasticity method. In 1911 he published formulas derived in this way which differed from the Sacerdote formulas and seemed to be in better accord with the experimental work. The present paper proposes to show that the formulas derived by the two different methods may be brought into agreement if the action of certain longitudinal forces due to the electric field in the neighborhood of the ends of the armature coatings be taken into account.[1] It will also show that the negative results of More and of Shearer are not to be interpreted as a proof that there is no such thing as electrostriction.

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[1] Sacerdote, Journal de Phys. (3), t. VIII., p. 457, 1899; t. X., p. 196, 1901.

[2] L. T. More, Phil. Mag. (5), I., p. 198, 1900; (6), VI., p. 1, 1903; (6), X., p. 676, 1905.

[3] J. S. Shearer, Phys. Rev. (1), XIV., p. 89, 1902.

[4] E. P. Adams, Phil. Mag. (6), XXII., p. 889, 1911.

The capacity of a cylindrical condenser of finite length is given approximately by the formula for the capacity of a section of an infinite cylindrical condenser having the same dimensions. To get the exact formula for the finite cylindrical condenser we must add to the approximate capacity an end correction independent of the length. This end correction becomes negligible in comparison with the whole capacity if the cylinder is very long. Reasoning by analogy one would be led to suppose that the end effect on the electrostriction would be negligible in the case of a long cylinder. But this is not the case. The longitudinal traction exerted by the electric field at the ends of the coatings is independent of the length of the coatings and consequently the elongation produced is proportional to that length.

We adopt the following notation:

x, y, z, Cartesian coördinates,
r, θ, z, cylindrical coördinates,
$\mathfrak{E}$, (Components $\mathfrak{E}_x$, $\mathfrak{E}_y$, $\mathfrak{E}_z$) electromotive force,
$\mathfrak{D}$, (Components $\mathfrak{D}_x$, $\mathfrak{D}_y$, $\mathfrak{D}_z$) electric displacement,
F, (Components F_x, F_y, F_z) ponderomotive force per unit volume,
f, (Components f_x, f_y, f_z) ponderomotive force per unit area,
K, specific inductive capacity,
E, Young's modulus,
σ, Poisson's ratio,
a, internal radius of dielectric tube,
b, external radius of dielectric tube,
d, thickness of dielectric tube ($b - a$),
L, length of coatings,
V, difference of potential between coatings,
ρ, volume density of electric charge,
δ_1, δ_2, constants characterizing the change in the specific inductive capacity of the dielectric when it is strained.

The general theory of electrostriction gives the following expression for the x-component of the ponderomotive force per unit volume in a dielectric which is isotropic at every point but not necessarily homogeneous:

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[1] Professor Adams has called my attention to the review of his article in the Beiblätter to the Annalen der Physik in which Pockels remarks that the discrepancy is probably due to the neglect of an end effect. So far as the present writer is aware this is the only published criticism of the work of Professor Adams.

$$\begin{aligned} F_x = \rho e_x - \frac{\epsilon^2}{8\pi}\frac{\partial K}{\partial x} - \frac{1}{8\pi}\Bigg\{ & \frac{\partial}{\partial x}\Big[\mathfrak{E}_x^2\delta_1 + (\mathfrak{E}_y^2 + \mathfrak{E}_z^2)\delta_2\Big] \\ & + \frac{\partial}{\partial y}\Big[\mathfrak{E}_x\mathfrak{E}_y(\delta_1 - \delta_2)\Big] + \frac{\partial}{\partial z}\Big[\mathfrak{E}_x\mathfrak{E}_z(\delta_1 - \delta_2)\Big]\Bigg\}. \end{aligned} \tag{1}$$[1]

The y- and z-components may be obtained by cyclic advancement. In the case of an uncharged homogeneous dielectric the above reduces to the simple vector expression

$$F = -\frac{(\delta_1 + \delta_2)}{16\pi}\,\text{grad}\,(\mathfrak{E}^2). \tag{2}$$[2]

By means of (1) and its companion equations the expanded Maxwell "fictitious" stress system

$$\begin{aligned} P_{xx} &= \frac{K}{8\pi}(\mathfrak{E}_x^2 - \mathfrak{E}_y^2 - \mathfrak{E}_z^2) - \frac{1}{8\pi}(\mathfrak{E}_x^2\delta_1 + \mathfrak{E}_y^2\delta_2 + \mathfrak{E}_z^2\delta_2), \\ & \cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\ P_{yz} &= \frac{K}{4\pi}\mathfrak{E}_y\mathfrak{E}_z + \frac{1}{8\pi}(\delta_2 - \delta_1)\mathfrak{E}_y\mathfrak{E}_z \\ & \cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \end{aligned} \tag{3}$$[3]

may be obtained. The expressions for the force exerted at a surface of discontinuity may be obtained either from (1) directly or from (3). Suppose that the x-axis coincides with the normal drawn from the medium A to the medium B. Then the components of the force per unit area exerted at the surface are

$$\begin{aligned} f_x &= 2\pi\mathfrak{D}_x^2\left(\frac{1}{K''} - \frac{1}{K'} - \frac{\delta_1''}{K''^2} + \frac{\delta_1'}{K'^2}\right) - \frac{(\mathfrak{E}_y^2 + \mathfrak{E}_z^2)}{8\pi}(K'' - K' + \delta_2'' - \delta_2'), \\ f_y &= \frac{\mathfrak{E}_y\mathfrak{D}_x}{2}\left(\frac{\delta_2'' - \delta_1''}{K''} - \frac{\delta_2' - \delta_1'}{K'}\right), \\ f_z &= \frac{\mathfrak{E}_z\mathfrak{D}_x}{2}\left(\frac{\delta_2'' - \delta_1''}{K''} - \frac{\delta_2' - \delta_1'}{K'}\right), \end{aligned} \tag{4}$$[4]

where primes refer to the medium A and double primes to medium B.

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[1] This is a specialization of the formulas given by Pockels, Encyk. der Math. Wiss., Band V 2, Heft 2, pp. 354 and 360.

[2] Pockels, l. c., p. 361.

[3] Pockels, l. c., equations (5) and (11′).

[4] Pockels, l. c., equation (13).

Adherent Armatures.

Now consider the case of a cylindrical condenser with adherent armatures. Let the z-axis coincide with the axis of the cylinder. Then the electric force at points far from the ends of the coatings is

$$\mathfrak{E}_0 = \frac{V}{r}\frac{1}{\log b/a} = \frac{S}{r}. \tag{5}$$

Substituting this value of $\mathfrak{E}$ into (2) we see that the ponderomotive force is radial and has the value

$$F_r = \frac{(\delta_1 + \delta_2)}{8\pi}\frac{S^2}{r^3}. \tag{6}$$

The surface forces are normal to the coatings and have the values

$$\begin{aligned} f_r \text{ (outer surface)} &= -\frac{(K - \delta_1)}{8\pi b^2}S^2, \\ f_r \text{ (inner surface)} &= +\frac{(K - \delta_1)}{8\pi a^2}S^2. \end{aligned} \tag{7}$$

Professor Adams assumes that the longitudinal stress is zero for an open tube and computes the elongation due to the forces specified on the assumption that the cylinder is thin. He obtains

$$\Delta_1 L = -\left(\frac{V}{d}\right)^2\frac{(\delta_1 + \delta_2)\sigma L}{8\pi E}. \tag{8}$$[1]

The formula of Sacerdote is

$$\Delta L = \left(\frac{V}{d}\right)^2\frac{[K + \delta_2 - (\delta_1 + \delta_2)\sigma]L}{8\pi E}. \tag{9}$$[2]

Both Adams and Sacerdote assume that the armature coatings are so thin that their tensile strength may be neglected.

Let us now compute the electrical ponderomotive forces exerted at the ends of the coatings and add the elongation which they produce to that already computed on the assumption that there is no longitudinal traction.

There are three possible sources of longitudinal stress.

(*a*) At the edges of the coatings there is a strong electric force directed along the axis of the cylinder. The intensity of this force is evidenced by the brush discharge which frequently takes place at the edges of the coatings when they are charged to high potentials. This strong electric field is accompanied by a mechanical force tending to stretch the coatings. This force is applied in the first place to the coatings but the stress is at once transmitted by them to the cylinder of dielectric.

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[1] E. P. Adams, Phil. Mag., 6, XXII., p. 892, equation (12), neglecting $\left(\frac{d}{2a}\right)\left(\frac{1}{1-\sigma}\right)$ in comparison with unity.

[2] E. P. Adams, l. c., equation (15); Pockels, l. c., equation (23).

(*b*) At points far from the ends of the cylinder the electric force in the dielectric is a function of r only, so that the body force, $-\frac{(\delta_1+\delta_2)}{16\pi}\,\text{grad}\,\mathfrak{E}^2$, has no component along the z-axis. But near the edges of the coatings the value of $\mathfrak{E}^2$ begins to diminish and drops nearly to zero a short distance beyond the ends of the armatures. In this region the force

$$F_z = -\frac{(\delta_1+\delta_2)}{16\pi}\frac{\partial(\mathfrak{E}^2)}{\partial z}$$

must be taken into account.

(*c*) The third source of longitudinal traction is the z-component of the force on that part of the surface of the cylinder which is not covered by the armature coatings. (Where the coating is in contact with the surface of the dielectric the ponderomotive surface force has no tangential component.)

For our present purpose it is not necessary to compute these forces separately. The resultant of all three may be obtained very simply from the expanded Maxwell stress system. The fundamental property of this system is that the surface integral of the stress over any closed surface is equal to the resultant ponderomotive force exerted on the material inside the surface. Consequently the total longitudinal pull exerted by the electric field in the neighborhood of the ends of the coatings may be obtained by integrating the z-component of the Maxwell stresses over a surface which encloses one end of the coatings.

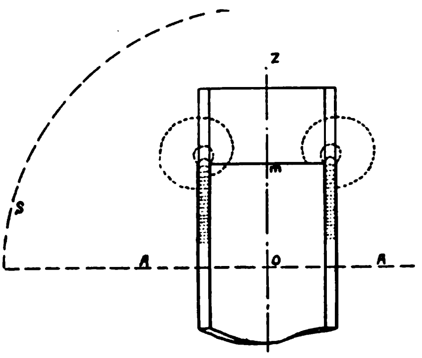

Fig. 1.

To make the computation as simple as possible we choose as our surface, S (Fig. 1), a hemisphere with its base plane A–A normal to the axis of the cylinder and cutting that axis at a point far from either end of the armatures. In the limit when the radius of the hemisphere increases indefinitely the integral of the Maxwell stress over S approaches the integral of the Maxwell stress over the plane A–A. Let S_1 denote the annular section in which the plane A–A cuts the cylinder. Let S_2 denote the rest of the plane A–A. As the distance $\overline{om}$ from the plane

A–A to the end of the coatings is indefinitely increased the integral of the Maxwell stress over S_2 will approach zero as a limit. Let P denote the longitudinal traction to be computed. Then to a close approximation P is equal to the integral of the z-component of the Maxwell stress over S_1 only.

$$P = \int\int_{S_1} P_{zz} ds = \int\int_{S_1} \frac{\mathfrak{E}^2(K + \delta_2)}{8\pi} ds = \frac{K + \delta_2}{4} \int_a^b \mathfrak{E}^2 r dr.$$

But

$$\mathfrak{E}^2 = \left(\frac{V}{r}\right)^2 \left(\frac{1}{\log b/a}\right)^2.$$

Hence

$$P = \frac{V^2(K + \delta_2)}{4(\log b/a)^2} \int_a^b \frac{dr}{r} = \frac{V^2(K + \delta_2)}{4 \log b/a}. \tag{10}$$

Now our preliminary analysis of the sources of this traction shows that the electrical field exerts no longitudinal pull except near the ends of the coatings. Hence we can obtain the elongation produced to a very close approximation if we treat P as a force applied directly to the ends of the coatings. Then

$$\frac{\Delta_2 L}{L} = \frac{P}{E\pi(b^2 - a^2)} = \frac{V^2(K + \delta_2)}{4\pi E(b^2 - a^2) \log b/a}.$$

If $(b - a)$ is small we have approximately

$$(b^2 - a^2) \log b/a = 2d^2.$$

Hence

$$\frac{\Delta_2 L}{L} = \frac{(K + \delta_2)}{8\pi E} \left(\frac{V}{d}\right)^2. \tag{12}$$

Adding this elongation to that obtained by Professor Adams we have

$$\frac{\Delta L}{L} = \frac{\Delta_1 L}{L} + \frac{\Delta_2 L}{L} = \frac{1}{8\pi E} \left(\frac{V}{d}\right)^2 [K + \delta_2 - (\delta_1 + \delta_2)\sigma], \tag{13}$$

which is the formula given by the energy method.

NON-ADHERENT ARMATURES.

Let us now suppose that the armatures are not in contact with the cylinder of dielectric but are separated from it by a nonconducting intermediary fluid of dielectric constant K'. Let g and f denote the radii of the outer and inner armatures respectively. The electric force at points in the interior of the dielectric and far from the ends of the condenser will be sensibly uniform. Denote its value by H.

The formula given by Sacerdote for this case is

$$\frac{\Delta L}{L} = \frac{H^2}{8\pi E}[\delta_2 - (\delta_1 + \delta_2)\sigma]. \qquad (14)^1$$

Professor Adams obtains

$$\frac{\Delta_1 L}{L} = -\frac{H^2}{8\pi E}(\delta_1 + \delta_2)\sigma. \qquad (15)^2$$

The discrepancy is

$$\frac{\Delta_2 L}{L} = \frac{\Delta L}{L} - \frac{\Delta_1 L}{L} = \frac{\delta_2 H^2}{8\pi E}. \qquad (16)$$

This case differs from the previous one in that the longitudinal pull on the edges of the armatures is no longer transmitted to the cylinder, but is born by the armature itself. The force on the end of the tube can no longer be computed by means of the fictitious stresses but must be obtained by the separate calculation of the resultant z-components of the body-force and the surface force which act directly on the dielectric.

In what follows it will be assumed that the cylinder of dielectric extends beyond the edges of the armatures far enough so that the stray electric field at the end of the tube is of negligible intensity. Otherwise the longitudinal traction would depend on the length of the tube.

We choose the axis of the condenser as the z-axis and the plane A–A as the xy-plane. Denote the value of the electric force in the xy-plane by $\mathfrak{E}_0$.

The body force is easily computed. It is

$$P' = -\frac{(\delta_1 + \delta_2)}{16\pi}\int\!\!\int\!\!\int \frac{\partial(\mathfrak{E}^2)}{\partial z}dv \qquad (17)$$

(Cf. equation (2).) The integral may be extended over the entire end of the tube although $\partial(\mathfrak{E}^2)/\partial z$ is sensibly equal to zero except near the ends of the armatures. Let L be the distance from the plane A–A to the end of the cylinder. Inserting the limits of integration we have

$$\begin{aligned} P' &= -\frac{(\delta_1 + \delta_2)}{16\pi}\int_0^{2\pi} d\theta \int_a^b r dr \int_0^L \frac{\partial(\mathfrak{E}^2)}{\partial z} dz \\ &= -\frac{(\delta_1 + \delta_2)}{8}\int_a^b r[\mathfrak{E}^2]_{z=0}^{z=L}\, dr \qquad (18) \\ &= \frac{(\delta_1 + \delta_2)}{8}\int_a^b r\mathfrak{E}_0^2 dr. \end{aligned}$$

The surface force will now be computed. Quantities referring to the

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[1] Sacerdote, J. de Phys. [3], XI., p. 196, 1901.

[2] Adams, l. c., equation 19.

inner and outer surfaces will be denoted by primes and double primes respectively. Let

$$\mathfrak{D}_x^2 + \mathfrak{D}_y^2 = \mathfrak{D}_r^2. \tag{19}$$

For a fluid

$$\delta_1 = \delta_2.$$

Consequently the expressions for the z-components of the forces per unit area on the inner and outer surfaces become

$$f_z' = \frac{\mathfrak{E}_z'\mathfrak{D}_r'}{2}\left[\frac{\delta_2 - \delta_1}{K}\right] \qquad (20)^1$$

and

$$f_z'' = -\frac{\mathfrak{E}_z''\mathfrak{D}_r''}{2}\left[\frac{\delta_2 - \delta_1}{K}\right].$$

Let $\mathfrak{E}_r'$ and $\mathfrak{E}_r''$ denote the radical components of the electric force in the tube of dielectric at its inner and outer surfaces respectively. Then

$$\left.\begin{aligned} f_z' &= \left(\frac{\delta_2 - \delta_1}{8\pi}\right)\mathfrak{E}_z'\mathfrak{E}_r', \\ f_z'' &= -\left(\frac{\delta_2 - \delta_1}{8\pi}\right)\mathfrak{E}_z''\mathfrak{E}_r''. \end{aligned}\right\} \tag{21}$$

The total traction exerted on the surface is

$$\begin{aligned} P'' &= \int_0^{2\pi} d\theta \int_0^L af_z'dz + \int_0^{2\pi} d\theta \int_0^L bf_z''dz \\ &= \frac{\delta_2 - \delta_1}{8\pi}\int_0^L dz \int_0^{2\pi} d\theta[a\mathfrak{E}_z'\mathfrak{E}_r' - b\mathfrak{E}_z''\mathfrak{E}_r''] \qquad (22) \\ &= -\frac{\delta_2 - \delta_1}{8\pi}\int_0^L dz \int_0^{2\pi} d\theta \int_a^b \frac{\partial(r\mathfrak{E}_z\mathfrak{E}_r)}{\partial r}dr. \end{aligned}$$

Both the curl and the divergence of the electric force vanish in the interior of the dielectric. Hence

$$\frac{\partial(r\mathfrak{E}_z\mathfrak{E}_r)}{\partial r} = r\mathfrak{E}_r\frac{\partial\mathfrak{E}_r}{\partial z} - r\mathfrak{E}_z\frac{\partial\mathfrak{E}_z}{\partial z} = \frac{r}{2}\frac{\partial}{\partial z}[\mathfrak{E}_r^2 - \mathfrak{E}_z^2]. \tag{23}$$

Therefore

$$P'' = \frac{\delta_1 - \delta_2}{16\pi}\int\int\int r\frac{\partial}{\partial z}(\mathfrak{E}_r^2 - \mathfrak{E}_z^2)\,drd\theta dz. \tag{24}$$

If we integrate with respect to z and θ, remembering that $(\mathfrak{E}_r^2 - \mathfrak{E}_z^2)$ vanishes at $z = L$ and reduces to $\mathfrak{E}_0^2$ at $z = 0$, we obtain

$$P'' = \frac{\delta_2 - \delta_1}{8}\int_a^b r\mathfrak{E}_0^2dr. \tag{25}$$

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[1] Cf. equations (4).

The total longitudinal traction is

$$P = P' + P'' = \frac{\delta_2}{4}\int_a^b r\mathfrak{E}_0^2 dr, \tag{26}$$

or, if the variation of ϵ_0 with r be neglected

$$P = \frac{ad\delta_2 H^2}{4}. \tag{27}$$

The corresponding elongation is

$$\frac{\Delta' L}{L} = \frac{P}{2\pi adE} = \frac{\delta_2 H^2}{8\pi E}. \tag{28}$$

This is the discrepancy we have already found between the formulas of Adams and Sacerdote.

LIQUID ARMATURES.

In our discussion of condensers having adherent armatures we have tacitly assumed that these armatures are thin metal sheets. It has been a common practice on the part of experimenters in this field to use a conducting liquid for one of the armatures. That we can use the same formula for this case as for that in which both armatures are of metal foil is not immediately obvious. In fact we can show that if both armatures are liquid in contact with the dielectric cylinder the elongation is the same as if the armatures were nonadherent.

In the case of the typical adherent armatures first discussed the longitudinal traction is

$$P = \frac{K + \delta_2}{4}\int_a^b \mathfrak{E}_0^2 r dr.$$

The longitudinal pull exerted directly on the tube is

$$P_1 = \frac{\delta_2}{4}\int_a^b \mathfrak{E}_0^2 r dr.$$

Consequently the traction exerted on the edges of the armatures must be

$$P_2 = \frac{K}{4}\int_a^b \mathfrak{E}_0^2 r dr.$$

(This could also be proved directly.) Now in the case of liquid armatures this force is not transmitted to the tube but is borne by the liquid itself. Consequently we may neglect P_2 in computing the elongation of the tube. In this way we get the same formula as for the case of nonadherent armatures.

If the two armatures are of the same length we may assume that each carries half of the traction P_2. Then if one of them is a conducting liquid we may compute the elongation from (9) if we substitute $K/2$ for K.

However, the usual experimental arrangement is to have the liquid armature very much longer than the coating of metal foil. With this arrangement the two armatures do not carry equal longitudinal tractions. In fact there is no longitudinal pull on the liquid dielectric at all, for the lines of force arrange themselves as indicated in Fig. 2, and the metal foil must carry the whole of the traction P_2. Consequently the whole stress is transmitted to the dielectric and the elongation is the same as if both armatures were metal sheets.

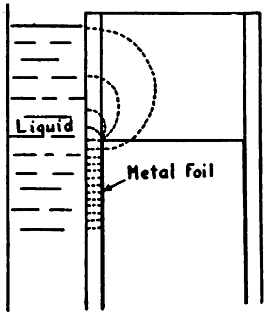

Fig. 2.

The Experiments of More and of Shearer.

The above work shows that if the general theory of electrostriction is correct the experimental results of More and of Shearer must be reconciled with the Sacerdote formula, (9). Moreover, it shows that the principal term of (9), $(KL/8\pi E)(V/d)^2$, is not due to a direct action of the electric field on the dielectric but to the longitudinal pull on the edges of the armatures. If there were no direct electrostrictive action (as has been suggested) we would still expect an elongation given by the formula

$$\frac{\Delta L}{L} = \frac{K}{8\pi E}\left(\frac{V}{d}\right)^2. \qquad (29)$$

Since the publication of the work of More and of Shearer, Professor Adams[1] has computed the value of the combination of constants $[\delta_1 - (\delta_1 + \delta_2)\sigma]$ for several different kinds of glass from the experiments of Wüllner and Wien on the change in the capacity of cylindrical glass condensers when they are stretched. The computed values vary from $+0.68$ to -2.85. For glasses for which this combination of constants has a negative value the formula (9) predicts a smaller elongation than does (29). Hence the experimental results of More and of Shearer are more easily explained if the general theory of electrostriction is assumed to be correct than otherwise.

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[1] E. P. Adams, l. c., p. 895.

The elongation predicted for tube No. 5 of Professor More's second series of experiments[1] by (9) is 3.6×10^{-6} cm. or 6 divisions of Professor More's micrometer scale if the extreme value -2.85 of the combination of constants $[\delta_2 + (\delta_1 + \delta_2)\sigma]$ is used. The smallest elongations observed for this tube were 5 and 7 micrometer divisions. Possibly an appreciable fraction of these minimum observed elongations was due to heat but the discrepancy between theory and experiment can hardly be considered to amount to a contradiction.

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[1] L. T. More, Phil. Mag. (6), 6, p. 13, 1903.

RETROGRADE RAYS FROM THE COLD CATHODE.

By Orrin H. Smith.

J. J. THOMSON,[1] as early as 1897, showed that a system of rays of an entirely different character from the cathode rays accompanies the cathode beam. He found that these rays proceed normally from the face of the cathode, that they are not appreciably deflected by a permanent magnet, and that they possess very little, if any, power of producing phosphorescence.

In 1906 Villard[2] gave an account before the French Academy of the rays accompanying the cathode beam which are not so readily deflected as the cathode beam but which were deflected in such a direction and by such an amount as would be expected of the "kanal strahlen." He noticed that in a mixture of oxygen and hydrogen (or water vapor) the cathode rays produced a luminescence characteristic of oxygen, but when these were deflected aside by a magnet there remained rays which produced a luminescence characteristic of hydrogen. He explained their presence by saying that they were the positive canal rays which fall against the cathode and rebound. To explain their rebounding beyond the limits of the cathode dark space, he assumed that the potential fall underwent rapid variations or was even discontinuous. A stroboscopic test showed this to be true; however, this was to be expected since he used a transformer to produce the discharge.

The following year Thomson[3] showed, independently, that these rays were deflected by strong electric and magnetic fields and that they possessed considerable mass. In a later work he observed that they were very feeble under the most favorable conditions of vacuum, discharge potential, etc., and were exceedingly feeble when the gas pressure in the discharge tube was very low. In this latter respect they were quite different from canal or positive rays. Employing a tube having an opening of about .5 mm. in diameter he obtained a photograph which showed that these rays contain (*a*) positively electrified atoms and molecules of hydrogen, (*b*) positively electrified atoms of oxygen, and (*c*) negatively electrified atoms of hydrogen and oxygen. The photograph showed the intensity of the lines corresponding to the negative ions to be greater than that of the positive ions. With the ordinary positive rays the positive lines are the more intense.

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[1] Proc. Camb. Phil. Soc., IX., p. 243, 1897.

[2] Comptes Rendus, CXLIII., p. 673, 1906.

[3] Phil. Mag., XIV., p. 359, 1907.

The conditions under which retrograde rays are produced are quite different from those that obtain for the ordinary positive rays and for this reason it seemed worth while to repeat and extend Thomson's investigations.

Thomson does not find the molecule of oxygen with the negative charge while in this investigation the molecule of oxygen and the molecule of hydrogen are the only carriers obtained with a negative charge, no atoms appearing at all. The presence of helium in the discharge chamber apparently makes no difference in the photographic result.

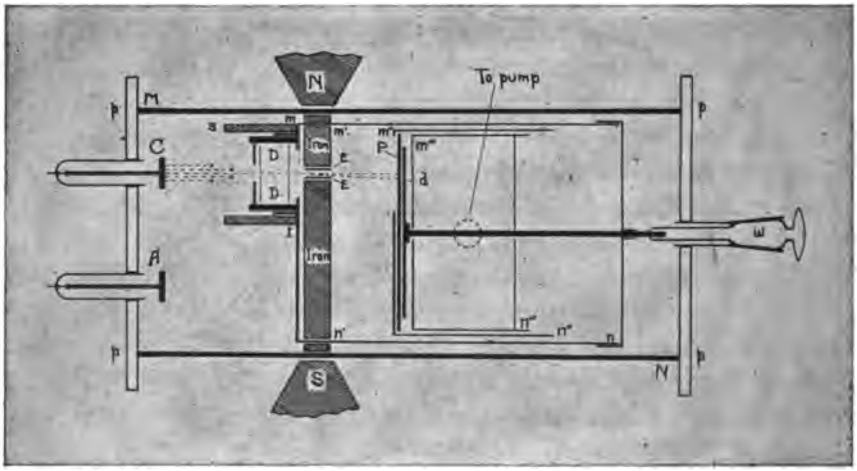

Fig. 1.

Top View. *MN*, containing vessel of glass; *pp*, glass end plates; *mn*, large brass cylinder; *m'n'*, magnetic field extensions; *EE*, electrostatic field plates, connections to which are not shown; *m''n''*, plateholder; *P*, photographic plate mounted on disc *d*, supported by telescoping cap *m'''n'''*, and turned by winch *w*, *DD*, aluminum diaphragms; and *SI*, iron shield.

It appears that Thomson was unable to use a tube of less than .5 mm. bore, while in this investigation traces were obtained with a tube and set of diaphragms having openings of about .05 mm. thus producing sharp lines on the plate which made possible more accurate measurements.

Owing to the short range at which these rays were obtained on the photographic plate, the increased sharpness of the lines, and the restricted range of their velocities due to a restricted cathode dark space, it was possible to obtain some evidence on the question as to whether the power of a particle to affect a photographic plate is a function of its velocity, momentum, or kinetic energy. This evidence seems to indicate that it is a function of the kinetic energy and that the mean value is about 7.4×10^{-9} ergs.

The apparatus, shown in Fig. 1, and the manipulation is essentially the same as that described by Knipp[1] except that a cold cathode was used instead of the Wehnelt cathode and the discharge was produced by an induction coil. The cathode was just like the anode and similarly placed facing the line of the tube and the diaphragms.

A large Leeds induction coil was operated on ten storage cells. The vacuum was maintained with the aid of a large charcoal bulb dipping into liquid air. In general the vacuum improved with sparking. After a few runs it was found that the liquid air could be removed after about ten minutes from starting, and, as the sparking and pumping continued, it could be dispensed with altogether. Finally the vacuum was so easily maintained that it was necessary to keep the pump itself turned off for about three fourths of the time.

There is a point of interest in connection with the charcoal bulb. It was left on the apparatus for weeks after its use was found unnecessary, remaining all the while at the nearly constant room temperature. The pumps were unable to produce a vacuum of .005 mm. in fully three hours' time when starting from atmospheric pressure, and this was the case whether the bulb was heated for an hour during that time or not. However, if it was pumped to a pressure of one or two mm. and left to stand for ten to fifteen hours then upon starting the pumps a vacuum of .005 mm. could be attained in twenty to thirty minutes. This, strangely, was true even when the vacuum had been let down for a very few minutes and then the pumps started again immediately.

The photographic plate used was Seed's Yellow Label lantern slide plate. This plate is very slow and hence produces great contrast which is the thing desired. Thomson[2] points out that the large ions affect only the surface of the film and do not penetrate like the faster moving electrons, into the film. Hence the plate best suited for this work is one that is slow and that has a thin film with a high percentage of silver. The best traces that could be gotten in this investigation were in many instances so thin that they could hardly be seen. They were obscured easily by the slightest fogging. For this reason a fast plate could not be used. Seed's Gilt Edge Number Twenty-seven plate was tried but in every instance fogging obscured the lines. The Double Coated Cramer Crown plate was tried and found to be entirely too sensitive. Some experience by another member of the department obviated the necessity of trying the Cramer X-ray plate. As an instance to show that these carriers affect only the surface of the film, the author gently stroked the film under water with a fine camel's hair brush to remove foreign particles and it was found that, in some cases, the lines were entirely obliterated. Further, after the negative had dried the lines could be obliterated by breathing on the film and wiping it gently with a soft cloth. In both cases, other than erasing the lines, no further change could be detected in the film. It was found advantageous to put some alum in the fixing bath to harden the film. The developer used was ordinary hydrochinon, the time of development being from six to twelve minutes.

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[1] Phys. Rev., XXXIV., p. 215, March, 1912.

[2] Thomson, Rays of Positive Electricity, p. 4.

The time of exposure varied from thirty minutes for the small to three hours for the larger deflections. There seems to be a limit to the intensity that is obtainable, for after a certain length of exposure the intensity of the lines did not apparently increase with further exposure. This was true for long or short development or even when they were exceedingly dim. This is in agreement, however, with the theory that they affect only the surface of the film.

Thomson found that the retrograde rays were best obtained when the gas pressure was not too low. The present photographs bear out that fact very well. If the vacuum was kept about .002 to .004 mm. scarcely any trace of the rays could be found on the plate. The best pressure for their production seems, from this investigation, to be between .015 and .008 mm.

There is always a central spot that is undeflected which is probably due to neutral carriers that were negative originally but which lost one electron before they got into the deflecting fields. It would seem from this that a moving particle need not be charged in order to affect a photographic plate. It is quite evident that the velocity of an uncharged particle must be above a certain value otherwise a plate would be affected by exposure to the air in a dark room due to no other agency than to the velocity of the air molecules produced by ordinary heat agitation. The mean of this velocity at 0° C. for the hydrogen molecule is about 2×10^5 cm./sec. and for the oxygen molecule about 4.5×10^4 cm./sec. Whether the ability of a moving particle to affect a photographic plate is due to its momentum or its kinetic energy, or simply to its velocity, is not definitely known. It seems reasonable to expect, however, that it should be a function of one of these. On a number of the plates the lines were distinct enough to locate approximately the place where the slowest ions would strike, *i. e.*, those that had just sufficient velocity to affect the plate. These points were found, in every case, to be well within the limits of the field, *i. e.*, so far as the limits of the apparatus are concerned the lines might have extended farther from the origin. It occurred to the author then to assume that there were particles which struck beyond the last points of the visible trace but whose velocity was not sufficient to cause them to affect the film. If the coördinates of the last visible point in each line be measured and v and e/m determined, then, for all such points, we should get a constant, showing whether this minimum effect on the plate is a function of the velocity, the momentum, or of the kinetic energy of the moving ion. Table I. shows values which are proportional to the velocity, momentum, and kinetic energy for the points in question on sixteen different lines. It can be seen that the values for the kinetic energy are nearly constant while the values for the velocity and the momentum are not constant. It thus appears that the power of a particle to affect a photographic film probably depends on its kinetic energy. The mean of these values of the kinetic energy is, from Table I., 7.4×10^{-9} ergs which is the minimum required. This value would probably be different for an electron because of its size. It is somewhat larger than the energy required to produce an ion which is 1.63×10^{-11} ergs. The above value (7.4×10^{-9}) was calculated from data obtained from this investigation, except for the value of e, by the formula

$$\text{kinetic energy} = 1/2 \cdot m/e \cdot e \cdot v^2.$$

The value of e was taken as 1.55×10^{-20}.

TABLE I.

Photographic Plate.	Line.	Constant × Velocity.	Constant × Momentum.	Constant × Kinetic Energy.
75	Upper	6.04	18.36	111.0
76	Upper	6.54	17.99	117.6
76	Lower	1.36	87.18	118.6
85	Upper	6.37	12.50	79.0
85	Lower	1.52	52.50	79.7
86	Upper	6.23	12.71	79.2
86	Lower	1.53	52.48	80.4
87	Upper	7.27	19.40	102.5
87	Lower	1.69	60.50	102.3
88	Lower	1.37	61.70	84.5
94	Upper	5.55	10.71	106.4
94	Lower	1.77	36.28	100.4
95	Upper	7.41	15.03	91.1
95	Lower	1.64	51.50	84.47
96	Upper	6.35	17.70	112.5
96	Lower	1.42	59.36	84.26

It can be seen from Table I. that, even though the values of the kinetic energy vary somewhat, the values for a given plate as a rule are more nearly alike. Plate ninety-six furnishes the greatest variation from this rule. It might be reasonable to expect that different emulsion numbers would reveal slightly different kinetic energies required to affect the film. Several emulsion numbers are represented in these data.

The photographs taken with the apparatus in the last refinement, while clear and capable of accurate measurement, do not lend themselves to reproduction and hence are omitted. The important dimensions are as follows:

Length of electrostatic field	1.10 cm.
Length of magnetic field	1.10 cm.
Distance from point of emergence to plate	1.58 cm.
Length of triangular test coil[1]	2.52 cm.
Base	.63 cm.
Number of turns	19.

The negative lines show distinctly the parabolic heads which are not in evidence on the positive lines. It was evident from nearly all the plates exposed that the negative carriers are in preponderance over the positive ones. This seems reasonable to expect since the distance to the plate is, for the lower pressures, within the limits of the mean free path and it is necessary to assume that every positive carrier has lost two electrons between the outer limits of the dark space and the deflecting fields. If this is true we should expect that the lines due to the positive carriers would not be as sharp as those due to the negative carriers, the ions being deflected somewhat from their true path in the process of losing an electron. Most of the photographs bear this out. It is somewhat surprising, in consideration of the foregoing, that this preponderance is not greater than the photographs seem to indicate unless the negative ion is more unstable than the positive ion. An additional suggestion in the same line comes from a study of Thomson's photographs of positive rays, in a great many of which the negative counterpart is very weak or cannot be seen at all on the prints when the positive lines are very pronounced. The positive lines do not have the distinct parabolic head that the negative lines have. They are also broader and more diffuse. Joining the parabolic head to the center is a line due to the secondary rays of Thomson. This is shown particularly in one exposure where the electric field overlapped the magnetic so that the secondary line does not join straight on to the head of the parabola. The data for exposure number eighty-five, are given in Table II. This indicates that the carriers which produced the two lines are the molecules of hydrogen and oxygen respectively. The measurements of the coördinates were made with an ordinator composed of a frame to which the plates could be fastened so that there was a movable point above the plate capable of being carried in either of two directions perpendicular to each other by micrometer screws. A Grassot fluxmeter was used to determine the strength of the magnetic field.

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[1] Thomson, Rays of Positive Electricity, p. 10.

TABLE II.

Photographic Plate Number 85.

Measurements for the Upper Line.

Position.	z in mm.	y in mm.	$v \times 10^{-7}$ cm./sec.	$e/m \times 10^{-4}$	Electric Atomic Weight.	Carrier.
1	3.34	6.42	6.37	.509	1.97	H_2
2	2.46	5.48	7.39	.504	1.99	"
3	1.88	4.77	8.96	.500	2.00	"
4	1.12	3.51	10.38	.458	2.18	"

Measurements for the Lower Line.

1	3.34	1.53	1.52	.029	34.5	O_2
2	2.46	1.31	1.77	.0288	34.8	"
3	1.88	1.16	2.04	.0296	33.8	"
4	1.12	.84	2.50	.0260	38.5	"

Time of exposure, 3.25 hours.
Gas pressure varied between .008 and .018 mm.
Electric deflecting field, 965 volts.
Magnet current, 4.25 amperes.
$A = 8{,}040$, $B = 267 \times 10^{6}$

All the photographs were exposed with residual air in the discharge chamber except number 88. In this instance it contained some helium but no traces appear in the photograph, in fact in no case does anything appear in any of the photographs except the lines due to the molecules of hydrogen and oxygen. In some cases the positive rays are not visible.

The data show very well how the velocity varies for the carriers striking at the various points along the parabola, that it decreases with increase of distance from the undeflected spot. The value of v and e/m obtained for the smaller values of the electric field are in general less reliable than for those for which the deflection is larger. The "electric atomic weight" of a carrier Thomson[1] has defined as the ratio of m/e for that carrier to m/e for the atom of hydrogen.

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[1] Phil. Mag., XXI., p. 234, Feb., 1911.

It was noticed in connection with these experiments that the discharge in the chamber passed more easily with the presence of a transverse magnetic field. Earhart[1] has shown that this is true for a longitudinal field.

SUMMARY OF CONCLUSIONS.

The results of this investigation may be summarized briefly as follows:

1. When obtaining retrograde rays in residual air the molecule of hydrogen appears on every plate accompanied by a heavier carrier which in most cases is the molecule of oxygen. The velocities obtained by the author are smaller than those obtained by Thomson. This is due to the position of the cathode with reference to the small canal through which the carriers pass, the dark space extending beyond the near end of this tube and hence the carriers not attaining their maximum velocity.

2. The negative lines are clearer and sharper than the positive; probably because of the disturbance to the path of the positive particles in the process of becoming positive.

3. Retrograde rays can be obtained with a canal having a bore of about .05 mm. diameter. The best range of pressures for their production is between .008 and .015 mm. of mercury.

4. The power of a moving particle to affect a photographic plate seems to be a function of its kinetic energy. The minimum required for the heavy carriers is of the order 7.4×10^{-9} ergs, which is larger than the energy required to produce an ion, however, there is evidence in favor of the view that this value may depend somewhat on the emulsion on the plate.

In conclusion I wish to express my thanks to Professor A. P. Carman for the excellent facilities placed at my disposal and to Dr. C. T. Knipp for his interest and help in carrying on the investigation.

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[1] PHYS. REV., Feb., 1914.

ON THE "CURRENT-DEFLECTION" METHOD FOR DETERMINING BALLISTIC CONSTANTS OF MOVING COIL GALVANOMETERS, WITH A NOTE ON THE NON-UNIFORMITY OF MAGNETIC FIELDS IN SUCH INSTRUMENTS.

By Paul E. Klopsteg.

I. Theoretical.

(*a*) *Statement of Formula for Method.*—The determination of the quantities involved in the equation[1]

$$K = \frac{T}{2\sqrt{\pi^2 + \Lambda^2}} k\rho^{\frac{1}{\pi}\tan^{-1}\frac{\pi}{\Lambda}}, \tag{1}$$

which reduces to

$$K = \frac{T}{2\pi} k\sqrt{\rho},$$

when Λ is small in comparison with π, is easily accomplished; and simple apparatus, found in any laboratory, is used. In these equations K represents the ballistic constant,[2] ρ is the ratio of one amplitude of the coil to the one next following, T is the complete period of the coil and Λ its logarithmic decrement; k is the current constant,[3] i/d, of the instrument, where, in the ordinary method of determination, d is the deflection from the null position caused by the current i.

(*b*) *Objection to the Use of k as Ordinarily Determined, and Proposal of Modified Method.*—It seems that equation (1) is not commonly used, except for instructional purposes in connection with the theory of the ballistic galvanometer, the presumable reason being that the values which it gives for the constant are not often in agreement with those given by the direct methods, the differences ranging from a fraction of one per cent. to as much as three or four per cent. The major part of the error when damping is small occurs in the value of k; with strong damping, large errors are as likely to occur in Λ as in k. The formulæ presuppose ideal conditions as to uniformity of field and symmetry of position of the coil with respect to the magnet; these conditions are far from being realized in most galvanometers, as has been previously indicated by the writer,[1] and as will be shown at greater length in this paper. Since, ordinarily, the field is by no means uniform, the value of k is not truly constant, but depends upon the angular position of the coil.

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[1] Kohlrausch, Lehrbuch der praktischen Physik, 12th edition, 1914, p. 525.

[2] The open coil type of instrument with a circular scale at 50 cm. from the mirror is here being considered; K in coulombs per centimeter.

[3] In amperes per centimeter.

The value of k to be used in formula (1) is the value which depends upon the average field intensity *at the null position of the coil*, which, of course, is the position occupied by the coil at the instant the greater part of the discharge passes through it. This fact suggests that it might lead to greater accuracy if, instead of producing a given deflection by means of a current, one were to produce a deflection by turning the torsion head from which the coil is suspended through a certain angle, and then to determine the current necessary to bring the coil back to its null position. The coil would then evidently be in the very position in the field which it occupies at the moment of the ballistic impulse. The question at once arises whether the current constant should, with this manipulation, have the same value as that which would be obtained by the ordinary method if the field were truly uniform. For it is evident that in the proposed procedure the spiral lower suspension is, when the coil has been returned to its null position, in the same condition of strain as it was before the coil was deflected by turning the torsion head.

To investigate this point, we may proceed as follows:

Let i_1 = current through coil equivalent to torque per unit angle of the upper suspension; and

i_2 = current equivalent to torque per unit angle of the lower suspension;

also let the upper torsion head be turned through an angle d_0 (in terms of centimeters on the circular scale), producing an angular deflection d of the coil. Then

$$i_2 d = i_1(d_0 - d),$$

from which

$$d = i_1 d_0/(i_1 + i_2).$$

Now let a current i be sent through the coil, just sufficient to bring it back to the null position; since the lower suspension is now free from strain, $i = i_1 d_0$. Consequently

$$k = i_1 + i_2.$$

The same result is obtained when the assumption is made that the lower suspension is given sufficient twist to produce the deflection d of the coil. By twisting either suspension, therefore, the restoring couple of the system is unchanged. This leads to the important conclusion in connection with the usual condition[1] of the suspensions in galvanometers, that within limits, no matter what the conditions of strain of the suspensions at the null position, the elastic constant of the suspensions is the same as though the suspensions were free from strain.[2]

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[1] PHYSICAL REVIEW, N.S., 5, 266, 1915.

(*c*) *Precautions to be Observed in Determining k by the Method Here Outlined.*—In determining the current constant by the modified method, one source of error should be kept in mind, namely, that in most instruments—unless special attention is paid to this point—the prolonged axis of the coil does not pass through the point of attachment of the upper suspension. This may, for example, be due to a bent terminal, or to eccentricity of the torsion head. When the torsion head has been turned through a certain angle and the coil brought back to zero deflection by means of a suitable current, the position of the coil is not usually identical with the "original" null position,[3] but may be shifted into a region of the field at which the intensity differs appreciably from that at the original null.[4] The current constant will then be affected by an error proportional to the difference between the field intensities at the two positions. The error from this cause, for a given deflection of the coil, is less with an upper suspension having large torque per unit angle than in one with small torque; for in the latter case the upper torsion head must be turned through a larger angle to produce a given deflection of the coil than in the former. The gain in accuracy, however, is offset by the smaller sensitivity with a stiff upper suspension. It should therefore be observed, especially when a galvanometer with weak control is being used, that the upper suspension be as nearly coincident with the vertical axis of symmetry of the instrument,[5] and the coil as near the ideal position, as careful levelling can bring them.

Another precaution to be observed is that of using the proper value of the logarithmic decrement corresponding to a given throw, especially when there is considerable damping; since the logarithmic decrement is not constant, its value must be determined for any given amplitude. A method for doing this is described in the experimental section of this paper.

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[1] This condition being that, at the null position of the coil, one of the suspensions is twisted, necessitating a twist of the other suspension in the opposite direction for equilibrium.

[2] It has been shown by Pealing (Phil. Mag., 29, 203, 1915) that "bifilar effect" of a phosphor bronze ribbon is negligible in its effect upon the elastic constant for torque.

[3] Meaning the position which the coil occupied before it was deflected by means of the torsion head.

[4] Klopsteg, loc. cit.

[5] Loc. cit

II. EXPERIMENTAL.

A. *Tests on Uniformity of Magnetic Field.*

(*a*) *By Means of Current Sensitivity.*—A 3-mil phosphor bronze strip upper suspension was selected with terminal wires such as are provided by the manufacturers. The upper terminal wire was bent so as to displace the suspension fiber about 2 mm. from the vertical axis of the instrument. If the plane of the coil, by suitable means, is kept parallel to its original position while the upper torsion head is turned through 360 degrees, each point of the coil describes a circle the radius of which is the amount of the eccentricity. To accomplish this end, the upper torsion head was turned through a small angle and sufficient current was sent through the coil to return it to the null reading. The angle and current were accurately measured. The process was then repeated sufficiently often to make a complete revolution of the torsion head. The current per degree, for different positions of the coil in the field, was found to vary almost 10 per cent. When the suspension terminals are very short, so as to eliminate the eccentricity mentioned in the first case, the current per degree is found constant over the whole range within which the instrument is used, showing that no matter what the twist in the fiber, the coil was, in each measurement, in the same position relative to the field. Similar results were obtained with a 1.5-mil phosphor bronze strip suspension.

(*b*) *By Means of Ballistic Sensitivity.*—The results of the preceding experiment make it seem probable that the differences which one frequently observes in the ballistic throws of an instrument with equal quantities of electricity but in opposite directions from the same null point[1] may be ascribed to the non-uniformity of the galvanometer field. To investigate this point, an experiment suggests itself, as follows: Using an upper suspension fiber with short terminals, so as to avoid eccentricity, one may, by means of the upper torsion head, shift the coil into various angular positions with respect to its usual null position; for each particular position one may take throws in opposite directions with equal quantities.

This was carried out, starting with a shift of about 10 degrees towards the "red" side of the scale, and, in small steps, working over to the "black" side. The results[2] are shown graphically in Fig. 1. The instrument was moderately damped, and the same quantity of electricity was used for every throw. Abscissas are given in degrees of displacement of the various positions from the ordinary null, ordinates represent centimeters of throw from each position. "*B*" refers to the "black" side of the scale, "*R*" to the "red" side. The two curves in the figure are seen to cross at a point about 0.8 degree from the null toward the "red," indicating that if one were to shift the coil into this position as a new null, equal throws would be obtained *with this particular quantity.* With larger or smaller quantities the curves would not necessarily intersect at the same abscissa.

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[1] Peirce, B. O., Am. Acad. Proc., 42, 161, 1906.

[2] The results here given are from a single instrument; but they may be observed, to a greater or smaller degree, in any instrument of the type described.

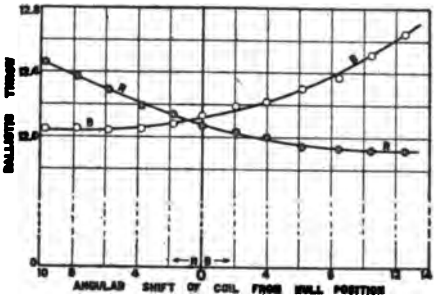

Fig. 1.

Clearly, with a given quantity, the magnitude of the throw depends upon two factors: the intensity of the field at the null position of the coil, and the amount of damping in the region through which the coil swings. The former is the same for any given position of the coil, regardless of the direction of swing; the latter depends upon the nature of the magnetic field. The experiment shows non-uniformity in two ways: first, differences in throws in the same direction from different null positions; and second, differences between throws in opposite directions from the same null.

B. Method of Finding Logarithmic Decrement to be Applied to a Throw of Given Magnitude.

When the damping is slight, it is quite sufficient to find the logarithmic decrement or the damping factor in the ordinary manner, using two successive elongations somewhere near the value of the throw to be corrected, remembering that these elongations are to be taken on the same side of the null on account of the probable difference in the damping on the two sides. For highly damped throws—the motion of the coil still being periodic—the above method may not give the value with sufficient accuracy because of the great difference between two successive elongations on the same side of the null. The procedure in this case is as follows: The resistance in the circuit having been adjusted to the same value as that which is to be used in the measurements, the coil is started swinging from a large displacement. As many sets of readings of successive elongations on the same side of the null are taken as is considered necessary for the desired degree of accuracy. One of these sets is first plotted, using the elongations as ordinates, and the time, expressed in terms of the complete period of the coil, as abscissas. The other sets may then be "fitted in," using the same time unit, and the curve drawn. To apply the curve to the determination of the logarithmic decrement corresponding to any particular amplitude, we find the intersection of the curve with the horizontal line at the amplitude in question; from this point we pass to the left a distance equivalent to a half time-unit, and find the amplitude corresponding to this instant. The natural logarithm of the ratio of the amplitude so found to the amplitude chosen is the logarithmic decrement sought. It is well to remember that this correction is slightly affected by the zero shift due to magnetic impurities;[1] for in an actual measurement one eliminates the zero shift as suggested by Zeleny, while the curve here described is obtained from readings of the positions of the coil as it executes complete swings. The amount of the error has not been determined, but in the ordinary instrument its effect upon the results of measurements is certainly less than 0.2 per cent.

C. *Test of Modified Method for Obtaining K.*

The results shown in Tables I. and II. were obtained in a series of determinations, the purpose of which was not to approach the limit of accuracy attainable by this method of finding the ballistic constant, but rather to determine the reliability of the method under ordinary laboratory conditions. Each quantity used in the formula was determined from but a single careful observation. The instruments were levelled, but it was not attempted to eliminate absolutely the possible eccentricity due to bent terminals. The galvanometers are the ones which are constantly in use in the undergraduate laboratory; no tests were made to see if the scales were accurately circular, or if they were exactly 50 cm. from the mirrors.

A comparison of the results shows that even without these precautions the modified method is superior to that commonly used. In the writer's opinion one is justified, when careful work is done and several determinations of each quantity are made, in assuming an accuracy of 0.2 per cent. in the results.

It would greatly facilitate the manipulation of returning the coil to its normal position from the deflection produced by turning the upper torsion head if the latter were provided with a tangent screw, permitting accurate adjustment at the first trial.

III. SUMMARY.

(*a*) A method is proposed for obtaining the current constant which is used in the equation for obtaining the ballistic constants of moving coil galvanometers, and possible sources of error are discussed.

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[1] A. Zeleny, PHYS. REV., O. S., 23, 400, 1906; PHYS. REV., O. S., 32, 297, 1911.

Table I.

Galvanometers provided with 3-mil phosphor bronze ribbon upper suspensions; damping sufficiently small to admit of use of approximate form of eq. (1).

Galv. No.	K, Coulombs/Cm. × 10^7.		
	Usual Method.	Modified Method.	Condenser[1] Method.
I	2.901	2.880	2.873
III	3.440	3.469	3.464
VI	2.957	2.931	2.938
VII	3.161	3.162	3.157
IX	5.314	5.326	5.324
XI	2.804	2.793	2.793
XIII	2.961	2.961	2.960
XV	3.009	2.993	2.994
XVI	2.814	2.799	2.802

Table II.

Galvanometers provided with 1.5-*mil phosphor bronze ribbon upper suspensions; smal damping.*

Galv. No.	K, Coulombs/Cm. × 10^7.		
	Usual Method.	Modified Method.	Condenser[1] Method.
II	1.561	1.561	1.563
IV	1.921	1.932	1.931
VIII	1.479	1.459	1.454
X	1.989	1.970	1.966
XII	1.650	1.669	1.665

(*b*) Experiments are described in which these sources of error are investigated, leading to suggestions for minimizing these errors. Principally the errors are due to non-uniformity of the magnetic field and to insufficient levelling of the instrument.

(*c*) Results of tests of the proposed method are given which indicate that with care the method is reliable to 0.2 per cent. accuracy.

(*d*) The suggestion is made that galvanometers to which the method is to be applied be provided with a tangent screw for fine adjustments of the torsion head.

Physical Laboratory,
University of Minnesota,
January 12, 1916.

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[1] Condenser with Bureau of Standards certificate, charged by means of cadmium standard cells, using free charge only. See A. Zeleny, Phys. Rev., O. S., 22, 1906, p. 65; O. S., 23, 1906, p. 409.

THE CORRECTION FOR THERMOELECTRIC CURRENT TO BE APPLIED TO THE THROW OF A BALLISTIC MOVING-COIL GALVANOMETER.

BY PAUL E. KLOPSTEG.

THEORETICAL.

ON account of the simplicity and convenience of the condenser method of determining the ballistic constant of a moving-coil galvanometer, one is likely to use it in preference to the more complicated inductance-coil methods in making an accurate determination of the constant. This is especially true when the instrument is used on open circuit, in which case there are no particular difficulties to be overcome in making a precise measurement. When the discharge to be measured takes place in the closed galvanometer circuit,[1] the constant should be determined under similar conditions, because of the dependence of the ballistic throw upon damping. When using a standard condenser for obtaining the constant on closed circuit it is necessary, immediately after the condenser has been discharged, to connect the galvanometer through a resistance such that the total resistance is the same as it is to be in the circuit with which the measurements are to be made. This is easily accomplished by means of special keys or switches.

Upon changing from open to closed circuit one is likely to encounter a thermoelectric current[2] which deflects the coil either in the same direction as or opposite to that of the throw caused by the impulsive discharge. The time of reaching the maximum elongation from the null point is not the same in the case of a current of very short duration as in the case of a deflection due to a steadily impressed electromotive force. Were these time intervals alike, the correction could be made by algebraic addition of the maxima. The relation between them depends upon the amount of damping in the circuit, and is therefore different for different values of the logarithmic decrement. Zeleny[3] has given an experimental curve for correcting the observed throw and obtaining from it the magnitude of the throw which would have resulted had there been no thermo-electromotive force in the circuit.

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[1] Zeleny and Erikson, Manual of Physical Measurements, 3d ed., p. 172; Smith, Electrical Measurements, p. 195.

[2] A. Zeleny, PHYS. REV., O.S., 23, 1906, 414. Another method of determining the constant, not yet published, obviates the necessity for making the correction for thermoelectric effect.

[3] Loc. cit.

It is the purpose of this paper to obtain a relation between the quantities involved, by means of which the correction may be made. The following notation will be used:

I_0 = moment of inertia of the coil;
$2f$ = proportionality constant between damping moment and angular velocity of the coil;
$M = nAH$, where n is the number of turns and A the mean area of the coil, and H the field intensity;
q^2 = elastic torque constant of the suspensions;
e = impressed electromotive force;
r = total resistance in the circuit;
i = current at instant t;
θ = angle of deflection;
Λ = logarithmic decrement of coil;
ρ = ratio between successive elongations;
T = complete period of coil.

When a steady electromotive force exists in the galvanometer circuit, the resulting motion of the coil may be expressed by the differential equation of moments

$$I_0 \frac{d^2\theta}{dt^2} + 2f\frac{d\theta}{dt} + q^2\theta = Mi, \tag{1}$$

one solution of which is

$$\theta = \epsilon^{-at}(A \cos bt + B \sin bt) + \frac{Mi}{q^2}, \tag{2}$$

the condition for this solution being $I_0q^2 > f^2$. A and B are the constants of integration, $a = f/I_0$ and $b = \sqrt{(q^2/I_0) - (f^2/I_0^2)}$. Imposing the initial conditions, $t = 0$, $\theta = 0$, $d\theta/dt = 0$, we find

$$\theta = \frac{Mi}{q^2}\left[1 - \epsilon^{-at}\left(\cos bt + \frac{a}{b}\sin bt\right)\right]. \tag{3}$$

This equation represents a damped motion of period $T = 2\pi/b = 2\Lambda/a$, θ having the value Mi/q^2 after a sufficiently long time has elapsed. The first maximum elongation is reached at the instant

$$t = \frac{\pi}{b} = \frac{T}{2}, \tag{4}$$

found by equating $d\theta/dt$ obtained from (3) to zero and solving for t.

Putting this value of t in (3) and solving for θ_d', the first maximum elongation,

$$\theta_d' = \frac{Mi}{q^2}(1 + \epsilon^{-\Lambda}). \qquad (5)$$

The equation of motion of the coil when there is no torque tending to produce a definite displacement of the coil from its null position is

$$I_0 \frac{d^2\theta}{dt^2} + 2f\frac{d\theta}{dt} + q^2\theta = 0, \qquad (6)$$

the solution of which, for the condition $I_0q^2 > f^2$, is the same as eq. (2) with the exception of the last term, which in this case is zero. Using the initial conditions $t = 0$, $\theta = 0$, $d\theta/dt = MQ/I_0$ to evaluate the constants of integration, the result is

$$\theta = \frac{MQ}{I_0}\,\epsilon^{-at}\left(\frac{1}{b}\sin bt\right). \qquad (7)$$

This is also a damped vibratory motion of the same period as that given by eq. (3), provided a and b have the same values, respectively. Its first maximum elongation occurs at the instant

$$t = \frac{1}{b}\tan^{-1}\frac{b}{a}. \qquad (8)$$

Suppose, now, that a quantity of electricity Q in a condenser is discharged through the galvanometer and that the circuit is immediately closed; and that, upon closing, a thermoelectric current flows. If the direction of the steady current is the same as that of the discharge, the resulting maximum is greater than it would be in the absence of the thermo-electromotive force; if the direction of the current is opposite to that of the discharge, the resulting throw is smaller. Clearly, the correction to be applied to the observed throw is the angle attained in a steady deflection[1] during the time interval required for a ballistic maximum, obtainable with the aid of equations (8) and (3). Designating this angle by ϕ, we find

$$\phi = \frac{Mi}{q^2}\left[1 - \epsilon^{-\frac{a}{b}\tan^{-1}\frac{b}{a}}\left(\cos\tan^{-1}\frac{b}{a} + \frac{a}{b}\sin\tan^{-1}\frac{b}{a}\right)\right],$$

or, in terms of the logarithmic decrement,

$$\phi = \frac{Mi}{q^2}\left[1 - \rho^{-\frac{1}{\pi}\tan^{-1}\frac{\pi}{\Lambda}} \cdot \frac{2\cdot\frac{\Lambda}{\pi}}{\sqrt{1 + \frac{\Lambda^2}{\pi^2}}}\right]. \qquad (9$$

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[1] Assuming steady deflection small in comparison with throw. Zeleny mentions 1/3 as he maximum permissible ratio. For further discussion see below.

In order to make this equation for the correction applicable to any galvanometer of the type under discussion, it is convenient to express ϕ in terms of either of two ratios: $R_1 = \phi/d$, giving its value as a fraction of the steady deflection; or, $R_2 = \phi/\theta_d'$, which gives the value as a fraction of the *first maximum due to the steady current.* The value of d, from eq. (3) (putting $t = \infty$) is seen to be Mi/q^2; consequently,

$$R_1 = 1 - \rho^{-\frac{1}{\pi}\tan^{-1}\frac{\pi}{\Lambda}} \cdot \frac{2 \cdot \frac{\Lambda}{\pi}}{\sqrt{1 + \frac{\Lambda^2}{\pi^2}}}. \tag{10}$$

Comparing equations (9) and (5), and noting that $\log_e \rho = \Lambda$, it is seen that

$$R_2 = R_1 \frac{\rho}{1 + \rho}. \tag{11}$$

Inasmuch as the actual motion of the coil is the result of the superposition of the motions defined by (3) and (7), the exact equations of the two possible kinds of motion are obtained, respectively, first, by adding the two equations, and second, by subtracting the one from the other. The observed maximum then occurs not at the instant given by eq. (8), but a trifle sooner or later than this, depending upon the relative directions of the current and impulsive discharge. By equating to zero the angular velocities obtained by differentiation of each of the combined equations and solving for t, we may write the result for both cases:

$$t = \frac{1}{b}\tan^{-1}\frac{b}{a \pm \frac{i}{bQ}}. \tag{12}$$

The upper or lower sign is taken according as the current and discharge are in the same or in opposite directions, respectively.

It would be a rare occurrence in an ordinary measurement to encounter an extraneous thermoelectric current which is appreciable as compared with bQ, considered in its effect upon the time required for the throw. Should it occur, however, the procedure would be to find the correction by several approximations, using (12) instead of (8) in eq. (3). To substitute (12) directly in the equations of motion obtained by the addition or subtraction of (3) and (7), and then to form the ratios to be used for correcting the observed throws, would lead to expressions which are too complex for practical application.

EXPERIMENTAL.

Procedure in Correcting Observed Throws.—Consideration of equations (10) and (11) shows that for a determination of either R_1 or R_2 for any particular case a single experimental determination suffices, namely of the quantity ρ. The factors $\rho^{1/\pi \tan^{-1} \pi/\Lambda}$ and $\sqrt{1 + (\Lambda^2/\pi^2)}$ occurring in the equations are obtainable from tables.[1] This makes the formulas readily applicable. To use the correction given by eq. (10), the observed steady deflection due to the thermoelectric current flowing in the closed circuit is multiplied by R_1 and the result is added to or subtracted from the observed throw, depending upon the relative directions of current and discharge. To apply eq. (11), the first maximum amplitude due to the thermoelectric current only is observed, and the correction obtained by multiplying this value by R_2. Having once obtained R_2, its application is more economical of time than the use of R_1, since the first maximum elongation may be observed $T/2$ seconds after closing the circuit. This obviates the necessity of waiting for the deflection to attain its steady value, or of using a short-circuiting key for bringing the coil to rest.

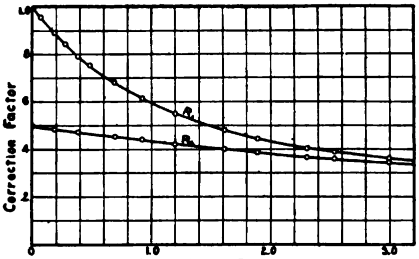

Fig. 1.

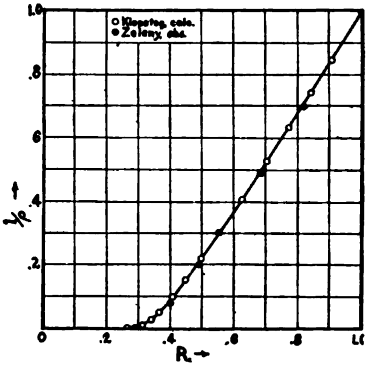

Fig. 2.

The curves of Fig. 1 show the variation of each of the ratios with logarithmic decrement. In these curves the abscissas have been carried to $\Lambda = 3.2$ only, since, in practice, Λ seldom exceeds 1 when the galvanometer is used ballistically. If the logarithmic decrement is large, it is most convenient to render the coil just aperiodic by diminishing the resistance in the circuit. In the critically damped case $R_1 = R_2 = 26.4$ per cent. This is shown in Fig. 2.

Test of the Equations.—The experimental test of the method here outlined was taken from the work of Zeleny.[2] From eq. (10) a set of corresponding values was obtained for R_1 and $1/\rho$, the abscissas and ordinates, respectively, of the experimental curve to which reference has been made. Fig. 2 shows the curve obtained by plotting these values, together with representative points taken from Zeleny's curve. There is good agreement between the theoretical and experimental curves.

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[1] Kohlrausch, Lehrb. d. prakt. Phys., 12th ed., p. 723.

[2] Loc. cit.

SUMMARY.

From a consideration of the equations of periodic motion of a galvanometer coil under the conditions, first, that a steady electromotive force exists in the circuit and second, that the electromotive force is zero, two expressions are deduced. Each expression gives the correction for thermoelectric current to be applied to the ballistic throw on closed circuit, the discharge having taken place on open circuit. The equations involve simply the determination of the ratio of one amplitude of the coil to the one next following, together with factors which are obtainable from existing tables. The calculated and observed corrections are found to be in good agreement.

PHYSICAL LABORATORY,
THE UNIVERSITY OF MINNESOTA,
February 2, 1916.

A RECORDING X-RAY SPECTROMETER, AND THE HIGH FREQUENCY SPECTRUM OF TUNGSTEN.

BY ARTHUR H. COMPTON.

ALTHOUGH the photographic method of obtaining the spectrum of a beam of X-rays reflected from a crystal has led to most interesting results in the hands of Moseley,[1] de Broglie[2] and others, it has been found possible to examine such a spectrum more thoroughly by ionization methods.[3] The reason for this is that while by the photographic method the intensity of the different spectrum lines can at best be only qualitatively measured, the ionization method is capable of giving quantitatively the relative intensity of the reflected beam of X-rays at different angles. The ionization method as usually applied, however, is open to the objection that a very large number of separate observations are necessary to obtain accurately a complete spectrum. In the present paper an apparatus will be described which gives a continuous record of the intensity of the beam of X-rays at different angles, and a study of the X-ray spectrum of tungsten will be made to illustrate the manipulation of the instrument.

This apparatus differs from the well-known Bragg X-ray spectrometer in two essentials: (1) The ionization current due to the reflected beam of X-rays, instead of charging up an electroscope directly, goes to one pair of quadrants of an electrometer and is shunted to earth through a high resistance. The electrometer thus acts as a highly sensitive galvanometer, a steady ionization current producing a steady proportional deflection. (2) The angle of the crystal and of the ionization chamber are varied continuously in such a manner that the ionization chamber is always in position to receive the reflected beam of X-rays, and the electrometer deflections corresponding to each particular angle of the crystal are recorded on a moving roll of photographic paper. In this manner a record of the complete spectrum can be obtained with a minimum of trouble and of exposure to the X-rays.

The arrangement of the apparatus is shown diagrammatically in Fig. 1. The X-rays pass from the anticathode *A*, through the slits *B* and *B′*, are reflected from the crystal *C* and pass through the slits *D′* and *D* into the ionization chamber *I*. The ionization chamber rests on an arm fastened to the spectrometer table *S*, and the table on which the crystal rests is geared to move with half the angular velocity of the ionization chamber, though its position can also be varied by means of the slow motion screw *F*. The ionization current is carried from the chamber *I* to one pair of quadrants of a highly sensitive electrometer *E*, and is shunted to the ground through a variable xylol-alcohol resistance *R*. The intensity of the primary beam is measured in a similar manner by means of the ionization chamber *I′*, the resistance *R′* and the electrometer *E′*. The mirrors of both electrometers reflect beams of light from the Nernst glower *N*, through a fine horizontal slit *k* onto a roll of bromide paper *P*. A motor-actuated driving clock *MQ* moves, by means of a system of pulleys and worm gears, both the spectrometer table and the roll of bromide paper at a constant, though adjustable, speed. The pointer *H* is so geared to the shaft which drives the spectrometer table that it makes one revolution for each degree through which the table turns, and by means of a mercury contact the lamp *L* is turned on for an instant at each degree or half degree. This illuminates the slit *K* and marks off the angle by a series of lines across the bromide paper, as is shown in the records, Figs. 4 to 11.

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[1] H. G. J. Moseley, Phil. Mag., 26, 1024 (1913).

[2] M. de Broglie, Compt. Rend., 157, 924 and 1413 (1913).

[3] Cf. W. H. Bragg and W. L. Bragg, X-rays and Crystal Structure, p. 66.

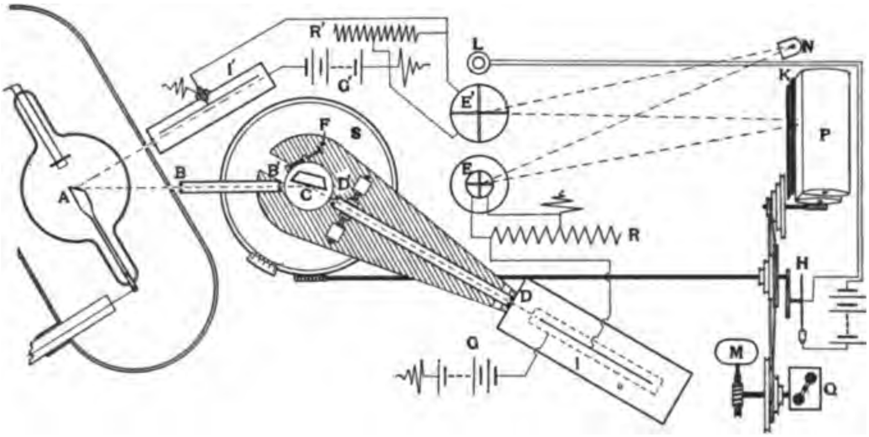

Fig. 1.

The construction of the system for measuring the reflected beam of X-rays is shown in more detail in Fig. 2. The ionization chamber *I* is closed at one end by a thin mica window *m* and at the other by a perforated brass cap covered with a plate of glass *g*. This arrangement allows the crystal to be adjusted optically to the angle of reflection.

The outside of the ionization chamber is grounded, but the wire framework *f* is raised to a sufficiently high potential to produce a saturation current to the wire *w* which is connected with the electrometer. The wire *w* passes through the ebonite plug e_1, is fastened to one terminal of the glass tube *R* containing a mixture of xylol and alcohol, and goes through the ebonite plugs e_2 and e_3 to the electrometer. The wire can be grounded directly by screwing up the mercury cup *O* until it touches a contact point, or it can be grounded through the desired high resistance by moving up one of the other mercury cups. If all the cups are down the system is insulated. The electric shielding consists of the rather heavy copper box *c* and the brass tube *b*. The electrometer *E* is placed directly over the axis of the spectrometer, so that the joint in the shield at *j* permits free motion of the spectrometer arm, while an oil seal renders the shield airtight.

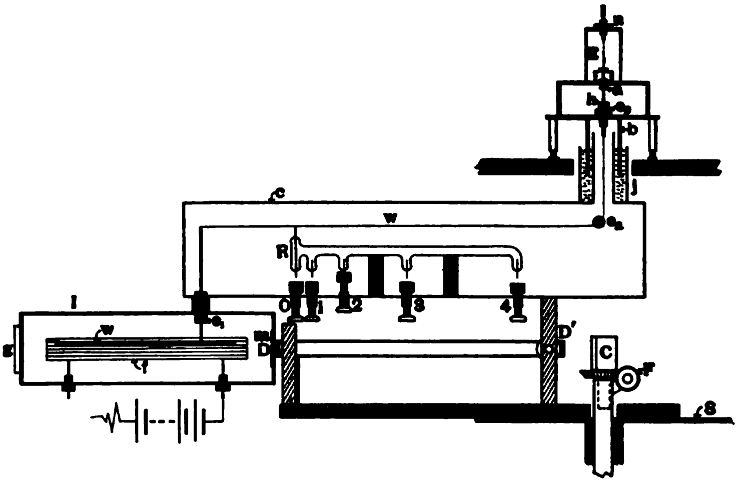

Fig. 2.

The electrometer *E* deserves particular mention, because it is so admirably adapted to this work. Its chief advantages lie in its flexibility, in its high sensitivity, in its perfect electrostatic shielding and in the use of a single piece of insulation for the sensitive quadrants. The quadrants of the electrometer are only 1.3 cm. in diameter. One pair of quadrants is supported by brass rods which fit into the electrometer case, and is thus permanently grounded; while the other pair is supported on a fine brass rod which passes through the amber insulation *a* and dips into the mercury cup *h*. The needle is suspended by a fine quartz fiber, and the whole system is sputtered with platinum. Most of the moment of inertia of the needle is due to the mirror, so this is made just large enough to reflect a sufficiently strong beam of light. The needle is "dead-beat," and comes approximately to rest in a few seconds. With ten volts on the needle the electrostatic control is small, and with the suspension used the sensitivity is about 1,000 divisions per volt. When the potential of the needle is above forty volts, the electrostatic control becomes the determining factor, and the sensitivity can be varied from 1,000 divisions to ∞ by adjusting the position of the quadrants and the height of the needle. The highest sensitivity that it has been found practicable to use is about 25,000 mm. per volt at a distance of a meter. The capacity of the electrometer is about 15 electrostatic units, so that it is capable of measuring a very small charge.

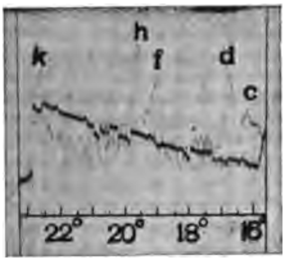

Fig. 3.

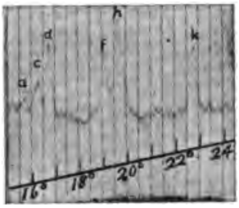

Fig. 4.

The Effect of Slight Radioactivity in the Ionization Chamber.—Fig. 3 shows an early attempt to obtain the X-ray spectrum of tungsten with this apparatus. The dark line represents the primary beam of X-rays and the light one the reflected beam. The electrometer in this case had a sensitivity of about 10,000 divisions per volt, and the shunting resistance was rather high. The electrostatic shielding at this time consisted of steel tubing and galvanized iron. Although the more prominent spectrum lines are easily distinguishable, there are so many spurious deflections of the electrometer which in no way correspond to variations of the primary beam of X-rays that it is impossible to make any accurate measurements. Supposing that these irregular motions were due to stray ionization currents within the electrostatic shield, a copper and brass shield was used instead, thus decreasing the contact potential difference between the shield and the wire. In this manner the relative size of the spurious deflections was much reduced, as is shown in Fig. 4, though these motions were still so large as to interfere seriously with the readings.

Investigation showed that the remaining irregular motions of the electrometer needle were due, in large measure at least, to real ionization currents in the ionization chamber, which were independent of the X-rays, as is shown in Fig. 5 in which the X-ray tube was not running. The magnitude of this current was found to be of the same order as the natural ionization current usually observed in brass vessels. It was also noticed that the deflections always occurred suddenly in the direction of the normal ionization current, and then died out gradually, which suggested ionization due to alpha particles.

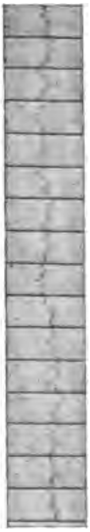

Fig. 5.

As will be shown later (equation 4 below), if the shunt resistance and the sensitivity of the electrometer are known, the total ionization corresponding to each hump on a record can be calculated from the area under the hump. When such a calculation is made on the record shown in Fig. 5, it is found that on the average each hump corresponds to the production of about 8×10^4 ions. Not all of the alpha particles will escape from the walls of the ionization chamber with their full velocity, since most of them will start at some distance from the surface of the metal. If we assume that the alpha particles have a definite range in the metal, a simple calculation shows that the average range of the emitted particles will be half the maximum range. Since, however, the ionization is somewhat greater per centimeter path for the more slowly moving particles, the average ionization will be somewhat more than half, about 0.6 of that produced by a particle which starts from the surface (assuming the emitted particles to be completely absorbed by the gas in the ionization chamber). A particle starting from the surface therefore produces about $8 \times 10^4/0.6 = 1.3 \times 10^5$ ions. An alpha particle from radium would produce 1.5×10^5 ions, which agrees well enough with the observed value to show that these spurious deflections are doubtless due to alpha particles given off either by the walls of the ionization chamber or by the gas within it.[1]

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[1] An effect exactly similar to that here described has been observed by Rutherford and Geiger (Proc. Roy. Soc., *A*, 81, 141 (1908)) while counting the alpha particles from radium by an ionization method. They attributed the effect to the natural radioactivity of the metal of which the ionization chamber is composed. Recently Shrader (PHYS. REV., 6, 292 (1915)) has found that if the electrode within the ionization chamber has a sharp point, and if the chamber is at high enough potential to produce ionization by collision, the "natural disturbances" depend only on the nature of the point, and can be practically eliminated if the point is properly treated. It is to be noticed, however, that the only disturbances which would affect his electroscope would be the ones which would ionize the air near the needle point of his electrode, where any ionization was greatly magnified by collision. It would seem that his apparatus was not sufficiently sensitive to be affected by the occasional production of alpha particles in other parts of the ionization chamber. His work cannot, therefore, be taken to indicate that the walls of his ionization chamber were not slightly radioactive, so that his results are not inconsistent with the writer's and those of Rutherford and Geiger.

A thorough cleaning of the ionization chamber with nitric acid seemed to reduce the amount of radioactivity but little. Its relative effect could be greatly diminished, however, by filling the ionization chamber with a dense gas, and thus increasing the ionization due to the X-rays. The spectrum shown in Fig. 6 was obtained in this manner, using a simple ionization chamber such as *I′*, Fig. 1, filled with dry hydrogen iodide. The effect of the alpha particles was still fruther reduced by changing the ionization chamber to the type shown in Fig. 2. The ionization due to the X-rays is produced chiefly within the wire network *f*, which is kept at a sufficiently high potential to produce a saturation current to the wire *w*. The outer casing of the ionization chamber is grounded, and enough air-space is left between the casing and the network to absorb all the alpha particles which leave the walls. Thus the particles from the walls of the vessel do not affect the current flowing to the wire *w*; this is affected only by the radioactivity of the wire netting and of the gas within it. By this means the number of spurious deflections of the electrometer was reduced to about 1/4 of the number when the simpler ionization was used, while the ionization due to the X-rays remained about the same. All of the spectra after Fig. 6 were obtained with this new ionization chamber.

A Study of the High Frequency Spectrum of Tungsten.—As a source of X-rays for the examination of the spectrum of tungsten, a Coolidge X-ray tube with a tungsten anticathode was used. In order that the tube should remain steady over long runs it was found best to heat the thermionic filament by means of batteries of rather large capacity, 175 ampere hours. The source of high potential was a Snook-Roentgen machine. This consists of a 10-kilowatt step-up transformer with a commutating device in the secondary circuit, which makes it possible to obtain a direct though intermittent high potential current. This apparatus was easily capable of running the Coolidge tube at its maximum capacity, and was found to be very satisfactory. In most of the runs a current of about 30 milliamperes was sent through the tube, with an alternate spark gap between sharp points of about 10 cm.

Fig. 6 shows a complete spectrum of the X-radiation from tungsten as analyzed by a crystal of zinc-blende. As in the other figures the heavy line indicates the strength of the primary beam, and the light one of the reflected beam of X-rays, though of course the two curves are on greatly different scales. Each vertical line corresponds to one degree on the spectrometer table, or 1/2 degree of rotation of the crystal. The angles marked on all of the spectra indicate the glancing angle θ of the beam of X-rays which strikes the crystal. The broken base line represents the zero point for the electrometer measuring the reflected beam, and the solid line that of the one measuring the direct beam of X-rays. At the angle $\theta = 30.5^\circ$ a zero reading was taken, by putting a lead screen between the slit B' and the crystal, to make sure that the zero point of the sensitive electrometer had not shifted. In obtaining this spectrum wide slits were used, the slit at D being about 1.0 cm. The shunting resistance was about 3.5×10^{11} ohms, and the sensitivity 1,050 mm. per volt. With this low resistance and wide slits it was possible to go over the complete spectrum in an hour and a half.

Owing to the comparatively great wave-length of the characteristic L-radiation from tungsten and to the closeness of the layers of atoms in the cleavage planes of zinc-blende, it is impossible to obtain a complete third order spectrum in this case, but the first and second orders stand out very clearly. The line marked k is the single line observed in the spectrum of tungsten by Moseley,[1] and the prominent lines c, d, f, h and k correspond to the five lines observed by Moseley and Darwin[2] in the spectrum of platinum. Recently Barnes[3] has detected also the lines g and i in the tungsten spectrum by a photographic method. Lines at a and l can also be seen plainly in this photograph, l being visible only in the second order spectrum because of its proximity to the prominent line k. An indication is also given of possible lines at e and j. These are confirmed by other records, though the line j is rather doubtful. A series of four records, such as Fig. 7, which is a part of the first order spectrum from rock-salt using a slit at D only 0.4 mm. wide, indicates that the line c is really a very close double. A line also appears con-

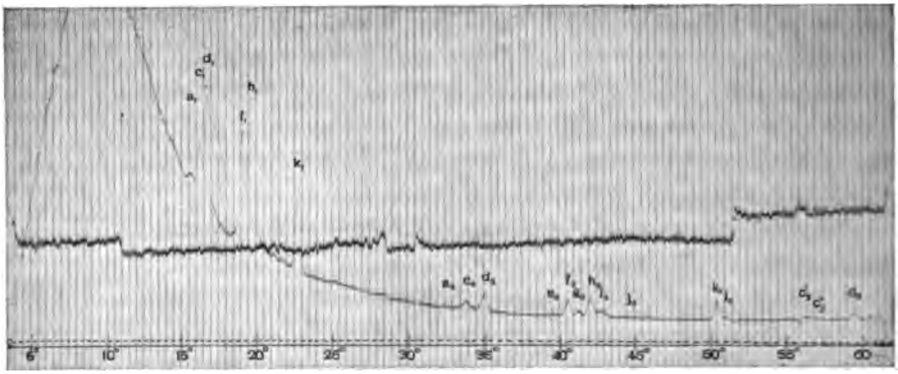

Fig. 6.

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[1] H. G. J. Moseley, Phil. Mag., 27, 703 (1914).

[2] H. G. J. Moseley and C. G. Darwin, Phil. Mag., 26, 211 (1913).

[3] J. Barnes, Phil. Mag., 30, 368 (1915).

sistently on a number of records between *a* and *c*. All of these lines are shown more or less distinctly in Fig. 8.

In order to make an accurate determination of the wave-length of the different lines, a crystal of calcite was used, as recommended by Professor Bragg, because of the extremely perfect faces obtainable. The X-ray tube was swung around until the rays which fell on the crystal left the target almost tangent to its face, the slits *B* and *D* were about 2 mm., and the slits *B′* and *D′* were made wide. The slit at *D* and the target of the X-ray tube were set as nearly as possible at equal distances, about 52 cm., from the crystal. With this arrangement it was found possible to measure the angles of reflection with considerable precision.

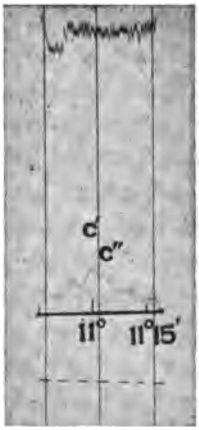

Fig. 7.

It was necessary first to determine the zero point accurately. This was done by taking a series of four records, two on either side of the zero point, one with the spectrometer moving toward and the other from zero. Fig. 8 is one of this series, taken with the spectrometer moving toward the zero point. The mean angle as measured from the three most prominent lines in the four records was taken as the true zero, and this could be determined with a probable error of less than ± 0.2′. The angle of the spectrum lines was then determined from a series of six records of the first order spectrum and two records showing the third order *k* and the fourth order *c*, *d*, *f* and *h* lines. Fig. 9 shows a part of one of these

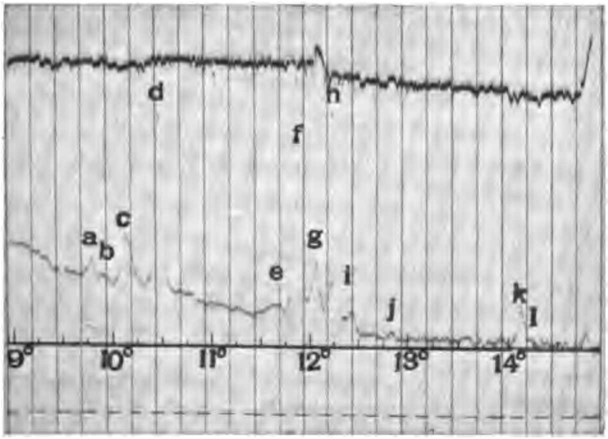

Fig. 8.

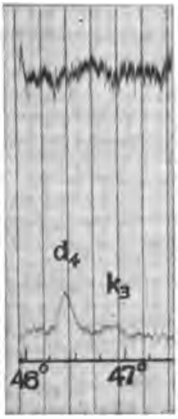

Fig. 9.

records, including the fourth order *d* and the third order *k* lines. The angles found for the higher order spectra did not correspond exactly with those calculated from the first order according to the formula

$$n\lambda = 2d \sin \theta.$$

The reason for the difference was traced to a combination of two causes: first, the front edge of the crystal projected about 0.5 mm. in front of the axis of the crystal table, and second, the slight penetration of the X-rays into the crystal made the observed angle slightly less than the true angle of diffraction. The latter effect is very small, being only about 0.2′ for tungsten rays diffracted by calcite. For X-radiation of short wave-length, however, and a crystal such as diamond, this error becomes of considerable magnitude, as has been noticed in the case of rhodium rays.[1] The correction due to the inaccurate adjustment of the crystal could be calculated accurately from a comparison of the position of the different orders of the same line. The sum of the two corrections was 3.15′ ± 0.10′, being practically equal for the different lines. The corrected angles of the lines of the first order spectrum are given in column 2 of Table I. The error in the angle here given for the more intense lines is probably less than ± 0.5′, though for some of the fainter lines the probable error is as much as ± 1.0′.

TABLE I.

Line.	θ	$\lambda \times 10^8$ Cm.	Barnes's Values.	Remarks.
a	9° 44.5′	1.0387		
b	9° 53.1′	1.0539		Line somewhat doubtful.
c′	10° 3.7′	1.0725	1.082	Very close double.
c″	10° 7.7′	1.0796		
d	10° 25.7′	1.1107	1.113	
e	11° 36.3′	1.2349		
f	11° 49.9′	1.2587	1.258	
g	12° 0.4′	1.2771	1.277	
h	12° 11.4′	1.2962	1.296	
i	12° 22.7′	1.3160	1.312	
j	12° 44.7′	1.3543		Line uncertain.
k	14° 4.7′	1.4933	1.477	Moseley gives 1.486.
l	14° 11.1′	1.5044		

The distance between the successive atomic layers in the cleavage planes of calcite may be calculated according to the formula[2]

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[1] W. H. Bragg, Phil. Mag. 27, 898 (1914).

[2] W. H. Bragg, Proc. Roy. Soc., *A*, 89, 468 (1914). "X-rays and Crystal Structure," p. 112. Professor Bragg uses the value $\phi(\beta) = 1.08$, which makes his value of *d* for calcite differ appreciably from that here obtained.

$$d = \left(\frac{1}{2}\frac{M}{\rho N\phi(\beta)}\right)^{\frac{1}{3}},$$

where M is the molecular weight of $CaCO_3$, ρ is the density of the crystal, N is the number of molecules per gram-molecule, and $\phi(\beta)$ is the volume of a rhombohedron the distance between whose opposite faces is unity and the angle between whose edges is β. This function may be shown to be

$$\varphi(\beta) = \frac{(1 + \cos\beta)^2}{\sin\beta(1 + 2\cos\beta)}.$$

For calcite, β is 101° 54′, which makes $\phi(\beta) = 1.0960$[1]. The density of the crystal used was carefully determined, and was found to be $\rho = 2.7116 \pm 0.0004$ g. cm^{-3}. (at 18°). Taking $M = 100.09$ and $N = 6.062 \times 10^{23}$ (Millikan), we find $d = 3.0279 \times 10^{-8}$ cm. From this value of d the wave-lengths can be immediately calculated according to the expression

$$\lambda = 2d\sin\theta.$$

Using the values of θ given in the second column of Table I., we obtain the wave-lengths given in the third column. For convenience in comparison the wave-lengths found by Barnes[1] are given in the fourth column.[2]

The Intensity of the Different Orders of Spectra.—It is difficult to obtain an accurate estimate of the relative intensity of the various lines in the same order of the spectrum, on account of the difference in the absorption of the corresponding wave-lengths. It is possible, however, to make a comparatively accurate determination of the relative intensity of the different order reflections of the same line. In order to do this, the slit at B' was made narrow, about 0.9 mm., slit D' was removed, and slit D was made 12 mm. wide, which was more than broad enough to take in all the rays reflected from the crystal. When the crystal and ionization chamber are moved at uniform speed past a spectrum line, the area under the curve representing the line on the resulting record is proportional to the integral of the ionization current produced by the line, and hence to the intensity of the line itself. Thus by comparing the areas under the humps on the record, the intensities of the different order lines may be compared.

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[1] J. Barnes, loc. cit.

[2] Note added May 12, 1916: In a preliminary report on this work, made at the meeting of the American Physical Society on February 26, the wave-lengths of the spectrum lines were calculated using as the grating space for calcite 3.0695×10^{-8} cm. instead of the value here found. The results thus obtained agreed acceptably with Barnes's measurements. W. S. Gorton has since shown (Phys. Rev., Feb., 1916), by comparison with rock-salt, that d for calcite is rather about 3.028×10^{-8} cm., which called my attention to an error in my previous calculation of the grating space. The wave-lengths here obtained are in much better agreement with those found by Gorton than with those due to Barnes.

Since the period of the needle and the time required for the electrometer to charge up to its maximum potential might also be expected to affect the area of the hump, it may be worth while to investigate the question in detail. The equation of motion of the spot of light reflected by the electrometer needle onto the photographic paper is

$$I\frac{d^2y}{dt^2} + D\frac{dy}{dt} + My = \frac{M}{a}V, \tag{1}$$

where y is the deflection of the spot of light, I is the inertia term, D the damping term, M represents the restoring force, $1/a$ is the sensitivity of the electrometer, and V is the potential difference between the two pairs of quadrants. On integrating this expression between the times t_0 and t_1 we obtain,

$$\frac{Ia}{M}\left[\frac{dy}{dt}\right]_{t_0}^{t_1} + \frac{Da}{M}\left[y\right]_{t_0}^{t_1} + a\int_{t_0}^{t_1} ydt = \int_{t_0}^{t_1} Vdt. \tag{2}$$

The potential of the system at any time is given by the expression

$$V = Ri - RCdV/dt,$$

where R is the resistance of the shunt to ground, i is the ionization current, and C is the electrostatic capacity of the system. Integrating as before,

$$\int_{t_0}^{t_1} Vdt = R\int_{t_0}^{t_1} idt - RC\left[V\right]_{t_0}^{t_1}.$$

Combining with equation (1) and substituting in (2) we get

$$\begin{aligned}\int_{t_0}^{t_1} idt = \frac{a}{R}\int_{t_0}^{t_1} ydt &+ \left(\frac{aD}{RM} + aC\right)\left[y\right]_{t_0}^{t_1} \\ &+ \left(\frac{aI}{RM} + \frac{aCD}{M}\right)\left[\frac{dy}{dt}\right]_{t_0}^{t_1} + \frac{aCI}{M}\left[\frac{d^2y}{dt^2}\right]_{t_0}^{t_1}.\end{aligned} \tag{3}$$

Since the intensity of a line is proportional to the total ionization produced in the ionization chamber as it moves past the line, expression (3) is evidently a measure of the intensity if the times t_0 and t_1 are taken just before and just after the line is traversed. If the displacement, velocity and acceleration of the spot of light are the same at the times t_0 and t_1, equation (3) reduces to

$$\begin{aligned}\int_{t_0}^{t_1} idt = \frac{a}{R}\int_{t_0}^{t_1} ydt &= \frac{a}{R}\frac{dt}{dx}\int_{x_0}^{x_1} ydx \\ &= \frac{a}{R}\frac{dt}{dx}A,\end{aligned} \tag{4}$$

where dx/dt is the velocity with which the roll of paper moves, and A is the area under the hump on the record. In practice it is easily possible to satisfy these conditions with sufficient accuracy, so that the intensity may be taken to be proportional to the area under the curve.

The records shown in Figs. 10 and 11 are examples of this method of comparing intensities. Fig. 10 shows the first and second order f and h lines as reflected from a cleavage plane of rock-salt, and in Fig. 11 the second and third orders of these lines are compared. In Fig. 10 the lighter line has been inked, in order to make a better reproduction. The areas under the humps were measured with a planimeter. The largest part of the error probably occurred in selecting the base line from which

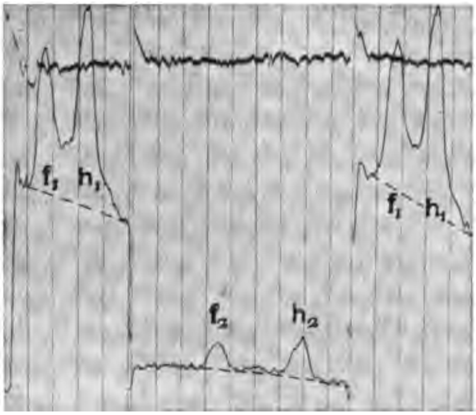

Fig. 10.

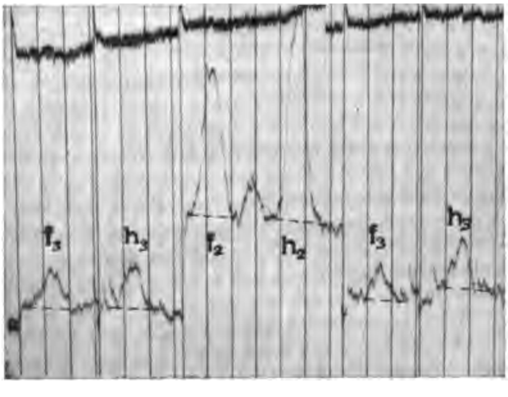

Fig. 11.

to measure the area. These records give the ratio of the intensities of the three orders as 100 : 18.9 : 3.9.

Bragg[1] has studied the relative intensities of the first three orders of reflection from rock-salt, using the rhodium line of wave-length 0.614 × 10^{-8} cm. He found that he could express his results within the probable error of his experiments by the formula

$$J_\theta = \frac{A(1 + \cos^2 2\theta)\, e^{-\beta \frac{\sin^2\theta}{\lambda^2}}}{\sin^2\theta},$$

where J_θ is the intensity of a given spectrum line when reflected at the angle θ, and A and B are constants. The factor $(1 + \cos^2 2\theta)$ was deduced theoretically by J. J. Thomson, and indicates a polarization of the reflected beam. Since in Bragg's measurements θ never became greater than about 20°, he was unable to decide experimentally whether this factor should really appear. In the writer's experiments the angle θ for the third order was 43°, so that the polarization should have been nearly complete. In the following table the first calculated values do not include the factor $(1 + \cos^2 2\theta)$, but this factor is included in the second calculation. The fact that the observed values agree much more closely with the values calculated when this factor is considered shows that the crystal grating does polarize the reflected beam and that this polarization is approximately complete when the angle of reflection is 45°.

TABLE II.

Order.	Mean Angle.	Intensities.		
		Calc.$_1$	Obs.	Calc.$_2$
1	13° 5′	100.0	100.0	100.0
2	26° 55′	21.6	18.9	16.2
3	42° 59′	7.5	3.9	4.2

Summary.—A recording X-ray spectrometer has been described by means of which a continuous record can be obtained of the ionization produced by a beam of X-rays reflected from a crystal as the angle of the crystal is varied.

A spurious and irregular current in the ionization chamber has been observed, which has been found to be due to a slight radioactivity within the ionization chamber. Methods are described whereby the effect of this radioactivity may be considerably reduced.

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[1] W. H. Bragg, Phil. Mag., 27, 893 (1914). also "X-rays and Crystal Structures," p. 198.

The X-ray spectrum of tungsten has been examined in some detail. In addition to the seven lines already known, an indication has been found of six others, two of which are, however, somewhat uncertain. The wave-lengths of the different lines have been carefully determined.

The relative intensities of the different order reflections of the same line have been shown to be proportional to the areas under the humps produced in the record by the spectrum line; and the relative intensities of the first three orders in the spectrum from rock-salt have been determined by measuring these areas.

The observed intensities of the different orders indicate that polarization occurs when a beam of X-rays is reflected by a crystal.

In conclusion I wish to thank Professor H. L. Cooke for many valuable suggestions in the design of the apparatus and for helpful advice in all parts of the work.

Palmer Physical Laboratory,
Princeton University,
February 16, 1916.

A NEW METHOD FOR ENLARGING PHOTOGRAPHS WITHOUT THE USE OF A LENS.[1]

BY ALFRED J. LOTKA.

AN observation of a peculiar distortion presented by the appearance of certain moving objects led the writer to the development of the method of enlarging photographs described below.

This method consists in moving the negative to be enlarged past a narrow slit source of light, and at the same time moving a sensitive plate under the negative at a speed equal to some constant multiple n of the speed of the negative.

On development of the sensitive plate a positive transparency is obtained, in which all lines of the original negative which were parallel to the slit during the exposure are unaltered, while all lines at right angles to the slit are magnified in the ratio $n : 1$.

This positive is subsequently subjected to a repetition of the process practised on the original negative, but with the motion now at right angles to the lines drawn out n-fold in the first operation. The result of the second operation is a negative geometrically similar to the original, but with the linear dimensions enlarged n times, or the area n^2 times.

The apparatus employed for carrying out the process has been constructed on exceedingly simple lines, with the use of material readily obtainable on the market. The slit is made with two strips of brass picked from the stock of a wholesale hardware store and having received no other treatment than a cleansing with a brass polish.

The ratio n of the velocities in the apparatus so constructed is 2, and is secured by the simple expedient shown diagrammatically in Fig. 1. In this drawing the original negative is represented at A, the sensitive plate at B, and the slit at S. The carrier C, on which the plate B rests, has attached to it at each end a silk thread D, which passes through an eye E, attached (with gummed paper) to the glass plate of the negative (when a celluloid film is used, such film may be attached with gummed tape to a glass plate). The further end of the silk cord D is attached to the end block of the box containing the negative and plate during exposure. The carrier C slides on the bottom of the box, between guides adjusted to fit the sides of the plate.

---

[1] Demonstrated at the New York meeting of the American Physical Society, February 26, 1916.

It will readily be seen that, in accordance with the principle of the single movable pulley, the velocity of the sensitive plate is always double that of the negative.

Any means, such as a hand-pull at P, may be employed to impart uniform motion to the negative. A cylinder containing water through which a fairly snug-fitting perforated piston is drawn makes a good dash-pot to regulate the motion. This and a still simpler and exceedingly compact device for this purpose is a subject of a pending patent application.

The adjustment of the lamp vertically above and in the plane of the slit is effected by looking down at the lamp and its reflection in the glass covering the slit. When the filament, its image, and the line of the slit coincide in the line of sight, the filament is in the plane of the slit. It was found that with a slit 6½ inches long good results were obtained with a filament of six or seven coils placed about one foot from the slit.

Among the advantages which the new method presents is uniformity of illumination over the entire field; geometric similarity to the original of the image produced, *i. e.*, absence of distortion such as spherical aberration introduces; compactness and simplicity of apparatus, replacement of lens by a simple slit and hence low cost of the apparatus.

The process necessitates two successive exposures, but this is true also of the ordinary method when a negative is to be prepared from a negative.

The example shown in the accompanying half-tone illustration gives an idea of the working of the process, though it does not represent the best that can be done in the way of definition.

The limiting separation which the apparatus will effect can be figured as follows:

Consider a point right on the edge of the negative. This will begin to be reproduced on the sensitive plate as soon as the negative and plate are in the position shown in Fig. *2a*. (The slit is here shown on a greatly enlarged scale.) The same point will continue to be reproduced until negative and sensitive plate are in the position shown in Fig. *2b*. It will therefore be drawn out into a line of length $ns - s = (n - 1)s$.

Now consider two points, a distance d apart in the negative. In the copy they will appear as two lines $(n - 1)s$ long and with their centers nd apart. These two lines will just merge at their ends when

$$nd = (n - 1)s,$$

$$d = \frac{n - 1}{n} s.$$

This distance

$$d = \frac{n - 1}{n} s$$

between two points on the original negative represents the limit of separation: Two points a distance greater than d apart will be separated, two points nearer together than d will not be separated.

For $n = 2$, as in the apparatus constructed, we have

$$d = \tfrac{1}{2}s.$$

Thus, for example, a comparatively coarse slit 1/100 inch wide, will still separate points just a little over 1/200 inch apart.

The process and apparatus here described are protected by U. S. Pat. 1,176,384 of March 21, 1916, issued to the writer.

GENERAL CHEMICAL COMPANY,
NEW YORK.

PHYSICAL REVIEW, VOL. VII., SECOND SERIES.
June, 1916.

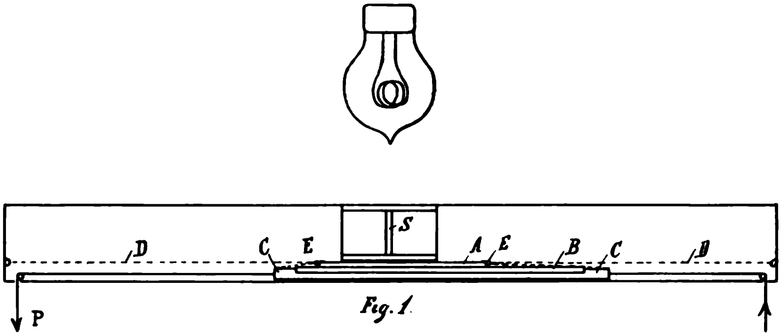

Fig. 1.

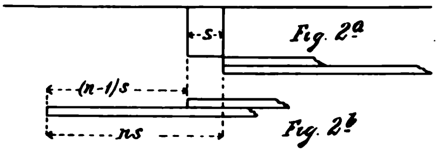

Fig. 2.

Figs. 1 and 2. Diagram of Apparatus.

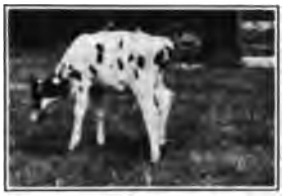

Fig. 3.

Half-tone copy of original photograph.

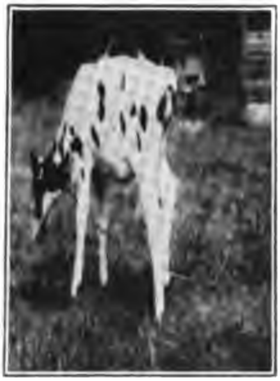

Fig. 4.

Copy of distorted positive transparency obtained in first step of process.

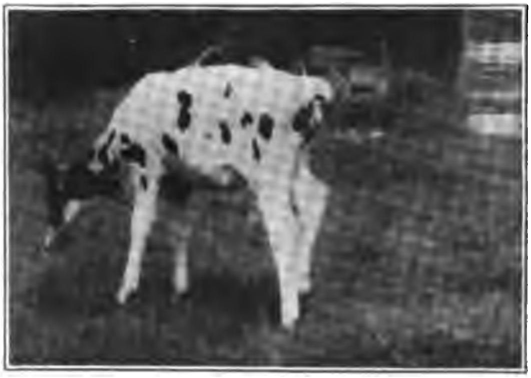

Fig. 5.

Copy of enlargement obtained in second step.

ALFRED J. LOTKA.

THE EFFECT OF A MAGNETIC FIELD ON THE CONDUCTIVITY OF FLAMES.

By C. W. Heaps.

WHEN an ionized gas is acted upon by an electric force the positive and negative ions drift in opposite directions and an electric current is produced. The velocities which the ions acquire under the influence of the electric field have been determined under different conditions and in a variety of ways. It is found that for small electric fields the velocity of the negative ion is proportional to the field and is in general greater than the velocity of the positive ion.

When the ionized gas is a Bunsen flame the negative ion appears to acquire a much greater velocity than the positive ion, and experiments which have been made give somewhat different values for this velocity. By a study of the Hall effect a value of 2,450 cm. per sec. for one volt per centimeter has been obtained;[1] measurements of conductivity under direct and alternating fields gave velocities varying from 5,300 to 12,900;[2] measuring the effect of a magnetic field on the resistance of a flame gave for the above quantity a value of about 5,000 cm. per sec. for one volt per centimeter.[3]

In the experiments described below the velocities of negative flame ions are determined for different values of the electric intensity and it is shown that when small electric forces act the velocity under unit force is smaller than that when larger forces act. Townsend has shown[4] that for gases at ordinary temperatures the velocity of the negative ion may be a somewhat complicated function of X/p, where p is the pressure of the gas and X is the electric intensity. When X/p is small the velocity varies as X/p; when X/p is between 0.01 and 0.2 the velocity increases rapidly; and when X/p is large the velocity increases more slowly than X/p. The experiments described below indicate that the velocity of the negative ions in a flame varies with X/p in a manner similar to that found in gases at ordinary temperatures. For small values of X/p the velocity is proportional to X/p; a stage is reached when the velocity increases rapidly with X/p; and for large values of X/p it becomes again nearly proportional to X/p.

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[1] H. A. Wilson, Phys. Rev., III., p. 375, 1914.

[2] H. A. Wilson, Electrical Properties of Flames. E. Gold, Proc. Roy. Soc., 79, p. 43, 1907.

[3] H. A. Wilson, Proc. Roy. Soc., 82, 1909. The above velocity is obtained by using the formula derived in the latter part of the present paper, and not by the formula of Wilson's paper.

[4] J. S. Townsend, Electricity in Gases.

The experiment consisted in determining the change of conductivity (which is defined as the ratio of the current to the potential gradient) produced in a flame by the presence of a magnetic field. In a similar experiment performed by H. A. Wilson,[1] in which a vertical flame and a horizontal current were used, complications were found to enter, probably because of the upward motion of the flame gases. In the present experiment, therefore, the electric current was made to flow vertically, in the same direction as the flame gases.

The flame was produced by burning a mixture of gasoline vapor and air flowing through a special type of burner. The air and the gas were well mixed in the proper proportions before entering the burner, the air coming from a reservoir where it was maintained at constant pressure. The burner consisted of a vertically mounted brass tube, 2.4 cm. in diameter, into the top of which an iron grid was fitted. This grid caused the ordinary green cone of the Bunsen flame to be broken up into a large number of very small cones. A large volume of homogeneous flame was thus available above the green cones for experimenting purposes. A brass tube 3 cm. in diameter and 6.5 cm. long was fitted loosely around the burner so as to project 1.2 cm. above the grid. This tube tended to make the flame steady. When the air and gas are mixed at some distance from the flame this type of burner gives a much steadier flame than the ordinary Bunsen burner, and in making measurements of conductivity steadiness is desirable if accuracy is to be secured.

The electrodes by which the current entered and left the flame were of platinum wire mounted on ebonite rods and bent to form a coarse grid about one centimeter in diameter. They were in series with a shunted galvanometer, sensitive to 3.8×10^{-8} amperes, a reversing switch, and a battery of small dry cells.

The potential difference between any two points in the flame could be measured by means of two platinum exploring wires connected to opposite quadrants of a Dolezalek electrometer. In order to secure steadiness of the electrometer the needle was equipped with a small rod projecting downward between the quadrants and immersing itself in a small insulated pot of castor oil. The electrometer gave a deflection of about 3 divisions per volt, and was calibrated by comparison with a Weston voltmeter.

The magnetic field was produced by a Weiss electromagnet with flat pole-pieces 10 cm. in diameter and 5.1 cm. apart, between which the flame was placed. The magnetic field was measured with a bismuth spiral. A diagram of the apparatus is given in Fig. 1, where *G* is a galvanometer, *E* the electrometer, and *F* a fuse of aluminium foil.

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[1] Proc. Roy. Soc., 82, 1909.

Experiment has shown[1] that when a current flows through a flame between platinum electrodes there is a large potential drop near the cathode and a small uniform gradient in the rest of the space. Covering the cathode with a coat of lime[2] or salt[3] causes this cathode fall of potential to disappear, the current through the flame is increased, and the previously small uniform gradient is made larger. When vertically placed electrodes are used in a flame it is necessary to make the top electrode the cathode if it is to be coated with salt or lime, otherwise the flame between the electrodes becomes filled with particles of salt or lime and its conductivity is changed. In the present experiment the top electrode—the cathode—was covered with the deposit formed by burning ordinary sealing wax on the platinum terminal. This deposit, consisting of metallic oxides, was found to produce the same effect on the potential gradient as a salt layer, and it had the added advantage of being much more lasting.

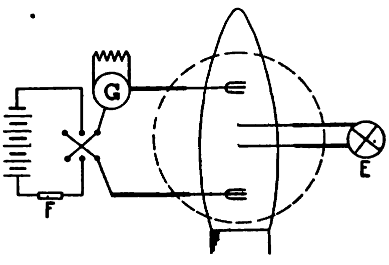

Fig. 1.

If the density of the ions varies from one part of the flame to another the conductivity will be different in different parts also. This variation in conductivity arising from change of position was examined by moving the exploring terminals of the electrometer along the flame between the electrodes and measuring the change of potential gradient. When the electric field was small the conductivity was found to be higher in the lower part of the flame. In one case it decreased about four times in moving from a point near the anode to a position not far from the cathode. For larger electric fields there was not such a large variation, and furthermore the conductivity was higher in the upper part of the flame near the cathode. When a transverse magnetic field acted the variation of conductivity was in general made greater.

The curves of Figs. 2, 3, 4, and 5 show the results of these measurements when different voltages were applied to the electrodes. The number on each curve is the value of the applied potential for which the curve was obtained. The exploring terminals of the electrometer were 0.8 cm. apart and the current electrodes were 4.8 cm. apart. The solid line gives the potential gradient when the magnetic field is zero and the broken line the gradient for a field of 3,600 gauss. It will be observed that the broken line always crosses the solid line—it must do this since the average potential gradient over the whole region between the electrodes is not changed appreciably by the magnetic field.

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[1] H. A. Wilson, Phil. Trans., *A*, 192, p. 499, 1899.

[2] Tufts, Phys. Zeitschr., Vol. V., p. 76, 1904.

[3] H. A. Wilson, Electrical Properties of Flames.

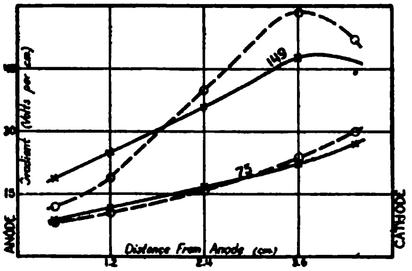

Fig. 2.

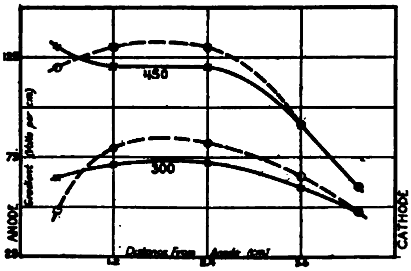

Fig. 3.

The curves shown cannot be taken as representing the exact variation of potential gradient, because if this gradient changes very rapidly electrometer terminals 0.8 cm. apart will not give the correct slope of the curve. A point on any curve gives the average potential gradient over a distance of 0.8 cm. When the exploring points were moved more closely together the existence of a fairly large decrease of potential gradient was detected near the cathode when 149 volts were applied.

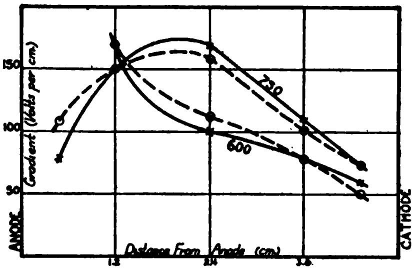

Fig. 4.

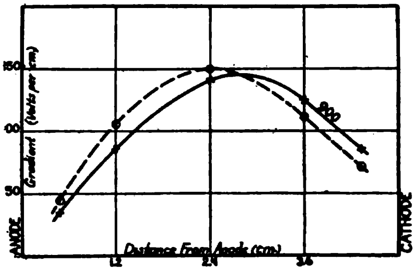

Fig. 5.

The electrometer was insensitive for very large potential differences. It would not measure a gradient larger than 160 or 170 volts, and points on the curve above 150 are not reliable. In view of this fact some observations were made with applied potentials of 450 and 600 volts, using a multicellular electrostatic voltmeter reading to 180 volts in place of the electrometer. When one terminal of this voltmeter was connected to the anode and the other to a single exploring point in the flame a large drop of potential was detected near the anode for the above applied potentials. With 600 volts there was a drop of over 180 volts inside of a millimeter distance from the anode, whereas the electrometer indicated a much smaller gradient. With smaller applied potentials this anode drop became much smaller.

It is evident, therefore, that the midpoints of the curves for 750 and 900 volts cannot be relied upon as being very exact. The electrometer did not give very accurate readings for the high points of the curves. In the curve for 900 volts a large value of the potential gradient must exist either at the electrodes or in the middle region, otherwise the average height of this curve would be less than that of the curve for 750 volts. This, of course, cannot be the case, so that these curves must be taken to represent the general way in which the gradient changes at some distance from the electrodes.

The curves of Figs. 2, 3, 4, and 5 are of interest because they indicate in a general way how the electric charge in the flame varies with applied potentials. The equation

$$\frac{d^2V}{dx^2}+\frac{d^2V}{dy^2}+\frac{d^2V}{dz^2}=-4\pi\rho$$

becomes in this case, approximately

$$\frac{dX}{dx}=4\pi(n_2e_2-n_1e_1),$$

where n_1 and e_1 are the number of positive ions per cubic centimeter and the charge on each, respectively; n_2 and e_2 are the corresponding quantities for the negative ions. From this equation we see that if the slope of the curve is positive there will be a negative charge in the flame. Inspection of the curves shows that for small applied potentials there is a negative charge in the flame, but that for potentials over 300 the charge becomes positive. This fact is in agreement with the idea that a fairly large field will remove the more mobile negative ions at a greater rate than the positive ions. It is also probable that the anode, when raised to a high positive potential, will remove negative ions from the gas streaming up through it.

For very large applied potentials (750 and 900 volts) a negative space charge appears near the anode. The reason for this charge will be clearer after a consideration of Fig. 6.

Fig. 6 shows the relation between the current through the flame and the applied potential, both with and without a magnetic field. The current was measured by the deflection of the galvanometer, which was shunted with a resistance of 20 ohms. The seven points on each curve correspond to the seven values of the applied potential for which Figs. 2, 3, 4, and 5 were obtained. Several other curves of the same type as those of Fig. 6 were obtained, and they all showed the same general characteristics. They resemble very much the ordinary curves obtained with gases at low temperatures when saturation and ionization by collision are obtained. In the case of the flame, however, a true saturation current for the whole region between the electrodes is not obtained when the curve approaches parallelism with the voltage axis. The existence of a larger drop of potential near the anode than in the rest of the flame indicates that the rest of the flame has a relatively large number of negative ions present. Moreover, the appearance of an anode fall of potential will explain the observed facts as follows:

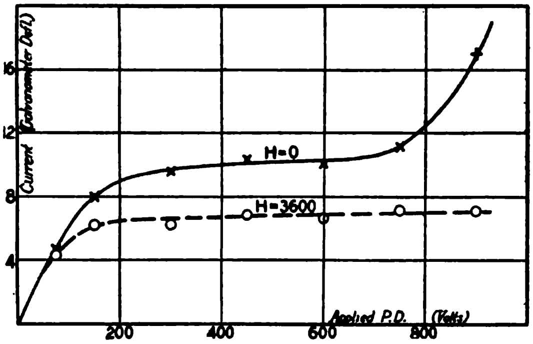

Fig. 6.

The current through the flame has been found[1] to obey the equation

$$V = Acd + Bc^2,$$

where V is the applied potential, c is the current, and d the distance between electrodes. A and B are constants. The term Bc^2 represents the potential drop near the electrodes, and Acd the drop in the intervening space. Under ordinary conditions with a small current there is a very large potential drop at the cathode and a small one at the anode. Putting lime on the cathode destroys the drop there so that Bc^2 becomes very small and the current is approximately proportional to the potential applied. With a larger current, however, the anode potential fall becomes large and Bc^2 is again large. It follows that the current increases slowly with V, as is the experimentally observed fact when V is over 300 or 400 volts.

For values of V greater than 750 volts the current begins to increase rapidly again, provided the magnetic field is zero. When the magnetic

[1] H. A. Wilson, Electrical Properties of Flames.

field acts there is a slower increase of current, not indicated in the curve of Fig. 6. This current increase is probably due to ionization by collision occurring in the regions where the ions have the highest velocity. The anode fall of potential disappears (see Figs. 4 and 5) because of the production of new ions near this region, and the new supply of negative ions being drawn to the anode produces a negative space charge. The appearance of this space charge is indicated by the change in the slope of the potential gradient curve.

From the data used in plotting Fig. 6 the velocity of the negative ions in the flame may be calculated. Townsend has derived[1] an expression for the average velocity, U_H, which an ion will acquire in the direction of an electric force, X, when a magnetic field, H, acts at right angles to X. Starting with the equations of motion

$$m\frac{d^2x}{dt^2} = Xe - He\frac{dy}{dt},$$

$$m\frac{d^2y}{dt^2} = He\frac{dx}{dt},$$

it is found that

$$U_H = \frac{U}{1 + H^2\left(\frac{e}{m}\right)^2 T^2}.$$

Here U is the velocity when the magnetic field is zero, and is equal to $X(e/m)T$, T being the mean free period of the ion. Hence

$$\frac{U}{U_H} = 1 + H^2\left(\frac{e}{m}\right)^2 T^2.$$

Let I and I_H be the currents with zero magnetic field and with field H, respectively. If the current is carried mainly by negative ions we may write

$$\frac{I}{I_H} = \frac{n_2e_2U}{n_2e_2U_H} = 1 + \frac{H^2U^2}{X^2},$$

$$\frac{I - I_H}{I_H} = \frac{H^2U^2}{X^2},$$

$$U = \frac{X}{H}\sqrt{\frac{I - I_H}{I_H}} = \frac{X}{H}\sqrt{\frac{\delta I}{I_H}}.$$

In general, the quantity under the radical depends on the value of X, so that U is not proportional to X in all cases.

In deriving the above relation it has been assumed that the electric field, X, is unchanged by the magnetic field, H. In applying the equation, therefore, X was determined from the curves of Figs. 2, 3, 4, and 5 at the point where the magnetic field did not affect the potential gradient. The magnetic field had a constant value of 3,600 gauss. In the fourth column of the table below are recorded the values of U as obtained in this experiment. In the fifth column X/p is tabulated. X is measured in volts per cm. and p is taken to be 760 mm.

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[1] Townsend, Electricity in Gases.

Applied Potential (Volts).	X (Volts).	$\frac{\delta I}{I_H}$	$U \times 10^{-5}$ (Cm. per Sec.)	$\frac{X}{p}$
75	19	0.09	1.58	0.025
149	31	0.29	4.64	0.04
300	69	0.54	14.1	0.09
450	125	0.52	24.8	0.16
600	168	0.50	33.0	0.22

Values of $\delta I/I_H$ are not calculated for larger values of the applied potential because of the presence of ionization by collision. The data in this table show that the velocity of the negative ion increases approximately in proportion to X/p except for the small values of X/p. If, in the equation $U = kX$, we consider the values of k, it is evident that k is approximately constant when X/p is large but begins to decrease when X/p grows small. It has generally been assumed heretofore that k in a flame has an approximately constant value whatever the magnitude of X/p.

Owing to the difficulty of measuring $\delta I/I_H$ accurately when X was small the velocity could not be determined for smaller values of X/p than 0.025. This limitation is unfortunate because the decrease in U and k cannot be followed out as completely as is desirable. However, from a study of the Hall effect in flames H. A. Wilson[1] has determined the velocity which the negative ion acquires under a gradient of one volt per cm., and the electric field used in his experiments was varied from 5 volts to 30 volts per cm. We are thus afforded a range in X/p of from 0.006 to 0.039. Throughout this range k was found to have a constant value of 2,450 cm. per sec., so that U would be proportional to X/p according to these experiments. The value of U varies from 0.12×10^5 cm. per sec. when $(X/p) = 0.006$, to 0.73×10^5 cm. per sec. when $X/p = 0.039$. There is a somewhat large discrepancy between Wilson's value of 0.47×10^5 when $(X/p) = 0.025$ and the value 1.58×10^5 obtained above, but this may be due to different flame conditions.

The general way, therefore, in which U varies with X/p is as follows:

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[1] PHYS. REV., III., p. 375, 1914.

(1) When X/p is greater than 0.09 U is approximately proportional to X/p, (2) when X/p is between 0.09 and 0.01 (approximately) U diminishes very rapidly with X/p and (3) when X/p is below 0.01 U has become relatively small and diminishes in proportion to X/p.

Townsend[1] has shown that the velocity of the negative ions in dry air at ordinary temperatures changes in very much the same way when X/p is varied. It will therefore be important to consider the effect of a high temperature on the velocity of the ions so that a comparison can be made between Townsend's results and the results obtained for flames.

We have

$$U = X\frac{e}{m}\frac{\lambda}{V},$$

where V and λ are, respectively, the velocity of agitation and the mean free path of the ion in the gas. Introducing the constant of proportionality, A, we may express the variation of λ with the absolute temperature, θ, and the pressure, p, by the equation $\lambda = A\theta/p$. When X/p is very small we also have $mV^2 = \alpha\theta$, where α is a constant, so that for small values of X/p,

$$U = AX\frac{e}{p}\sqrt{\frac{\theta}{\alpha m}}.$$

When X/p is large it has been shown by Townsend that V becomes greater than the ordinary value given by $mV^2 = \alpha\theta$, and he has determined the factor, f, by which V^2 is increased. Hence $mV^2 = f\alpha\theta$, where f is a function of X/p. If the subscript A refers to air at 27° C. and F to the flame which has a temperature of about 2000° C., we have, when X/p is large:

$$U_A = \frac{AXe}{p}\sqrt{\frac{300}{\alpha m_A f_A}},$$

$$U_F = \frac{AXe}{p}\sqrt{\frac{2273}{\alpha m_F f_F}},$$

so that for a given value of X/p

$$\frac{U_A}{U_F} = \frac{k_A}{k_F} = \sqrt{\frac{m_F f_F\, 300}{m_A f_A\, 2{,}273}};$$

where k is defined by the equation $U = kX$. The velocity of agitation in the flame is much greater than in air at ordinary temperatures, so that the effect of the electric field on V would be comparatively unimportant.

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[1] Townsend, Electricity in Gases.

We might, therefore, expect f_F to be approximately equal to 1, when the equation becomes

$$\frac{k_A}{k_F}\sqrt{\frac{2,273 f_A}{300}} = \sqrt{\frac{m_F}{m_A}}.$$

The following table gives some of the data obtained by Lattey[1] and by Townsend and Tizard[2] in their experiments on air at ordinary temperatures, together with the corresponding data secured in the present experiment on flame ions:[3]

Air

x/p	0.04	0.07	0.08	0.2	0.3[4]	0.5
$U_A \times 10^{-5}$	0.001	0.005	0.01	5.0	6.3[4]	9.0
k_A	3.5	9.5	15.6	3,290	2,980[4]	2,370
f_A	—	—	—	2.5	3.5[4]	5.7
$k^A \sqrt{2,273 f_A 300}$	—	—	—	14,400	15,200[4]	15,640

Flame.

X/p	0.007	0.025	0.04	0.09	0.16	0.22
$U_F \times 10^{-5}$	(0.12)	(0.46)	(0.74)			
		1.58	4.64	14.1	24.8	33.0
k_F	(2,450)	(2,450)	(2,450)			
		8,300	15,000	20,400	19,800	19,700

The numbers in parentheses are obtained from Wilson's experiments on the Hall effect.

If m_F is equal to m_A, k_F should be equal to $k_A \sqrt{(2,273 f_A/300)}$. It appears that the maximum values observed for these quantities are 20,400 and 15,640 respectively, which do not differ very much. The

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[1] Proc. Roy. Soc., *A*, 84, p. 173, 1910.

[2] Proc. Roy. Soc., 88, p. 336, 1913.

[3] We may calculate the value of k_A from the relation $k = (e/m)(\lambda/V)$. If the ion is an electron we have $e/m = 1.7 \times 10^7$, $\lambda = 4.2 \times 10^{-5}$ (if the free path of an air molecule at 27° C. is 10.5×10^{-6}), and

$$V = 1.84 \times 10^5 \sqrt{\frac{2 \times 300 \times 1800}{273}} = 1.1 \times 10^7$$

Hence, $k_A = 6,500$ cm. per sec. This agrees very well with the highest observed value of $k_A \sqrt{f_A}$ which is about 5,680. For an electron in the flame k_F may also be calculated. Taking $e/m = 1.7 \times 10^7$, $\lambda = 31.6 \times 10^{-5}$ (four times the free path of a nitrogen molecule at 2000° C.),

$$V = \frac{1.84 \times 10^5 \sqrt{2 \times 2273 \times 1800}}{\sqrt{273}} = 3.2 \times 10^7,$$

we get $k_F = 16,800$ cm. per sec. This result agrees as well as could be expected with the largest value of k_F above.

[4] Obtained by interpolation.

mean free path of an electron in the flame is probably longer than it would be in pure air at the same temperature, a fact which would explain why the maximum value of k_F in the flame is greater than the maximum value of $k_A \sqrt{(2{,}273 f_A/300)}$.

For the smaller values of X/p the values of k_F are much greater than those of $k_A \sqrt{(2{,}273 f_A/300)}$, so that it appears that the negative ions in the flame are electrons for smaller values of X/p than in air. This conclusion was to be expected, because a high velocity of agitation would be unfavorable to the formation of aggregations of molecules about the negative ion.

The way in which the velocity of the ions varies with X/p is brought out very clearly by the curves of Fig. 7, where k_A and $2.75\sqrt{f_A}\cdot k_A$ (dotted) are plotted against X/p for air, and k_F is plotted against X/p for the flame. The name of the quantity plotted along the ordinate for each curve is written on the curve instead of along the axis. If the mean free paths in the flame and in air at the temperature of the flame were equal, and if $f_F = 1$ in the flame, then the curves for $2.75\sqrt{f_A}\, k_A$ and for k_F would coincide if the mass of the air ion and the flame ion were the same.

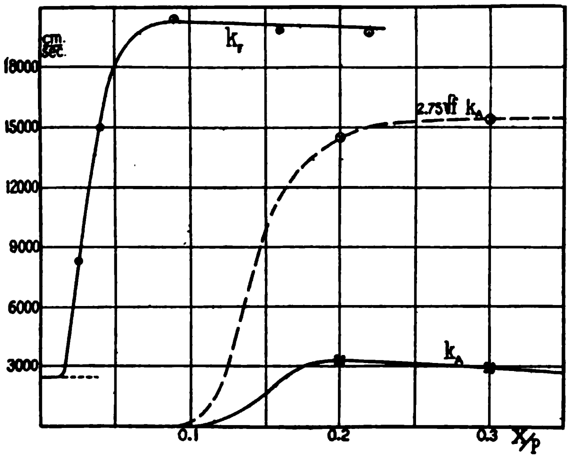

Fig. 7.

The curve for k_A in air brings out[1] the following points: (1) When X/p is less than 0.01, U_A is very small and is proportional to X/p, so that k_A is constant; (2) when X/p is between 0.01 and 0.2 the value of k_A (and therefore of U_A) increases rapidly with X/p; (3) when X/p is greater than 0.2, U_A is comparatively large but does not increase in proportion to X/p, so that k_A in this case decreases as X/p increases.

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[1] The scale on which the curve is plotted is so small that (1) and (2) are not evident. A consideration of the data is necessary (see Townsend, Electricity in Gases).

Townsend explains this variation of velocity as follows: when X/p is less than 0.01 the negative ions consist of aggregations of molecules; as X/p increases a transition stage is passed through, and when the value of 0.2 is reached the ions are free electrons. The electrons do not acquire a velocity proportional to X/p when the latter is large because a large electric field increases the velocity of agitation of the ions, thus causing k_A to decrease.

In the case of the flame it is probable that very much the same phenomena occur as in the gas at ordinary temperatures. However we cannot conclude that for small values of X/p the negative flame ion never exists in an electronic state. The experiments on the Hall effect show that the negative flame ion has a much higher velocity for small values of X/p than has the ion in air. This fact indicates that its average mass is smaller. Also, even when a very small potential difference is applied to electrodes in a flame there is a greater fall of potential at the cathode than at the anode. This cathode fall of potential indicates that the negative ions have a higher velocity than the positive ions and hence a smaller mass. The conclusions to be drawn are as follows:

The electrons in the flame are, under ordinary conditions, in a state of statistical equilibrium with the molecules; that is, each flame ion spends a certain part of its life as a free electron, the rest of its life being spent as a molecular aggregate. When X/p is small this equilibrium condition is not much disturbed. As X/p changes from about 0.01 to 0.09 a transition stage is traversed, when the ion spends more and more of its time as a free electron. For values of X/p greater than 0.09 the ion is a free electron during its whole life. A rough idea of the time that an electron is associated with molecules when X/p is small may be obtained as follows:

Let k be the velocity of the free electron under unit potential gradient and k' the velocity when associated for part of the time with molecules. From the curve of Fig. 7 we then have, approximately, $k'/k = 1/8$. If t is the time during which the electron is free and t' the time it is associated we may write

$$k'(t + t') = kt + k''t'.$$

The last term on the right hand side may be neglected, because k'', the velocity of the ion when loaded with a molecule, is small when compared with k. Hence

$$\frac{k'}{k} = \frac{t}{t + t'} = \frac{1}{8},$$

$$t' = 7t.$$

The electron, therefore, is free about one eighth of the time, even when X/p is very small.

The following is a recapitulation of the main points of the experiment:

1. The potential gradient between vertical electrodes in a flame has been studied in a series of cases, the cathode being covered with a deposit of metallic oxides for the production of electrons.

2. The current-voltage curve indicates an apparent saturation current. This, however, is mainly due to the appearance of an anode fall of potential. Ionization by collision occurs for large values of the electric field.

3. By measuring the effect of a magnetic field on the conductivity of the flame the velocity of the negative ions has been determined, and k, the velocity under unit potential gradient, is found to have a maximum value of 20,400 cm. per sec.

4. The value of k for a flame is found to vary with X/p in much the same way as does k for air at ordinary temperatures. The conclusion is drawn that when X/p is above 0.09 the flame ions are free electrons, when X/p varies from 0.09 to 0.01 (approximately) the free electrons begin to associate with gas molecules, and when X/p is below 0.01 the electrons are free only about one eighth of the time. This variation of k with X/p probably explains the lack of agreement between the values of k as determined by different experimenters.

The writer is indebted to Professor H. A. Wilson for valuable suggestions and much helpful advice regarding the working out of the experiment.

The Rice Institute,
Houston, Texas.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE NEW YORK MEETING, FEBRUARY 26, 1916.

THE Society held its eighty-second meeting in Fayerweather Hall, Columbia University, New York City, at 10:15 a.m., Vice-President Bumstead presiding. In the absence of the secretary, George B. Pegram was elected secretary pro tem.

The following program of papers was presented:

Morning Session.

On Possible Planetary Magnetic Effects. (Illustrated.) L. A. BAUER.

Note on the End Effect in the Electrostriction of Cylindrical Condensers. EDWIN C. KEMBLE.

A New Method and Apparatus for Enlarging Photographs without the Aid of a Lens. ALFRED J. LOTKA.

The X-Ray Spectrum of Tungsten. ARTHUR H. COMPTON.

A New Mercury Vapor Pump. HORATIO B. WILLIAMS.

Distribution of Velocities of the Electrons in Vacuum Tubes. K. T. COMPTON.

Reversed Sounds. JOHN B. TAYLOR.

Theory of Ionization of Gases by Elastic and Inelastic Impact. K. T. COMPTON.

Fundamental Ideas in Photography. F. M. STEADMAN.

Osmotic Pressure and Temperature. W. F. MAGIE.

Afternoon Session.

Behavior of a Stable Aeroplane in Longitudinal Gusts. E. B. WILSON.

Some Afterglow Phenomena in Nitrogen and Argon. IRVING LANGMUIR AND C. U. FERGUSON.

Rate of Evaporation of Small Spheres in Air. IRVING LANGMUIR.

Theory of Crystal Structure. A. C. CREHORE.

The question of endorsing a bill now before Congress proposing to make the Centigrade thermometer scale the official scale in the publications of the various government bureaus and to abolish by 1920 the use of the Fahrenheit scale in government publications, was discussed at some length and it was finally decided to postpone action upon it until the Washington meeting in April.

G. B. Pegram, *Secretary pro tem.*,
A. D. Cole, *Secretary.*

Minutes of the Washington Meeting, April 21 and 22, 1916.

The Eighty-third Regular Meeting of the Physical Society was held at the National Bureau of Standards, Washington, April 21 and 22, 1916. The following papers were presented:

Friday, April 21, 9:30 A.M.

Convection Currents and Calorimeter Design. Walter P. White.

The Application of the Moving Coil Galvanometer to the Measurement of Time. Paul E. Klopsteg. Read by A. Zeleny.

On the Influences Contributing to the Variation of Contact Electromotive Force with Time. Otto Stuhlmann, Jr.

The E. M. F. Generated by the Rotation of a Cylindrical Magnet About its Axis. G. F. Hull and A. B. Meservy.

A High Vacuum Mercury Vapor Pump of Extreme Speed. Irving Langmuir.

The Effect of a Magnetic Field Upon Solid Solutions of Iron and Carbon. F. L. Bishop. (By title.)

The Combination of a Series and a Parallel Resistance for Reducing the Voltage Sensitivity of a Galvanometer. F. Wenner.

An Alternating Current Method for the Measurement of Mutual Inductance. Harvey L. Curtis.

The Resistance of Thin Films Produced by Evaporation. S. Dushman.

On a Chemically Active Modification of Hydrogen Produced by Alpha-Rays. William Duane and G. L. Wendt.

The Volume Effect in the Silver Voltameter. E. B. Rosa and G. W. Vinal.

The Law of Absorption of X-Rays at High Frequencies. Albert W. Hull.

Stoletow's Constant: Its Theoretical Derivation and Application to Measurements of Electronic Free Paths. K. T. Compton. (By title.)

The Calculation of Planck's Constant C_2. J. H. Dellinger.

Note on the Nature of the Absorbers of Characteristic X-Rays. R. A. Millikan and E. C. Watson.

The Low Potential Discharge Spectrum of Mercury Vapor in Relation to Ionization Potentials. John T. Tate.

2.30 P.M.

Photo-currents at Different Wave-lengths as a Function of Incident Energy. Wilmer Souder.

On the Conduction of Electricity through an Ionized Gas, more particularly in its Relation to Bronson Resistances. W. F. G. Swann and S. J. Mauchly.

An Experimental Determination of the Mean Free Paths of Electrons in Rarified Gases. K. T. COMPTON AND J. M. BENADE. (By title.)

Some Properties of Mercury Spheres Important in Electronic Charge Measurements. L. W. MCKEEHAN.

On the Mobilities of Gas Ions in High Electric Fields. L. B. LOEB.

Wave-length Energy Distribution in the Continuous X-Ray Spectrum. BERGEN DAVIS.

On Spectroscopic Resolving Power. C. M. SPARROW.

The Planetary System of Convection as Observed by Means of Sounding Balloons. WM. R. BLAIR.

The Vertical Distribution of Air Temperature and of Air Pressure in Middle Latitudes. WM. R. BLAIR.

Saturday, April 22, 9:30 A.M.

The Constants of Radiation of a Black Body. W. W. COBLENTZ.

On the Use of the Thermopile in Photographic Photometry, with Special Reference to Spectroscopy and the Determination of Stellar Magnitudes. H. T. STETSON.

The Light Absorption Coefficient of Solutions. E. O. HULBURT, J. F. HUTCHINSON AND H. C. JONES.

The Candle-power of the Black Body and the Mechanical Equivalent of Light. E. P. HYDE, F. E. CADY AND W. E. FORSYTHE.

The Under-water Spark as an Ultraviolet Source. H. E. HOWE.

The Rotating Sector in Photographic Photometry. H. E. HOWE.

The Law of Reflection of Gas Molecules from Liquid and Solid Surfaces. R. A. MILLIKAN.

The Efficiency of Thermostats. WALTER P. WHITE.

Rates of Evaporation and Vapor Pressures of Iron, Nickel, Silver and Copper. IRVING LANGMUIR AND G. M. J. MACKAY.

A Synchrono-phosphoroscope. EDWARD L. NICHOLS AND H. L. HOWES. (By title.)

Note on the Action of Calcium in a Discharge Tube. HAROLD B. C. ALLISON.

A Misconception of the Criterion for Gray-body Radiation. PAUL D. FOOTE AND C. O. FAIRCHILD.

The Thermal Expansion of Tungsten at Incandescent Temperatures. A. G. WORTHING.

American Nickel Steels of Low Thermal Expansivity. A. W. GRAY, D. H. SWEET AND L. W. SCHAD.

Some Effects of Magnetization on the Length of a 36 Per Cent. Nickel Steel. A. W. GRAY, D. H. SWEET AND L. W. SCHAD.

The Intensity of X-Ray Spectra, and the Distribution of the Electrons in Atoms. ARTHUR H. COMPTON.

Experiments on the Joule-Thomson Effect in Air at Moderate Temperatures and Pressures. L. G. HOXTON.

An Automatic Pressure Regulator. L. G. HOXTON.

An Investigation of the Laws of Plastic Flow. E. C. BINGHAM.

Dynamics of Electron Fields. G. B. PEGRAM.

Note on a Method of Measuring Thermocouple E.M.F.'s. E. F. NORTHRUP.

During the Saturday session several items of business were presented by the Council to the Society. First they presented a report from a committee on the extension of the influence of the Society among teachers of physics, consisting of G. W. Stewart, A. G. Webster and W. S. Franklin, and moved the adoption of the following recommendations of this report:

"1. That the Council be instructed to formulate and to recommend such amendments to the By-laws as are necessary to establish a *student membership.*"

"2. That the Council be authorized to establish, if feasible, a special PHYSICAL REVIEW subscription rate for individuals who are members of societies that are interested in the teaching of physics, the subscriptions being made through the secretaries of such organizations."

"3. That the Council be authorized to appoint a representative of the Society who shall prepare for each issue of School Science and Mathematics a record of some of the most interesting recent achievements in physics, to make such an appointment annually and to discontinue such representation at its own discretion."—Motion carried.

The second item recommended by the council for adoption by the Society was the following:

"The American Physical Society shares the desire of scientific men in general for international and world-wide uniformity in units of measurement of all kinds, and with this object in view it favors the introduction of the Centigrade scale of temperature, as a standard in the publications of the United States government.

"It must be recognized that considerable initial expense must be incurred by the U. S. Weather Bureau in changing its apparatus to conform to the proposed act. Furthermore, on account of the more open scale of the Centigrade system that Bureau will be subject to a continued increased cost of publication, owing to the necessity of printing the first decimal place in order to maintain the present accuracy. The use of negative temperatures and minus signs entails greater liability to errors, and more clerical labor would be required in checking the accuracy of the reports of coöperative observers of the Weather Bureau, and in computing monthly and other mean temperatures.

"Notwithstanding the foregoing, the American Physical Society is in favor of legislation to make the Centigrade scale of temperatures the standard in publications of the United States government, and funds should be made available by Congress to accomplish the desired result.

"The Society favors Bill H. R. 528, 'To Discontinue the Use of the Fahrenheit Thermometer Scale in Government Publications,' but recommends that it be amended by the addition of the following:

"*Sec. 4.* When in the publication of tables containing several meteorological and climatic elements, the use of data in Centigrade temperatures leads to manifest incongruities, the chief of the Weather Bureau is directed to publish related data in such units as are necessary to make the tables homogeneous and to secure international uniformity as far as practicable.

"*Sec. 5.* Nothing in this act shall prevent the use of the absolute Centigrade scale of temperature in publications of the government."

The motion was adopted without a dissenting vote.

The Committee on Technical Physics (F. K. Richtmyer, Chairman) also made an extended report of progress, neostyle copies of which were distributed. This report is unfortunately too long to be included here; it suggested seven lines of activity in which the Society might engage with the object of increasing the usefulness to the physicists of the country of the physical investigation now being conducted in the laboratories of industrial organizations. Seven further suggestions were made of ways in which individual members might help to secure better coöperation between university and industrial laboratories.

A vote of thanks was extended to the Washington members of the Society and to the Bureau of Standards for numerous courtesies extended to visiting members. It was announced that the meeting of the Society next October will probably be held at the Nela Research Laboratory of the General Electric Company, Cleveland, O.

A considerable number of members of the Physical Society took advantage of the special invitations extended to them to attend the open sessions of the National Academy of Sciences, April 17, 18, and 19, and the address before the Philosophical Society of Washington by Professor R. A. Millikan the evening of April 20.

On both days of the meeting visiting members of the Society were guests of the Washington members of the Society at luncheon at the Bureau of Standards. On Friday evening a subscription dinner at the Cosmos Club was participated in by about fifty. The attendance at each of the three sessions for the reading of papers was large—probably 125 to 150 at each session.

A. D. COLE,
Secretary.

In accordance with a vote of the Council of the American Physical Society, the ARTICLES OF INCORPORATION of the Society, adopted Dec. 30, 1914, are here recorded:

Articles of Incorporation of The American Physical Society.

Know All Men by These Presents, that we, all citizens of the United States, whose names are hereto undersigned, a majority of whom are residents of the District of Columbia, being desirous to form a society and body corporate under the laws of the said District, and in particular under sub-chapter III of Chapter XVIII of the Code of Law in force in the said District, do hereby declare and certify as follows, that is to say:

First. The name of the Society hereby formed shall be The American Physical Society.

Second. The term of existence of the said society shall be perpetual.

Third. The said society shall have no capital stock.

Fourth. The object and purpose of the said society is, and shall be, to promote the advancement and diffusion of the knowledge of physics, to publish a periodical and other publications for that purpose, and to do such other things as may be conducive to the said purpose.

Fifth. The business affairs and concerns of the said society shall be, during the first year of the existence, managed and directed by a board of trustees to be called the council and consisting of twelve persons, the first members of which council, who shall act as such until the membership may be changed by an election held in accordance with the constitution and by-laws of the society, shall be:

Ernest Merritt, Ithaca, New York;
Karl E. Guthe, Ann Arbor, Michigan;
Alfred D. Cole, Columbus, Ohio;
Joseph S. Ames, Baltimore, Maryland;
Robert A. Millikan, Chicago, Illinois;
John Zeleny, Minneapolis, Minnesota;
Henry A. Bumstead, New Haven, Connecticut;
Theodore Lyman, Cambridge, Massachusetts;
Howard T. Barnes, Montreal, Canada;
William J. Humphreys, Washington, D. C.;
Dayton C. Miller, Cleveland, Ohio;
George K. Burgess, Washington, D. C.

The said society reserves the right and power to amend, alter, replace and add to any and all of the provisions contained in this certificate in such manner and to such extent as may be authorized by law.

In witness whereof, we have affixed our signatures and our seals this the fourth day of November in the year 1914.

Arthur L. Day, (seal)
W. J. Humphreys, (seal)
S. W. Stratton, (seal)
George K. Burgess, (seal)

(Sworn statements before Notary Public and Deputy and Acting Recorder of Deeds accompany the original.)

CONVECTION CURRENTS AND CALORIMETER DESIGN.[1]

BY WALTER P. WHITE.

IN calorimeters surrounded by air the cooling factor ("thermal leakage" factor) varies with the temperature difference between the calorimeter and the inclosing jacket. To avoid this difficulty it is necessary either to work with small differences or else to make certain corrections which, while not especially troublesome, are preferably to be avoided. The variation can be made negligible by a suitable proportioning of the calorimeter and its surrounding jacket. The heat transfer by conduction through the air and by radiation is, under ordinary conditions, practically proportional to temperature difference; these sources of thermal leakage contribute practically nothing to the variation in the leakage factor. The whole trouble comes from the convection currents, which, in thin air layers, transfer heat at a rate which is practically proportional to the square of the temperature difference. By making the space between calorimeter and jacket narrow enough the effect of these currents can be made negligible and the desired constancy in the thermal leakage factor can be sufficiently approximated. This construction, however, while diminishing the convection currents, increases conduction, and therefore should not be carried too far.

In order to get data for determining the optimum distance between calorimeter and jacket the convection through vertical and horizontal spaces was investigated. The apparatus consisted of two silvered plates of copper placed back to back within a rectangular copper box. The box was heated in an oil bath and the rise in temperature of the copper plates served as a measure of the heat flowing to them through the air spaces from the walls of the box. The temperatures were measured by thermoelements soldered to the copper surfaces. 0.1 microvolt, the smallest quantity read, corresponded to 0.0025°, and the final agreement was frequently, though not always, as good as this. Conduction and radiation were determined for a small temperature difference where convection was practically absent, and the heat flow due to these was then subtracted to get the convection in the determinations where the temperature difference was greater. The advantages of this method are: its simplicity and relative rapidity, the ability to work accurately for very small temperature differences, and the ability to work over a rather wide temperature range. The absence of guard rings is perhaps an advantage, since conduction and radiation entered only as a correction term, and the convection currents deliver their heat at the very edge of the plate.

Vertical flow was mainly studied. For distances profitable in calorimetry stream line flow prevails. The fundamental fact, therefore, about the convection currents is that, flowing parallel to the solid surfaces, they carry no heat to and from these except when they first strike or when they leave. It follows that for surfaces not too low (1) convective transfer is independent of height, that is, the transfer per square cm. is inversely proportional to height; (2) the stream velocity is proportional to temperature difference; (3) the velocity for infinite plates is theoretically proportional to the third power of the distance separating them. Practically, owing to end effects, etc., it proved more nearly proportional to the fourth power, and its behavior in this regard would depend on the special conditions of the particular apparatus used. The variation from Newton's law, therefore, is inversely proportional to height, proportional to temperature difference and approximately proportional to the fourth power of the width of the air gap.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21, 1916.

The convection between horizontal surfaces is not so simple, but its magnitude was found to be for a diameter of 8 cm. approximately double that between the same surfaces put vertically. Since convection takes place from but one of the horizontal surfaces of the calorimeter at a time, these horizontal surfaces may for all practical purposes be calculated as if they were vertical and 8 cm. high.

On account of the rapid increase of convection with distance between the plates the profitable limits for that distance are narrow. As far as the lateral surface is concerned, for a calorimeter 16 cm. high and an air gap of 12 mm. the variation from constancy in the thermal leakage factor is approximately 6 per cent. when the temperature interval is raised to 10°, and this variation would be doubled by a further increase of 3 mm., which would only decrease the total cooling rate about 20 per cent.

With a very large calorimeter, however, in which the temperature difference is likely to be considerably less, the air gap may often profitably be increased in proportion to the other linear dimensions, and such an increase in the air gap makes the relative performance of a large calorimeter far more advantageous than it has generally been considered to be, since the discussion of such performance has usually and unconsciously assumed an invariable width of gap. It further follows that the lag effect due to the convection currents, discussed by several writers on calorimetry, is altogether negligible. This conclusion confirms, but also extends, one announced several years ago. The lag effect due to the air itself can be calculated from the specific heat and thermal conductivity. I must also retract a statement I once made to the effect that there is no disadvantage in having a calorimeter chamber much larger than the calorimeter. The disadvantage, however, since it concerns only the variation from Newton's law, is not serious.

Geophysical Laboratory,
Carnegie Institution of Washington,
Washington, D. C.
March 28, 1916.

SOME EFFECTS OF MAGNETIZATION ON THE LENGTH OF A 35.25 PER CENT. NICKEL STEEL.[1]

BY A. W. GRAY, D. H. SWEET, AND L. W. SCHAD.

WHILE investigating the thermal expansivity of a 35.25 per cent. nickel steel, interesting and valuable results were obtained by studying the effects of magnetic treatments at temperatures ranging from – 75° C. to over 300° C. Combined thermal and magnetic treatment was made to effect a permanent increase in the length at room temperatures that amounted to more than 55 microns per meter. This length change is in the same direction as that produced by Guillaume's aging process but is far more rapid. Guillaume found a hot-forged bar left to itself at room temperature to elongate 38 microns per meter in the course of 12.4 years. Artificial aging by gradual cooling from 150° C. for nearly four months produced in a similar bar an elongation of 47 microns per meter, which increased to 53 microns per meter after remaining at room temperature 4.6 years longer. The greatest aging effect recorded by Guillaume seems to be 66 microns per meter, which was produced by the above heat treatment followed by over thirteen years rest at room temperatures. Since considerable elongation had already taken place before we began to observe the 55 microns mentioned above, it would appear that our combined thermal and magnetic treatment is more rapid and more effective than the aging process of Guillaume. In addition to producing this elongation at room temperatures, the magnetic treatment raises the expansion curve as a whole at all temperatures below that at which ferromagnetic properties disappear on heating. Also, the length assumed at any temperature immediately after magnetic treatment is very nearly independent of whether this temperature is reached by warming or by cooling.

Measurements of linear magnetostriction at temperatures distributed between – 75° C. and 300° C. have confirmed and extended the observations of Nagaoka, Honda, and Shimizu, which appear to be the only magnetostriction measurements so far made on nickel steels. Complete magnetostriction cycles showing hysteresis, similar to those first observed by Nagaoka in the case of iron but hitherto not investigated in the case of nickel steels, have been taken at a series of temperatures up to that at which the magnetostriction vanishes, and magnetic elongation exceeding 45 microns per meter have been measured. This appears to be by far the largest longitudinal magnetostriction effect on record for any substance, exceeding by 45 per cent. the greatest noted by Honda and Shimizu with a nickel steel cooled in liquid air, and by 18 per cent. the maximum contraction recorded by Nagaoka and Honda for nickel.

BUREAU OF STANDARDS,
WASHINGTON, D. C.
April 5, 1916.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21, 1916.

AMERICAN NICKEL STEELS OF LOW THERMAL EXPANSIVITY.[1]

By A. W. Gray, D. H. Sweet, and L. W. Schad.

IN order to assist American manufacturers in producing nickel steels of specified thermal expansivity combined with other desirable properties, the Bureau of Standards has coöperated with the Midvale Steel Company in an investigation of alloys containing about 36 per cent. nickel. The expansivity has been studied at many temperatures extending from about − 150° C. to about 665° C., and data have been obtained that show effects of mechanical, thermal, and magnetic treatments, as well as of slight variations in the constituents. The investigation has already resulted in the production of specimens which, as far as small expansivity is concerned, compare favorably with good French invar; but sufficient time has not yet elapsed to show how stable they can be made as regards length.

In conformity with the experience of Guillaume it has been found that cold-drawing and quenching lower somewhat the expansivity at room temperatures, while annealing raises it. Also, the curve representing the expansion on heating was found to lie above that representing the contraction on return to room temperature, except in the case of a cold-drawn specimen, in which the relation was reversed. At any temperature, however, above that at which the alloy assumes ferromagnetic properties on cooling, the length was always found to be the same whether the temperature in question was reached by warming or by cooling. At very low temperatures the expansion curve rises slowly with slight convexity upwards. At ordinary temperatures this convexity flattens out as the curve passes through a region of inflection that has a very gentle slope. In the neighborhood of 200° C. the direction of curvature is reversed, giving place to a strong concavity upwards as ferromagnetism rapidly disappears. At higher temperatures the concavity is slight but the slope is steep.

Bureau of Standards,
Washington, D. C.
April 5, 1916.

THE E.M.F. GENERATED BY THE ROTATION OF A CYLINDRICAL MAGNET ABOUT ITS AXIS.[1]

By G. F. Hull and A. B. Meservey.

AN electromagnet with poles in the form of hemispherical shells is made to rotate about its axis of symmetry. A ring of brass or iron is held at rest between the poles. Brushes rotating with the magnetic shells make contact with the inner and outer edges of the ring and are connected to insulated metal rings on the axis. When the magnet rotates, an E.M.F. is picked up by sliding contact on these metal rings. The amount is equal to that which is computed from the product Nn when N is the total magnetic flux through the fixed ring and n is the number of revolutions per second. If we adhere to the idea of magnetic lines of force, quantitatively the result is in agreement with the view that the lines of force rotate with the magnet and sweeping across a fixed conductor produce in it an E.M.F. equal to Hlv. The experiment is being continued.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21, 1916.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21, 1916.

DARTMOUTH COLLEGE,
HANOVER, N. H.

THE LOW POTENTIAL DISCHARGE SPECTRUM OF MERCURY VAPOR IN RELATION TO IONIZATION POTENTIALS.[1]

BY JOHN T. TATE.

IT has been shown by Franck and Hertz that collisions between electrons accelerated in an electric field and atoms of mercury vapor are elastic for velocities of the electrons less than 4.9 volts. At this velocity the electrons are capable of giving up all their energy at collision. They were also able to show that part, at least, of this energy was emitted by the vapor as monochromatic radiation of wave-length 253.67 $\mu\mu$—the frequency to be expected if we put

$$h\nu = eV.$$

Incidentally the writer has repeated the measurements of Franck and Hertz very carefully and has obtained a value of 4.90 ± .03 volts for the critical velocity. Indications of secondary maxima in the current-potential curves were found showing the possibility of other types of collision at higher potentials.

Franck and Hertz interpret their results to mean that an atom of mercury vapor is ionized by collision with an electron possessing a velocity of 4.9 volts. It should be noted however that their experimental evidence does not definitely prove this. All that is clearly demonstrated is that the collisions become inelastic at that point. It is readily conceivable, for instance, that the energy lost by the colliding electron merely goes over into energy of agitation of the electrons bound in the atom and not necessarily into completely separating one or more electrons from the atom.

However that may be, it is certain that a secondary critical velocity of the electrons is to be expected at which the energy they give up is emitted as radiation of many frequencies corresponding to the many-lined spectrum of mercury. The present paper deals with an attempt to fix the value of this secondary critical velocity and to demonstrate whether or not ionization takes place at this point. McLennan and Henderson, investigating the character of the single- and many-lined spectra of various elements state that the many-lined spectrum of mercury appears when the applied potential is greater than 12.5 volts. This value however can only be a rough approximation.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21, 1916.

The apparatus used in this investigation consisted of a discharge tube into which a mercury barometer column projected giving a continuous supply of mercury vapor when heated by a hot cathode situated about 2 cm. above the surface of the mercury. A platinum net screening the mercury surface served as anode. To further concentrate the field a second platinum net was introduced between hot cathode and anode. The space between nets was focused on the slit of a quartz spectrograph in such a way that the lines of the spectrum were limited in length by the nets. The initial velocity of the electrons was obtained in the usual way.

It was found that with velocities corresponding to total effective potentials of more than about 5 volts and less than 10 volts the spectrum consisted of a single line of wave-length 253.67 $\mu\mu$. At about 10.3 volts the many-lined spectrum of mercury began to appear in the region of the anode and with increasing potentials the lines gradually increased in length. To determine this critical potential more accurately, and at the same time to demonstrate whether or not ionization took place there, current-potential curves were taken. A very sharp break in the curves was found at 10.0 volts. No break in the curves was detected in the region of 4.9 volts.

The results of the investigation show (1) that a marked ionization occurs in mercury vapor when the velocity of the colliding electrons reaches a critical value of 10.0 volts (possible error about .3 volt). This is very nearly the value (10.2 volts) to be expected on the basis of Bohr's theory of the atom as McLennan has pointed out. (2) That the energy lost by the electrons at these collisions is radiated out as the many-lined spectrum of mercury. (3) That although ionization of mercury vapor at 4.9 volts is not definitely disproved it is certainly much less complete than the ionization taking place at 10.0 volts.

The Brace Laboratory of Physics,
The University of Nebraska.

The Calculation of Planck's Constant C_2.[1]

By J. H. Dellinger.

THIS important constant can be calculated directly and simply from the observed ratio of the energies at any two wave-lengths and temperatures. A rigorous solution of Planck's radiation equation is readily obtained, and the following approximate expression is sufficiently exact for most cases.

$$C_2 = \frac{\lambda_1\lambda_2\theta}{\lambda_2 - \lambda_1}\left[\log\frac{J_2}{J_1} + 5\log\frac{\lambda_2}{\lambda_1} - e^{-C_2/\lambda_2\theta}\right]$$

An approximate value of C_2 always suffices for the last term. The various other relations which have been used for obtaining C_2 from energy data are deducible from the general relation as special cases. The equation for isothermal curves is of particular interest. This general method of solution is very much superior to the method of equal ordinates. No curve has to be drawn, and the calculations are not limited to particular pairs of points. The method is more powerful in determining whether an observed curve fits the Planck equation. In fact, curves which give normal values for C_2 by the method of equal ordinates are found to give very high values when calculations are made by this method for two points both on the same side of the maximum.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21, 1916.

Bureau of Standards,
Washington.

Note on the Action of Calcium in a Discharge Tube.[1]

By H. B. C. Allison.

During the course of some experiments on gaseous conduction, a numbef of Plücker tubes containing various gases were made. The action or one of these tubes was considered worthy of note.

The tube was of the usual type, having a capillary connecting the two electrode chambers, and was about eight inches in length. The electrodes were aluminum rods mounted on platinum wires. In one of the electrode chambers some calcium chips were placed before the electrode was sealed into the tube.

The usual lamp exhaust was given, and the tube filled with argon of at least 99.8 per cent. purity. After washing out the tube twice it was sealed off at 2 mm. pressure and connected with a high voltage transformer. A spectroscope having been put in position, the spectrum was observed to be a combination of argon lines and nitrogen bands, with four prominent hydrogen lines.

As the tube continued to run the nitrogen disappeared first, and then the hydrogen, until only the pure argon spectrum remained. During this time the calcium chips were in the path of the discharge, but not in contact with the electrode.

The tube was then inverted, and shortly the hydrogen lines appeared as brilliantly as at first. The calcium was then in contact with the electrode. Upon returning to the original position the clean up of the hydrogen again took place. It was possible to repeat this reversal a large number of times, but as long as any calcium remained in contact with the electrode, hydrogen was always present in the spectrum.

This is undoubtedly due to the formation of calcium hydride in the first position, and its subsequent decomposition when in contact with the electrode.

Research Laboratory,
General Electric Co.,
Schenectady, N. Y.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21, 1916.

A CHEMICALLY ACTIVE MODIFICATION OF HYDROGEN PRODUCED BY ALPHA-RAYS.[1]

By William Duane and G. L. Wendt.

IT is well known that the rays from radioactive substances produce chemical reactions. They decompose water, whether the water is in the solid, liquid or gaseous phase; they generate ozone when they pass through oxygen, etc.

We performed the experiments described in this paper for the purpose of determining whether or not the rays cause a chemical change in pure hydrogen.

The hydrogen was prepared by the electrolysis of a ten per cent. solution of redistilled hydrochloric acid contained in a sealed two-liter Wolff bottle. A sheet of platinized platinum formed the cathode and bars of purest commercial zinc immersed in redistilled mercury, the anode. Such a generator furnishes its own current. It evolves hydrogen of exceptional purity, since the chlorine is absorbed by the zinc amalgam to form zinc chloride and never forms the gaseous phase. The hydrogen was purified by passing it through three Emmerling towers filled with lumps of potassium hydroxide which had been fused in an iron dish with potassium permanganate to remove organic material. Thence it flowed through a hard glass tube filled with platinized asbestos and kept at a red heat. Passing then through three additional KOH towers it was finally dried in a long tube of phosphorus pentaoxide. This process completely freed it from acid spray and vapors, from CO_2, Cl_2, from oxygen and from water. Any arsine which may have been formed from an impurity of arsenic in the zinc of the generator would have been decomposed by the red-hot tube, but long experience with this type of hydrogen generator has shown that no arsine is developed even with very impure zinc. The hydrogen could have been contaminated then only with nitrogen and the rare gases coming from air dissolved in the acid of the generator. These were obviated by exhausting the whole system by means of a water aspirator to very close to the vapor pressure of the solution. Hydrogen was then generated until the pressure reached atmospheric value. The exhaustion was then repeated three times. Following this treatment the entire system was swept out by its own hydrogen for 40 hours before any hydrogen was used. Contamination from dissolved air at the end of this time was out of the question. No impurities from stop-cock grease were possible because only mercury-sealed U-tubes were used in place of stop-cocks and all stoppers were sealed with mercury.

When a five c.c. bulb of this hydrogen was radiated from the interior with the alpha radiation from 35 millicuries of radium emanation in an alpha-ray tube a decrease in volume resulted. We measured the decrease by the change in level of the mercury in two branches of a U-tube, one connected to the radiated bulb, the other to a similar bulb filled with the same hydrogen and mounted with the first bulb in a water-bath to equalize their temperatures. We must use caution in interpreting this change of volume. It may be due to an agglomeration of hydrogen atoms and molecules, and it may also be the result of some effect on the surface of the mercury.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21, 1916.

To investigate the chemical properties of the gas a stream of hydrogen was passed through a somewhat larger bulb and during its passage through the bulb was exposed as before to the radiations from radium emanation. From this ionization chamber the hydrogen was passed through a glass tube containing flowers of sulfur and thence over a strip of filter paper moistened with acetate of lead. In the course of five or six hours the paper gave a decided record of the formation of hydrogen sulfide, though in quantities less than a milligram. When the emanation was withdrawn the stream of hydrogen passed for days over the sulfur without producing the slightest trace of sulfide. This form of hydrogen, then, is much more active chemically than is the ordinary form. Furthermore it is unstable, as would be expected, because the amount of sulfide detected could be varied at will by regulating the speed of the stream of hydrogen. Reducing to one half the time elapsing between the exposure to the rays and the contact with the sulfur considerably increased the amount of H_2S formed. If more than a minute was allowed to elapse between the formation of the new gas and its action on the sulfur very little sulfide could be detected.

Before the gas reached the sulfur it passed through a plug of clean glass wool. This would indicate that the chemical effect is not due to charged ions that had not yet recombined. To make certain that ions are not concerned, the hydrogen after exposure to the ionizing rays was passed through electric fields up to of 2,000 volts per centimeter in such a way that the gas remained in each field for at least half a minute. This effectually removed all ions, since the mobility of the hydrogen ion at atmospheric pressure is 9 cm. per sec. per volt per cm. The sulfide test was unaffected by this treatment; and the activity is therefore due to electrically neutral molecules.

To dispose further of the objection that the chemical effect may be due to an impurity in the hydrogen which becomes active under the influence of the alpha rays—the rays effect the activation in any case—hydrogen prepared by the electrolysis of a weak solution of potassium hydroxide containing barium hydroxide between platinum electrodes was used, as well as hydrogen bought in the market as electrolytic. Both were purified as was the first hydrogen. Both gave exactly the same effects. In many experiments the hydrogen was passed through a long spiral immersed in liquid air before entering the ionizing bulb, and this also had no effect in modifying the observations.

The reaction with sulfur is not the only one possible, although it was used preferably as a test because the reaction with lead acetate is so singly a test for sulfide. If any oxygen, for instance, had passed with the hydrogen and had become ozonized it would have attacked the sulfur to form oxides, but these when they reached the lead acetate paper would in every case have turned it white, whether through the formation of sulphites or sulfates. There is no misinterpreting the black color produced with the lead salt.

The active hydrogen also attacks phosphorus, as evinced by the blackening of silver nitrate paper by the phosphine formed. Similarly arsine is formed by its action on powdered gray arsenic, the test here being the Gutzeit test, blackening of mercuric chloride paper. It was thought that perhaps the unknown hydride of bismuth could be prepared in the dry way by this hydrogen, but no evidence for it was obtained, using Marsh's test by decomposition in a warmed glass tube. It is conceivable but hardly probable that BiH_3 was formed but not decomposed at the temperature used in the test. When passed over the surface of pure mercury the active hydrogen forms a lustrous yellow scum which may be the hydride, though not enough was obtained to make an analysis.

Neutral potassium permanganate solution is at once reduced by this new hydrogen to manganese dioxide. It was not possible, however, to reduce the organic dyestuffs indigo carmine or methyl violet by bubbling the gas through their water solutions.

Passing the hydrogen through a spiral immersed in liquid air after the exposure to ionization but before contact with sulfur destroyed its ability to attack the sulfur, although exposure to liquid air before ionization in no way affected the test. It may be that the active gas is not truly liquefied but only adsorbed by the glass walls at the low temperature. Experiments to test the effect of red heat on the gas were not successful. The ionized hydrogen was passed over a spiral of fine platinum wire wound on a quartz rod and heated to about 600°, then through a cooling coil and over the sulfur, but it has not yet been possible to heat the gas to 600° and cool it down again within a minute to a temperature at which ordinary hydrogen does not attack sulfur. Even at 200° ordinary hydrogen attacks sulfur appreciably.

The atomic mechanism of this phenomenon is not yet clear. Atoms of hydrogen may be fired into other atoms or molecules by the direct bombardment of the alpha particle or the new form may be the result of the breaking down of a large aggregate of molecules which had collected on a charged molecular fragment. It is conceivable that such an aggregate, formed by the ionization, while disintegrating may become chemically more active than in the normal state. No evidence is at hand to decide this point. An attempt was made to place so high an electrostatic field across the ionizing field that ions would be removed at once without having opportunity to produce large aggregates A field of 2,000 volts per cm. however, placed directly across the region of greatest ionization had no appreciable effect. This may indicate that the primary bombardment is responsible for the production of the active state.

Whether or not the rays produce the compound H_3 remains undetermined.

HARVARD UNIVERSITY.

AN ALTERNATING CURRENT METHOD OF MEASURING MUTUAL INDUCTANCE.[1]

BY HARVEY L. CURTIS.

THE mutual inductance between two coils is defined as the electromotive force induced in one when a current in the other changes at the rate of one ampere per second. If an alternating current $i_1 = I_1 \sin \omega t$ flows in one coil then the induced electromotive force e_2 is given by the equation

$$e_2 = I_1 M \omega \cos \omega t \tag{1}$$

provided that no current due either to leakage or capacity flows between the two coils or between the turns of either coil. If such currents do flow, then the secondary electromotive force e_2, measured at the terminals of the secondary with the secondary circuit open, may be represented by the equation

$$e_2 = I_1 M_e \omega \cos \omega t + I_1 \sigma \sin \omega t \tag{2}$$

where M_e is the effective mutual inductance and σ a small factor having the dimensions of a resistance.

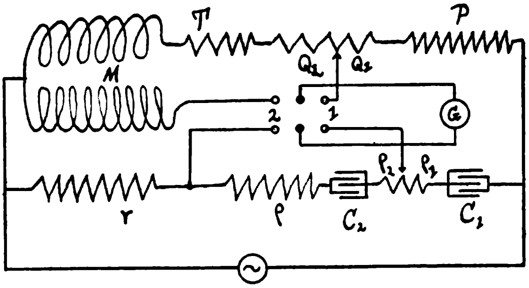

Fig. 1.

The open-circuit values of σ and M_e may be measured by means of the set-up of Fig. 1. The detecting instrument G is a vibration galvanometer or telephone, depending upon the frequency of the alternating current used. The inductances of the resistances r and P should be small and known. The resistances Q_1 and Q_2 are so designed that in varying Q_1, the sum $Q_1 + Q_2$ is always the same. This same condition also applies to ρ_1 and ρ_2; viz., $\rho_1 + \rho_2 =$ a constant.

With the galvanometer switch in position 2, T and ρ are alternately adjusted until no current flows through the galvanometer. The balance can only be obtained provided the self inductance in T is greater than M, so that it may be necessary to add inductance to this arm. The switch is then changed to position 1, and Q_1 and ρ_1 adjusted for zero current in the galvanometer circuit. Then, neglecting small correction terms,

$$M_e = C_1 r (P + Q_1)$$

and

$$\sigma = \omega^2 C_1 \rho_1 M_e.$$

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21, 1916.

If σ is very small, it may be impossible to make ρ_1 sufficiently small to obtain a balance. In such a case, a small inductance l is introduced in P. Then $\sigma = \omega^2 C_1(M_e\rho_1 - rl)$ and M_e is unchanged.

One of the advantages of the method is that the resistances r, P and Q_1 upon which the value of M_e depend can be so constructed that the change of resistance with frequency and with temperature is a minimum. This in a measure compensates for the disadvantage of having to make four independent adjustments.

Bureau of Standards,
Washington, D. C.

The Coefficient of Total Radiation of a Uniformly Heated Enclosure.[1]

By W. W. Coblentz and W. B. Emerson.

A recent determination of the coefficient, or so called Stefan-Boltzmann constant, of total radiation of a black body gave a value of $\sigma = 5.72 \times 10^{-12}$ watt cm.$^{-2}$ deg.$^{-4}$. The radiometer used in measuring the radiant energy in absolute value consisted of a bismuth-silver thermopile at a short distance in front of which was placed a thin strip of metal of manganin or platinum, from 0.008 to 0.012 mm. in thickness, from 2 to 8 mm. in width and 30 mm. in length. Potential terminals of very thin platinum wire were attached at a distance of 2 to 3 mm. from the ends. This thin strip of metal, which was blackened electrolytically with platinum black, or painted with lamp black and then smoked, functioned (1) as a receiver for absorbing radiant energy, (2) as a source of radiation which can be produced by heating the strip electrically and (3) as a standard of radiation to test the galvanometer sensitivity, by heating the strip electrically by a standard current.

The results obtained with various receivers have a range of 2 to 3 per cent.; but for any one receiver the variations in the measurements are considerably less than 1 per cent. It was found that the presence of atmospheric humidity had a marked effect upon the value of the radiation constant. The general performance of the radiometer is sufficiently satisfactory to warrant the attempt to operate the whole radiometric outfit in an evacuated enclosure, thus eliminating absorption by carbon dioxide and water vapor. Another difficulty (in addition to atmospheric absorption) is the temperature scale. Much time has been spent in attempting to obtain an accurate agreement in the measurements of the temperature of a black body as determined by means of thermocouples and an optical pyrometer, both of which were calibrated at the melting points of metals. In view of this difficulty, the coefficient of total radiation is to be determined by measuring the radiation from suitably constructed black bodies heated to the melting points of metals. At the rear of the radiator is a thermopile (with a circular receiver) which is used in maintaining a constant temperature radiometrically instead of thermometrically, with thermocouples, as has, heretofore been the custom. Equipped with this outfit, as soon as a vacuum spectrometer can be provided the facilities will be at hand to redetermine the constant (C_2) of spectral radiation.

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[1] Abstract of a paper presented at the Columbus meeting of the Physical Society, December 27, 1915.

The radiometer is a *null* instrument which evaluates radiation energy in absolute measure. This enables one to determine various subsidiary physical constants. Auxiliary apparatus has therefore been constructed for operating the device as a physical photometer by means of which it will be possible to determine the luminosity, and the radiant luminous efficiency of a black body at various temperatures; also the mechanical equivalent of light—all of which constants have been discussed recently by Pirani and others on the basis of the pioneering work primarily of Lummer and Pringsheim done some twenty years ago.

All the subsidiary measurements are in terms of the radiation of a black body at a known temperature so that the exact value of the coefficient, σ, is of secondary importance. The forthcoming paper contains also a summary of the most reliable determinations of this constant as obtained by various observers. It is shown that all the measurements give a value of the constant of total radiation which lies close to $\sigma = 5.7 \times 10^{-12}$ watt cm.$^{-2}$ deg.$^{-4}$

BUREAU OF STANDARDS,
WASHINGTON, D. C.

THE CONSTANTS OF RADIATION OF A BLACK BODY.[1]

BY W. W. COBLENTZ.

THE present communication gives the results of a recomputation of the constants of spectral radiation of a black body, which data were presented at previous meetings of this Society. This recomputation was necessitated as the result of the adoption of a new and apparently more reliable calibration curve of the fluorite prism used in the work, and as a result of the discovery of a small error which was found in the previous computations. The results of the present computations give a value of $C = 14{,}369$ and $A = 2{,}894$.

The data of other investigations are summarized and it is found that they lie close to $C = 14{,}350$.

At the Columbus meeting the value of the coefficient of total radiation was reported to be $\sigma = 5.72 \times 10^{-12}$ watt cm.$^{-2}$ deg.$^{-4}$ Hence, if theoretical considerations are to be relied upon, this value of the coefficient of total radiation indicates that the coefficient of spectral radiation is of the order of $C_1 = 14{,}322$ as compared with the experimentally determined value of $C = 14{,}369$. This is a difference of only 0.3 per cent. The small outstanding discrepancy is probably due to uncertainties in the calibration curve, and in the determination of the value of σ.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21, 1916.

From a consideration of the data now available it appears that the values of the constants of spectral radiation are close to

$$C = 14{,}350 \text{ micron deg.,}$$
$$A = 2{,}890 \text{ micron deg.,}$$

and that the coefficient of total radiation is of the order of $\sigma = 5.7 \times 10^{-12}$ watt cm.$^{-2}$ deg.$^{-4}$ This indicates that the constant h, of the quantum theory is of the order $h = 6.56$ to 6.57×10^{-27} erg sec., which is Millikan's latest value.

Washington, D. C.
March 24, 1918.

ERRATA AND ADDENDA.

Vol. V., March, 1915, p. 238, article by D. L. Webster, entitled "The Intensities of X-Ray Spectra"; the author adds the following correction:

In the REVIEW for March, 1915, I published an article showing that if the length of a wave train of X-rays is small compared to the distance in which a crystal is uniform enough to give good interference, one might expect the intensities at corresponding points in X-ray spectra to diminish with increasing order somewhat faster than the inverse square. More recently, however, a more detailed examination of the question, by Fourier's series as well as by the method of that paper, has shown that under these circumstances the widths of the reflections should appear proportional to the order. Since, instead of that, the widths appear constant, this hypothesis cannot be correct, and the wave trains must be distinctly longer than the uniform stretches of crystal structure. Undoubtedly the correct explanation of the intensities must be that of Bragg's Bakerian Lecture of 1915, based on the distribution of reflecting electrons in the atom.

Index to Volume VII., Series II.

A.

Absorption, The Effect of Temperature upon the Coefficient of, of Certain Glasses of Known Composition, *K. S. Gibson*, 194.

Absorption Spectra, The, of the Blue Solutions of Sodium and Magnesium in Liquid Ammonia, *G. E. Gibson and W. L. Argo*, 33.

Absorption, An Investigation of the Transmission, Reflection and, of Sound by Different Materials, *F. R. Watson*, 125

Acoustical Properties, An Investigation of the, of the Armory at the University of Illinois, *F. R. Watson*, 400.

Allen, S. J. M., The Radioactive Deposit from the Atmosphere of an Uncharged Wire, 133.

Allison, H. B. C., Note on the Action of Calcium in a Discharge Tube, 688.

Allotropy of Copper, On a Supposed, *G. K. Burgess and I. N. Kelberg*, 281.

Alpha-Rays, A Chemically Active Modification of Hydrogen Produced by, *William Duane and G. L. Wendt*, 689.

Alternating Current Method, An, of Measuring Mutual Inductance, *Harvey L. Curtis*, 692.

American Physical Society:
  Abstracts, 141, 281, 397, 497, 583, 682.
  Minutes, 139, 278, 676, 677.

Argo, W. L., The Absorption Spectra of the Blue Solutions of Sodium and Magnesium in Liquid Ammonia, 33.

Atomic Models in Whose Behavior a Magnetic Field is an Important Factor, *Ernest Merritt*, 285.

B.

Baker, Will C., A Method for the Measurement of Small Capacities, 112.

Barker, E. F., Selective Radiation from Osmium Filaments, 451.

Barnett, S. J., On Ewell's Method of Measuring Single Piotental Differences, 321

Barus, Carl, The Distance between two Parallel Transparent Plates, 79.
  The Interferences of the Direct Spectra of Two Gratings, 587.

Bauer, L. A., On Possible Planetary Magnetic Effects, 500.

Bismuth, Tests of Thermoelectric Formulæ Based on, and Bismuth-Tin Alloys, *A. E. Caswell*, 269.

Book Reviews, 159, 416.

Bridgman, P. W., The Velocity of Polymorphic Changes under Pressure, 150.
  On the Effect of General Mechanical Stress on the Temperature of Transition of Two Phases, with a Discussion of Plasticity, 215.

Brown, S. LeRoy, Measurements of Electric Currents by their Heating Effect, 398.

Burgess, G. K., On A Supposed Allotropy of Copper, 281.

C.

Calcium, Note on the Action of, in a Discharge Tube, *H. B. C. Allison*, 688.

Calorimeter Design, Convection Currents and, *Walter P. White*, 682.

Capacities, A Method for the Measurement of Small, *Will C. Baker*, 112.

Caswell, A. E., Tests of Thermoelectric Formulæ Based on Bismuth and Bismuth-Tin Alloys, 269.

Cathode Measurements with Glow Current in Hydrogen, *W. L. Cheney*, 241.

Cathode Measurements with Glow Current in Oxygen and Nitrogen, *W. Neuswanger*, 253.

Cheney, W. L., Cathode Measurements with Glow Current in Hydrogen, 241.

Clark, Harry, Sub-Helmholtzian Vibrations of a Rubbed String, 561.

Coblentz, W. W., The Coefficient of Total Radiation of a Uniformly Heated Enclosure, 693.
  The Constants of Radiation of a Black Body, 694.

Cold Cathode, Retrograde Rays from the, *Orrin H. Smith*, 625.

Compton, Arthur H., A Physical Study of the Thermal Conductivity of Solids, 341.

  On the Location of the Thermal Energy of Solids, 349.

  The X-Ray Spectrum of Tungsten, 498.

  A Recording X-Ray Spectrometer, and the High Frequency Spectrum of Tungsten, 646.

Compton, K. T., The Temperature Coefficient of Contact Potential, 209.

  The Theory of Ionization by Collision, I. Distribution of the Velocity of the Electrons, 489.

  The Theory of Ionization by Collision, II. Case of Inelastic Impact, 501.

  The Theory of Ionization by Collision, III. Case of Elastic Impact, 509.

Condensers, Note on the End Effect in the Electrostriction of Cylindrical, *Edwin C. Kemble*, 499.

Condensers, A Resonance Method for Measuring the Phase Difference of, *H. L. Dodge*, 584.

Condensers, Note on the End Effect in the Electrostriction of Cylindrical, *Edwin C. Kemble*, 614.

Conductivity, A Physical Study of the Thermal, of Solids, *Arthur H. Compton*, 341.

Conductivity of Flames, The Effect of a Magnetic Field on the, *C. W. Heaps*, 663.

Contact Potential, The Temperature Coefficient of, *K. T. Compton*, 209.

Convection Currents and Calorimeter Design, *Walter P. White*, 682.

Corona, A Theory of the Initial Conditions of the, *Jakob Kunz*, 285.

Copper Oxide, The Light Sensitiveness of, *A. H. Pfund*, 289.

Current, Distribution of, between a Gas Column and a Metallic Shunt, *R. W. Earhart*, 447.

Current, A New Law Relating Ionization Pressure and, in the Corona of Constant Potentials, *Earle H. Warner*, 399.

Current-Deflection Method for Determining Ballistic Constants of Moving Coil Galvanometers, with a Note on the Non-Uniformity of Magnetic Fields in such Instruments, *Paul E. Klopsteg*, 633.

Curtis, Harvey L., An Alternating Current Method of Measuring Mutual Inductance, 692.

Crystal Detectors, Some Characteristics of, *Victor A. Hunt and Laurens E. Whittemore*, 518.

Cylindrical Magnet, The E.M.F., Generated by the Rotation of a, about its Axis, *G. F. Hull and A. B. Meservey*, 685.

D.

Darrow, Karl K., On the Velocity of Sound and the Value of the Ratio of the Specific Heats for Hydrogen, 413.

Dellinger, J. H., The Calculation of Planck's Constant C_2, 687.

Dieterich, E. O., The Relation between the Pressure Effect and the Light Effect in Selenium Crystals, 415.

Dieterich, Kathryn Johnstone, The Effect of Temperature on the Resistance, the Light-Sensitiveness, and the Rate of Recovery of Certain Crystals of Metallic Selenium, 551.

Diffraction Ring Pattern, The, in the Shadow of a Circular Object, *Mason E. Hufford*, 545.

Dodd, L. E., The Distribution of Electrons in a Substance that can Exist in Both the Conducting and the Non-Conducting States, 282.

Dodge, H. L., A Resonance Method for Measuring the Phase Difference of Condensers, 584.

Doubt, Thomas E., Talbot's Bands and the Resolving Power of Spectroscopes, 283.

Droplet, Comparison of the Fall of a, in a Liquid and in a Gas, *O. W. Silvey*, 87.

Droplets, The Fall of Mercury, in a Viscous Medium, *O. W. Silvey*, 106.

Duane, William, Planck's Radiation Formula Deduced from Hypotheses Suggested by X-Ray Phenomena, 143.

  A Chemically Active Modification of Hydrogen Produced by Alpha-Rays, 689.

Dulong, The Exceptions to the Law of, and Petit, *J. E. Siebel*, 288.

E.

Earhart, R. W., Distribution of Current between a Gas Column and a Metallic Shunt, 447.

Einstein's Photoelectric Equation and Contact Electromotive Force, *R. A. Millikan*, 18.

Electric Currents, Measurements of, by their Heating Effect, *S. LeRoy Brown*, 398.

Electrodes, Arcs in Gases between Non-Vaporizing, *G. M. J. McKay and C. F. Ferguson*, 410.

Electrons, Reflection of Slow Moving, by Copper, *Albert W. Hull*, 1.

Electrons, The Distribution of, in a Substance that can Exist in Both the Conducting and the non-conducting States, *L. E. Dodd*, 282.

Electrons, Distribution of the Velocity of the, Theory of Ionization by Collision, *K. T. Compton*, 489.

Electrostriction, Note on the End Effect in the, of Cylindrical Condensers, *Edwin C. Kemble*, 614.

Emerson, W. B., The Coefficient of Total Radiation of a Uniformly Heated Enclosure, 693.

Energy, Equipartition of, and Radiation Theory, *W. F. Swann*, 154.

Energy, On the Location of the Thermal, of Solids, *Arthur H. Compton*, 349.

Enlarging Photographs, A New Method for, without the Use of a Lens, *Alfred J. Lotka*, 660.

Errata and Addenda, 696.

Ewell's Method of Measuring Single Potential Differences, *S. J. Barnett*, 321.

F.

Fall of a Droplet, Comparison of the, in a Liquid and in a Gas, *O. W. Silvey*, 87.

Ferguson, C. F., Arcs in Gases between Non-Vaporizing Electrodes, 410.

Flicker Photometer, The Theory of the, II. Unsymmetrical Conditions, 149.

Foote, Paul D., A New Relation Derived from Planck's Law, 224.

Forman, A. H., The Effect of Magnetization on the Opacity of Iron to Rontgen Rays, 119.

G.

Galvanometers, Note on the Damping Factor used in Ballistic Constants of Moving-Coil, *Paul E. Klopsteg*, 543.

Gas Molecules, Demonstration of a Model Visibly Representing, *E. F. Northrup*, 148.

Gibson, G. E., The Absorption Spectra of the Blue Solutions of Sodium and Magnesium in Liquid Ammonia, 33.

Gibson, K. S., The Effect of Temperature upon the Coefficient of Absorption of Certain Glasses of Known Composition, 194.

Gorton, A. F., Reflection from, and Transmission through Rough Surfaces, 66.

Gorton, W. S., The X-Ray Spectrum of Tungsten, 203.
  A Mica X-Ray Spectrometer, 334.

Gray, A. W., Some Effects of Magnetization on the Length of a 35.25 Per Cent. Nickel Steel, 684.
  American Nickel Steels of Low Thermal Expansivity, 685.

Gray, Frank, The Optical Activity of Liquids and Gases, 472.

H.

"*h*," the Value of, Determined Photoelectrically from the Ordinary Metals, *W. H. Kadesch and A. E. Hennings*, 147.

"*h*," A Direct Photoelectric Determination of Planck's, *R. A. Millikan*, 355.

Hall Effect, The, and Allied Phenomena in Tellurium, *Peter I. Wold*, 169.

Hartman, L. W., The Heat Losses from Incandescent Filaments in Air, 431.

Heaps, C. W., The Effect of a Magnetic Field on the Conductivity of Flames, 663.

Heat Conductivity of Tungsten at High Temperatures and the Wiedemann-Franz-Lorenz Relation, *Irving Lanmuir*, 151.

Heat Losses, The, from Incandescent Filaments in Air, *L. W. Hartman*, 431.

Hennings, A. E., The Relations of the Photo Potentials Assumed by Different Metals when Stimulated by Light of a Given Frequency, 147.

Hennings, A. E., The Value of "*h*" Deterined Photoelectrically from the Ordinary Metals, 147.

Howes, E. L., The Fluorescing Sodium Uranyl Phosphate, 394.
  A Synchrono-phosphoroscope, 516.

Hufford, Mason E., The Diffraction Ring Pattern in the Shadow of a Circular Object, 545.

Hull, Albert W., Reflection of Slow Moving Electrons by Copper, 1.
  Negative Resistance, 141.
  The Maximum Frequency of X-Rays at Constant Voltages between 30,000 and 100,000, 156.
  A Powerful Source of Constant High Potential, 405.

Hull, G. F., The E.M.F., Generated by the Potation of a Cylindrical Magnet about its Axis, 685.

Hunt, Victor A., Some Characteristics of Crystal Detectors, 518.

Hydrogen, Cathode Measurements with Glow Current in, *W. L. Cheney*, 241.

Hydrogen, On the Velocity of Sound and the Value of the Ratio of the Specific Heats of, *Karl K. Darrow*, 413.

Hydrogen, A Chemically Active Modification of, Produced by Alpha-Rays, *William Duane and G. L. Wendt*, 689.

I.

Impact, Case of Inelastic, The Theory of Ionization by Collision, II., *K. T. Compton*, 501.

Impact, Case of Elastic, The Theory of Ionization by Collision, III., *K. T. Compton*, 509.

Incandescent Filaments, The Heat Losses from, in Air, *L. W. Hartman*, 431.

Induction Unipolar, and Absolute Rotation, *E. H. Kennard*, 399.

Ingersoll, L. R., The Measurement of Optical Rotation in the Infra-red Spectrum, 284.

Ionization Pressure, A New Law Relating, and Current in the Corona of Constant Potentials, *Earle H. Warner*, 399.

Ionization, The Theory of, by Collision. I., Distribution of the Velocity of the Electrons, *K. T. Compton*, 489.

Ionization, The Theory of, by Collision. Case of Inelastic Impact, II., *K. T. Compton*, 501.

Ionization, The Theory of, by Collision. Case of Elastic Impact, III., *K. T. Compton*, 509.

Ionization Potentials, The Low Potential Discharge Spectrum of Mercury Vapor in Relation to, *John T. Tate*, 686.

Ions, The Free, in the Atmosphere of the Tropics, *J. R. Wright and O. F. Smith*, 49.

Ives, Herbert E., The Theory of the Flicker Photometer, II., Unsymmetrical Conditions, 149.

Ives, James E., On the So-called Magnetic Rays of Righi, 407.

K.

Kadesch, W. H., The Relations of the Photo Potentials Assumed by Different Metals when Stimulated by Light of a Given Frequency, 147.
  The Value of "*h*" Determined Photoelectrically from the Ordinary Metals, 147.

Kelberg, I. N., On A Supposed Allotropy of Copper, 281.

Kemble, Edwin C., Note on the End Effect in the Electrostriction of Cylindrical Condensers, 499.
  Note on the End Effect in the Electrostriction of Cylindrical Condensers, 614.

Kennard, E. H., Unipolar Induction and Absolute Rotation, 399.

Kingsbury, E. F., The Theory of the Flicker Photometer, II., Unsymmetrical Conditions, 149.
  A Spectral Luminosity Curve Equation and its Use, 161.

Klopsteg, Paul E., Note on the Damping Factor used in Ballistic Constants of Moving-Coil Galvanometers, 543.
  On the "Current-Deflection" Method for Determining Ballistic Constants of Moving Coil Galvanometers, with a Note on the Non-Uniformity of Magnetic Fields in such Instruments, 633.
  The Correction for Thermoelectric Current to be Applied to the Throw of a Ballistic Moving-Coil Galvanometer, 640.

Kunz, Jakob, On the Construction of Sensitive Photoelectric Cells, 62.
Sensitive Photoelectric Cells and a Photoelectric Relay, 282.
A Theory of the Initial Conditions of the Corona, 285.
On the Properties of Matter at Low Temperatures, 399.

L.

Langmuir, Irving, Heat Conductivity of Tungsten at High Temperatures and the Wiedemann-Franz-Lorenz Relation, 151.
Radiation from Tungsten Filaments and the Mechanical Equivalent of Light, 152.
The Characteristics of Tungsten Filaments as Functions of Temperature, 302.
Low Potential Discharge Spectrum, The, of Mercury Vapor in Relation to Ionization Potentials, *John T. Tate*, 686.
Low Temperatures, On the Properties of Matter at, *Jakob Kunz*, 399.
Lotka, Alfred J., A New Method for Enlarging Photographs without the Use of a Lens, 660.

M.

McKay, G. M. J., Arcs in Gases between Non-Vaporizing Electrodes, 410.
Magnetic Effects, On Possible Planetary, *L. A. Bauer*, 500.
Magnetic Rays, On the So-called, of Righi, *James E. Ives*, 407.
Magnetization, The Effect of, on the Opacity of Iron to Röntgen Rays, *A. H. Forman*, 119.
Magnetization, Some Effects of, on the Length of a 35.25 Per Cent. Nickel Steel, *A. W. Gray, D. H. Sweet and L. W. Schad*, 684.
Matter, On the Properties of, at Low Temperatures, *Jakob Kunz*, 399.
Mechanical Equivalent, Radiation from Tungsten Filaments and the, of Light, *Irving Langmuir*, 152.
Mercury Vapor Pump, A New, *H. B. Williams*, 583.
Merritt, Ernest, Atomic Models in Whose Behavior a Magnetic Field is an Important Factor, 285.
Meservey, A. B., The E.M.F., Generated by the Rotation of a Cylindrical Magnet About its Axis, 685.
Millikan, R. A., Einstein's Photoelectric Equation and Contact Electromotive Force, 18.
A Direct Photoelectric Determination ot Planck's "*h*," 355.
Moving Coil Galvanometers, On the "Current-Deflection" Method for Determining Ballistic Constants of, with a Note on the Non-Unifoımity of Magnetic Fields in such Instruments, *Paul E. Klopsteg*, 633.
Moving-Coil Galvanometer, The Correction for Thermoelectric Current to be Applied to the Throw of a Ballistic, *Paul E. Klopsteg*, 640.
Mutual Inductance, An Alternating Current Method of Measuring, *Harvey L. Curtis*, 692.

N.

Nathanson, J. B., The Reflecting Power of Alkali Metals in Contact with Glass as Determined by the Photo-electric Cell, 403.
Neuswanger, W., Cathode Measurements with Glow Current in Oxygen and Nitrogen, 253.
New Books, 159, 416.
Nichols, Edward L., A Synchrono-phosphoroscope, 516.
Nickel Steel, Some Effects of Magnetization on the Length of a 35.25 Per Cent. Nickel Steel, *A. W. Gray, D. H. Sweet, and L. W. Schad*, 684.
Nickel Steels, American, of Low Thermal Expansivity, *A. W. Gray, D. H. Sweet, and L. W. Schad*, 685.
Nitrogen, Cathode Measurements with Glow Current in Oxygen and, *W. Neuswanger*, 253.
Northrup, E. F., Demonstration of a Model Visibly Representing Gas Molecules, 148.

O.

Optical Activity, The, of Liquids and Gases, *Frank Gray*, 472.
Optical Properties, On the, of an Isolated Crystal of Selenium, *Charles H. Skinner*, 285.

Optical Rotation, The Measurement of, in the Infra-red Spectrum, *L. R. Ingersoll*, 284.

Osmium Filaments, Selective Radiation from, *E. F. Barker*, 451.

Oxygen, Cathode Measurements with Glow Current in, and Nitrogen, *W. Neuswanger*, 253.

P.

Page, Leigh, The Distribution of Energy in the Normal Radiation Spectrum, 229.

Petit, The Exceptions to the Law of Dulong and, *J. E. Siebel*, 288.

Pfund, A. H., The Light Sensitiveness of Copper Oxide, 289.

Phase Difference, A Resonance Method for Measuring the, of Condensers, *H. L. Dodge*, 584.

Phosphoroscope, A Synchrono, *Edward L. Nichols and H. L. Howes*, 516.

Photoelectric Cells, On the Construction of Sensitive, *Jakob Kunz and Joel Stebbins*, 62.

Photoelectric Cells, Sensitive, and a Photoelectric Relay, *Jakob Kunz and J. Stebbins*, 282.

Photoelectric Equation, Einstein's, and Contact Electromotive Force, *R. A. Millikan*, 18.

Photo Potentials, The Relations of the, Assumed by Different Metals when Stimulated by Light of a Given Frequency, *A. E. Hennings and W. H. Kadesch*, 147.

Photometry, Some Devices used in Evaluating Certain Trigonometric and Exponential Functions Occurring in, *Irwin G. Priest*, 414.

Potential, A Powerful Source of Constant High, *Albert W. Hull*, 405.

Potential Differences, On Ewell's Method of Measuring Single, *S. J. Barnett*, 321.

Planck's Constant C_2, the Calculation of, *J. H. Dellinger*, 687.

Planck's Law, A New Relation Derived from *Paul D. Foote*, 224.

Planck's Radiation Formula Deduced from Hypotheses Suggested by X-Ray Phenomena, 143.

Plasticity, On the Effect of General Mechanical Stress on the Temperature of Transition of Two Phases, with a Discussion of, *P. W. Bridgman*, 215.

Polymorphic Changes, The Velocity of. under Pressure, *P. W. Bridgman* 150.

Priest, Irwin G., Some Devices used in Evaluating Certain Trigonometric and Exponential Functions Occurring in Photometry, 414.

R.

Radiation, The Coefficient of Total, of a Uniformly Heated Enclosure, *W. W. Coblentz and W. B. Emerson*, 693.

Radiation, The Constants of, of a Black Body, *W. W. Coblentz*, 694.

Radiation from Tungsten Filaments and the Mechanical Equivalent of Light, *Irving Langmuir*, 152.

Radiation, Selective, from Osmium Filaments, *E. F. Barker*, 451.

Radiation Spectrum, The Distribution of Energy in the Normal, *Leigh Page*, 229.

Radiation Theory, Equipartition of Energy and, *W. F. Swann*, 154.

Radioactivity, The Constants of, *Gerald L. Wendt*, 389.

Radioactive Deposit, The, from the Atmosphere of an Uncharged Wire, *S. J. M. Allen*, 133.

Ramsey, R. R., The Cause of the Variation of the Emanation Content of Spring Water, 284.

Reflection from, and Transmission through Rough Surfaces, *A. F. Gorton*, 66.

Reflection, An Investigation of the Transmission, and Absorption of Sound by Different Materials, *F. R. Watson*, 125.

Reflection, The, of Sound, *F. R. Watson*, 287.

Reflecting Power, The, of Alkali Metals in Contact with Glass, as Determined by the Photo-electric Cell, 403.

Resistance, Negative, *A. W. Hull*, 141.

Resolving Power, Talbot's Band and the, of Spectroscopes, *Thomas E. Doubt*, 283.

Resonance Method, A, for Measuring the Phase Difference of Condensers, *H. L. Dodge*, 584.

Retrograde Rays from the Cold Cathode, *Orrin H. Smith*, 625.

Righi, On the So-called Magnetic Rays of, *James E. Ives*, 407.

Ripple Wave Motion, A Study of, *F. R. Watson and W. A. Shewhart*, 226.

Röntgen Rays, A Note on the Spectrum of, *O. J. Zobel*, 580.

Roop, W. P., The Magnetic Susceptibility of Gases, 529.

Rotation, Absolute, Unipolar Induction and, *E. H. Kennard*, 399.

Rotation, The E.M.F., Generated by the, of a Cylindrical Magnet about its Axis, *G. F. Hull and A. B. Meservey*, 685.

S.

Schad, L. W., Some Effects of Magnetization on the Length of a 35.25 Per Cent. Nickel Steel, 684.

American Nickel Steels of Low Thermal Expansivity, 685.

Selenium, On the Optical Properties of an Isolated Crystal of, *Charles H. Skinner*, 285.

Selemum, An Attempt to Detect a Change in the Specific Heat of, with a Change in the Illumination and also with the Application of an Electric Field, *L. P. Sieg*, 397.

Selenium Crystals, The Relation between the Pressure Effect and the Light Effect in, *E. O. Dieterich*, 415.

Selenium, The Effect of Temperature on the Resistance, the Light-Sensitiveness, and the Rate of Recovery of Certain Crystals of Metallic, *Kathryn Johnstone Dieterich*, 551.

Shewhart, W. A., A Study of Ripple Wave Motion, 226.

Siebel, J. E., The Exceptions to the Law of Dulong and Petit, 288.

Sieg, L. P., An Attempt to Detect a Change in the Specific Heat of Selenium with a Change in the Illumination, and also with the Application of an Electric Field, 397.

Silvey, O. W., Comparison of the Fall of a Droplet in a Liquid and in a Gas, 87.

The Fall of Mercury Droplets in a Viscous Medium, 106.

Skinner, Charles H., On the Optical Properties of an Isolated Crystal of Selenium, 285.

Smith, O. F., The Free Ions in the Atmosphere of the Tropics, 49.

Smith, Orrin H., Retrograde Rays from the Cold Cathode, 625.

Sound, An Investigation of the Transmission Reflection and Absorption of, by Different Materials, *F. R. Watson*, 125.

Sound Intensity, Certain Cases of Variation of, with Distance, *G. W. Stewart*, 442.

Specific Heat, An Attempt to Detect a Change in the, of Selenium with a Change in the Illumination, and also with the Application of an Electric Field, *L. P. Sieg*, 397.

Specific Volume, A Study of Apparent, in Solution, *L. D. Weld*, 421.

Spectra, The Interferences of the Direct, of Two Gratings, *Carl Barus*, 587.

Spectral Luminosity Curve Equation, A, and its Use, *E. F. Kingsbury*, 161.

Spectrometer, A Mica X-Ray, *W. S. Gorton*, 334.

Spectrum of Tungsten, A Recording X-Ray Spectrometer, and the High Frequency, *Arthur H. Compton*, 646.

Spring Water, The Cause of the Variation of the Emanation Content of, *R. R. Ramsey*, 284.

Stebbins, Joel, On the Construction of Sensitive Photoelectric Cells, 62.

Sensitive Photoelectric Cells and a Photoelectric Relay, 282.

Stewart, G. W., Certain Cases of the Variation of Sound Intensity with Distance, 442.

Stress, On the Effect of General Mechanical, on the Temperature of Transition of Two Phases, with a Discussion of Plasticity, *P. W. Bridgman*, 215.

Swann, W. F. G., Equipartition of Energy and Radiation Theory, 154.

Sweet, D. H., Some Effects of Magnetization on the Length of a 35.25 Per Cent. Nickel Steel, 684.

American Nickel Steels of Low Thermal Expansivity, 685.

Susceptibility of Gases, The Magnetic, *W. P. Roop*, 529.

Synchrono-phosphoroscope, A, *Edward L. Nichols and H. L. Howes*, 516.

T.

Talbot's Bands and the Resolving Power of Spectroscopes, *Thomas E. Doubt*, 283.

Tate, John T., The Low Potential Discharge Spectrum of Mercury Vapor in Relation to Ionization Potentials, 686.

Tellurium, The Hall Effect and Allied Phenomena in, *Peter I. Wold*, 169.

Thermoelectric Current, The Correction for, to be Applied to the Throw of a Ballistic Moving-Coil Galvanometer, *Paul E. Klopsteg*, 640.

Thermo-electric Formulæ, Tests of, Based on Bismuth and Bismuth-Tin-Alloys, *A. E. Caswell*, 269.

Thermal Expansivity, American Nickel Steels of Low, *A. W. Gray, D. H. Sweet and L. W. Schad*, 685.

Transmission, An Investigation of the, Reflection and Absorption of Sound by Different Materials, *F. R. Watson*, 125.

Transmission through, Reflection from and, Rough Surfaces, *A. F. Gorton*, 66.

Transparent Plates, The Distance between two Parallel, *C. Barus*, 79.

Trigonometric Functions, Some Devices used in Evaluating Certain, and Exponential, Occurring in Photometry, *Irwin G. Priest*, 414.

Tribo-Luminescence, The, of Manganese-Zinc Compounds, *C. W. Waggoner*, 402.

Tungsten, Heat Conductivity of, at High Temperatures and the Wiedemann-Franz-Lorenz Relation, *Irving Langmuir*, 151.

Tungsten, The X-Ray Spectrum of, *W. S. Gorton*, 203.

Tungsten Filaments, The Characteristics of, as Functions of Temperature, *Irving Langmuir*, 302.

Tungsten, The True Temperature Scale for, and its Emissive Powers at Incandescent Temperatures, *A. G. Worthing* 497.

Tungsten, The X-Ray Spectrum of, *Arthur H. Compton*, 498.

U.

Uranyl Phosphate, The Fluorescing Sodium, *H. L. Howes and D. T. Wilber*, 394.

V.

Velocity, On the, of Sound and the Value of the Ratio of the Specific Heats for Hydrogen, *Karl K. Darrow*, 413.

Vibrations, Sub-Helmholtzian, of a Rubbed String, *Harry Clark*, 561.

W.

Waggoner, C. W., The Tribo-Luminescence of Manganese-Zinc Compounds, 402.

Warner, Earle H., A New Law Relating Ionization Pressure and Current in the Corona of Constant Potentials, 399.

Watson, F. R., An Investigation of the Transmission, Reflection and Absorption of Sound by Different Materials, 125.

A Study of Ripple Wave Motion, 226.

The Reflection of Sound, 287.

An Investigation of the Acoustical Properties of the Armory at the University of Illinois, 400.

Webster, David L., The Emission Quanta of Characteristic X-Rays, 403.

Experiments on the Emission Quanta of Characteristic X-Rays, 599.

Weld, L. D., A Study of Apparent Specific Volume in Solution, 421.

Wendt, G. L., The Constants of Radioactivity, 389.

A Chemically Active Modification of Hydrogen Produced by Alpha-Rays, 689.

White, Walter P., Convection Currents and Calorimeter Design, 682.

Whittemore, Laurens E., Some Characteristics of Crystal Detectors, 518.

Wiedemann-Franz-Lorenz Relation, Heat Conductivity of Tungsten at High Temperatures and the, *Irving Langmuir*, 151.

Wilber, D. T., The Fluorescing Sodium Uranyl Phosphate, 394.

Williams, H. B., A New Mercury Vapor Pump, 583.

Wold, Peter I., The Hall Effect and Allied Phenomena in Tellurium, 169.

Worthing, A. G., The True Temperature Scale for Tungsten and its Emissive Powers at Incandescent Temperatures, 497.

Wright, J. R., The Free Ions in the Atmosphere of the Tropics, 49.

X.

X-Rays, The Emission Quanta of Characteristic, *David L. Webster*, 403.

X-Rays, Experiments on the Emission Quanta of Characteristic, *David L. Webster*, 599.

X-Rays, The Maximum Frequency of, at Constant Voltages between 30,000 and 100,000, *Albert W. Hull*, 156.

X-Ray Spectrum, The, of Tungsten, *W. S. Gorton*, 203.

X-Ray Spectrum, The, of Tungsten, *Arthur H. Compton*, 498.

X-Ray Spectrometer, A Recording, and the High Frequency Spectrum of Tungsten, *Arthur H. Compton*, 646.

Z.

Zobel, O. J., A Note on the Spectrum of Röntgen Rays, 580.

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